AN AUTOMATIC SURFACE ELEMENT GENERATOR FOR CALCULATING MEMBRANE AND BENDING STRESSES FROM THREE DIMENSIONAL FINITE ELEMENT RESULTS

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SUMMARY

The purpose of this paper is to present a method for calculating nominal membrane and bending stresses from a set of discrete point stresses within a three-dimensional finite element model. Most ASME stress limits govern stress categories such as membrane and bending. However, finite element programs provide total stresses valid only at discrete points. An automatic surface element generator enables membrane and bending (average and linearized) stresses to be calculated on any selected cross-sectional surface within a finite element model. An application of this method has produced accurate results for surfaces with complex geometry.

The calculation of membrane and bending stresses over a general three-dimensional surface is complicated by the irregular density of points at which finite element stresses are computed and by the irregular geometry of the selected surface. Membrane stress computation requires averaging the stress over the surface, which can be accomplished by summing the products of stress and area over the entire selected surface and dividing the result by the total surface area. Closed form integration is impossible due to the general boundaries and to the lack of defined stress functions at all surface locations. The effect of the irregular stress point density and surface geometry is that both the total surface area and the area to be multiplied by each stress value can only be calculated economically from a supplementary mesh for numerical area integration.

An automatic surface element generator has been developed to cover a selected surface with triangular elements. Element vertices are located at points of intersection of solid element edges with the selected surface. This element generator program is unique in that all input necessary for element coverage of an arbitrary surface within a three-dimensional grid can be calculated from the grid geometry and coefficients of the equations defining the selected surface. Algorithms developed for the generator have demonstrated accuracy and efficiency on a wide variety of geometries. Generated surfaces may include any number of holes and concentric or separated regions.

The stress state for each surface element is assumed to be the average of stress values at the vertices. Vertex stresses are computed from a least-squares fit of integration point stresses within adjacent solid elements of the grid model. Membrane stresses are calculated normal to the best fit planar approximation to the total surface, and membrane shears are calculated along and normal to an in-plane axis chosen by the analyst. Bending stresses are computed similarly to membrane stresses except that (a) the element stresses used are total stresses less the membrane stress, and (b) before summation over all elements, the products of stress and area for each element are multiplied by a moment arm from the element centroid to an arbitrary line parallel to the bending axis.

Accuracy is determined principally by the finite element solution accuracy. In refined grid examples for which exact stress values were supplied, solution inaccuracies were insignificant.

Three-dimensional finite element stresses processed by this method can now be compared directly to code limits on membrane and bending stresses. This represents an expansion of such capabilities from previous two-dimensional limitations.
Introduction

A method for automatic two-dimensional finite element stress classification for comparison with ASME stress limits was presented by W. C. Kroeneke [1]. As the use of three-dimensional finite element programs expands, the need for analogous three-dimensional classification capabilities is increasing. In addition to the total stress component values computed at points throughout the finite element mesh, certain analyses require calculation of the membrane and bending stresses over selected surfaces within the mesh.

The procedure presented here provides numerical computation of membrane and bending stresses over any surface represented by discrete points lying on the surface. The range of potential surface geometries permissible is indicated in Figures 1, 8, and 9, where each surface may be non-planar in a three-dimensional view. Stress component values at each point and boundary point connectivity indicators are the only additional data necessary. This data set can be provided manually for simple surfaces or by a finite element program post-processor for large or complex surfaces. A post-processor for this purpose includes the following operations:

1. Define the classification surface (usually a plane) by equation coefficients.
2. Locate intercepts of the surface and element edges within the mesh.
3. Identify intercept points located on mesh boundaries.
4. For each intercept point on the mesh boundary, find the two adjacent intercept points on the same boundary.
5. Calculate stress components at each intercept point by extrapolation or interpolation of adjacent element stresses.

Each intercept point is designated by an integer and has associated with it the three coordinates of the point and the six components of stress at the point. In addition, mesh boundary intercept points have associated with them the numbers of the two adjacent boundary intercept points.

The method by which membrane and bending stresses can be obtained from such a set of discrete point data is based on representation of the surface by numerically generated piecewise-continuous surface elements covering each connected set (see Figure 1 for connected set examples). The average stress state for each surface element is calculated and multiplied by the element area to obtain the forces on the element. All element forces in a specified direction are summed and divided by the total surface area to provide the membrane stress. Bending moments are calculated by summing element moments about reference axes described later.

Generation of elements on a three-dimensional surface using only intercept point data is the key operation required for the approach described. An automatic mesh generator developed by Frederick, Wong and Edge [2] requires additional input in the form of "ghost points" for boundary definition. This additional input has been eliminated to automate the process more fully. A surface element generation strategy, which has been successfully applied to a wide variety of complex surfaces, is described below.

TRANSFORMATION FROM THREE-DIMENSIONAL SURFACE TO A PLANE

Element generation in three dimensions requires more complex and time-consuming algorithms than are necessary on a planar surface. Since any surface useful for stress classification can be mapped onto a plane, element generation algorithms are required only for a plane. This mapping process is dependent on the nature of the surface selected. Most practical classification surfaces can be mapped onto a plane using a projection tech-
nique. The plane must be selected such that the image produced on the plane has a one-to-
on correspondence with the original surface.

If \( K, L, \) and \( M \) are the three points defining the image plane orientation, then the
vector product \( \vec{N} \) of vectors \( \vec{p} \) from \( K \) to \( L \) and \( \vec{q} \) from \( K \) to \( M \) is normal to the image plane

\[
\vec{N} = a\hat{x} + b\hat{y} + c\hat{z}
\]  

(1)

where \( \hat{x}, \hat{y}, \hat{z} \) are unit vectors along global axes so that

\[
a = p_{y}q_{z} - p_{z}q_{y}
\]

(2)

\[
b = p_{z}q_{x} - p_{x}q_{z}
\]

(3)

\[
c = p_{x}q_{y} - p_{y}q_{x}
\]

(4)

are the components of the normal vector. The equation of a plane through the origin and
perpendicular to \( \vec{N} \) is

\[ax + by + cz = 0\]

(5)

where coefficients \( a, b, \) and \( c \) are identical to the components of \( N \). The plane so defined
is used as the image plane.

The projection of a point with coordinates \((x, y, z)\) parallel to \( \vec{N} \) onto the image plane
must satisfy both Eq. (5) and the parametric equations of a line through \((x, y, z)\) in the
direction of \( \vec{N} \):

\[
x_{1} = x - at
\]

(6)

\[
y_{1} = y - bt
\]

(7)

\[
z_{1} = z - ct
\]

(8)

where \((x_{1}, y_{1}, z_{1})\) are the coordinates of the projected point. Simultaneous solution of
Eqs. (5) through (8) gives

\[
t = ax + by + cz
\]

(9)

which is substituted into Eqs. (6), (7), and (8) to obtain \((x_{1}, y_{1}, z_{1})\). Solution of
Eqs. (6) through (9) for every intersection point on the original surface completes the
mapping onto the image plane. Each intersection point then has two sets of coordinates
associated with it...the original surface coordinates and the image plane coordinates.

NUMERICAL SURFACE ELEMENT GENERATION

Numerous approaches to automated generation of elements covering an arbitrary surface
are possible using the set of intersection points and list of sequential adjacent boundary
points already described. The strategy presented here has been shown to introduce negli-
gible error in the membrane and bending stresses produced for a variety of examples and to
be rapid compared to most other techniques.

The strategy consists basically of two sets of algorithms. The first set provides
general guidelines for element formation. The second set includes restrictions which may
not be violated by proposed elements. If a restriction is violated, the offending element
is disallowed, and the formation guidelines are applied to define an alternate element.

ELEMENT GENERATION GUIDELINES

The choice of element type for the strategy presented here was based on simplicity,
accuracy, and low numerical solution cost. For simplicity, planar elements are used to
avoid surface fitting of intersection points. In particular, triangular elements are
selected, since only triangular planar elements fit a general distribution of points in
three dimensions. Accuracy is maintained if stress gradients between adjacent intersection
points are not severe.

A systematic element generation procedure is invaluable during manual grid review.
This strategy employs a wavefront progression technique such that element numbers increase
as a function of the number of elements that must be crossed to reach the starting point.
The starting point for element generation must be an exterior boundary point in order for certain algorithms employed in this method to be successful for all geometries. Since boundary points are not initially identified as being interior or exterior, the simplest method of selecting an exterior boundary point is to locate the furthest point from the origin on the image plane. This point must be on the exterior boundary if the surface mapping is valid.

Once a starting point is chosen, priority levels are assigned to all image points (hereafter "points" will replace "image points") to simplify the bookkeeping and promote wavefront progression of element formation. Four priority levels are established.

Level 0: contains all points completely surrounded by elements and exterior region (whereby the sum of interior adjacent element angles plus the included angle of exterior region about the point equals 2) so that the points are unavailable for use in further element formation.

Level 1: contains a single point which must be used in each new element (or set of elements, as will be explained) until it is completely surrounded by elements and exterior region, at which time it is moved to level 0 and a new point is assigned to level 1.

Level 2: contains all points (except those in levels 1 and 0) used as corners of any previously defined elements; points are maintained in the order in which they enter this level, and the first point in this level at the time of a vacancy in level 1 is assigned to level 1.

Level 3: contains all points available for element formation but not yet used to define any element corner.

Initially levels 0 and 2 are empty, the starting point is in level 1, and all other points are in level 3.

If elements are defined one at a time by three corner points, the probability is high that adjacent elements will vary considerably in area. To reduce this probability, sets of four mutually adjacent corner points are selected whenever possible to define two or three elements simultaneously. The majority of such sets of corner points allow a choice in the assignment of element corner points based on the areas of the potential elements. Details on the variety of possible cases resulting from this approach are presented later. The selection of the sets of four nodes is guided primarily by proximity considerations. Obviously, unattached points may not be imbedded in elements. Selections of immediately adjacent sets of corner points eliminates this possibility.

Each new set of corner points begins with the priority level 1 point, called K. Distances from K to all points in levels 2 and 3 are calculated and ranked from minimum to maximum in association with the point number. The closest point not encountering any restriction (presented below) against combination with K is labeled point L, the second member of the set. The sums of distances from both K and L to all other residents of levels 2 and 3 are computed and ranked from minimum to maximum. Point M is the closest point to K and L not violating a restriction in combination with either K or L. M is the third member of the corner point set. The fourth member, N, is the closest point (excluding M) to K and L, which in combination with any of K, L, or M, does not violate a restriction. If no satisfactory choice for N can be found, the corner point set is permitted to
have only three members.

For the set (K, L, M, N) two general orientations of the members with respect to each other are possible. The first orientation is such that one point lies within the area of a triangle formed by the other three points. For this case three elements are formed as illustrated in Fig. 3a. The second general orientation includes two subcases, each forming two elements. If any three of the points lie in a straight line, the intermediate point on the line and the fourth point must define the common side of the two elements (see Figure 3b) to prevent the formation of elements with zero area. Any other arrangement of the second general orientation allows a choice between two pairs of potential elements (see Figure 3c). The majority of all random distribution of four points on a plane fall in this category.

Recognition of the orientation case by a numerical program is accomplished using the following procedure. For each of the four possible single triangles (KLM, KLN, KMN, LNM), which can be formed from three or four points, the areas are calculated as the absolute vector products of vectors p and q along any two sides of each triangle

$$A = \left| \mathbf{p} \times \mathbf{q} \right| / 2 = \left( (p_y q_x - p_x q_y)^2 + (p_y q_y - p_y q_y)^2 + (p_y q_y - p_y q_y)^2 \right)^{1/2}$$

where the subscripts indicate vector components along global coordinate axes.

If a zero area is computed, the corner points of that triangle lie in a straight line. For each corner point of the "triangle" the sum of distances to the other two corner points is calculated. The corner point having the lowest distance sum must be the intermediate point, and is therefore paired with the point not on the straight line to form the common side of the two resulting elements.

If no zero areas are found above, point N must lie in one of six regions outside triangle KLM, since, due to the selection algorithm, L and M are closer than N to K. Figure 4 illustrates the six regions, arbitrarily labeled I through VI, formed by extending infinite lines along each side of triangle KLM. Shaded regions II, IV and VI involve orientations of the first type which define three elements, and regions I, III and V are orientations of the second type which produce two elements. Determination of the region occupied by N is accomplished using the following algorithm.

A convention is established whereby positive and negative normal directions to triangle KLM are defined. The positive direction is chosen to be in the direction of the vector product $\mathbf{T}_{KLM}$ of a vector from K to L crossed into a vector from K to M. Let $s_{KLM}$ be the sign of the first nonzero component of the normal vector $\mathbf{T}_{KLM}$. Three vector products involving point N are now calculated.

$$\mathbf{T}_{KLN} = \mathbf{T}_{KL} \times \mathbf{T}_{KN}$$

$$\mathbf{T}_{LMN} = \mathbf{T}_{LM} \times \mathbf{T}_{LN}$$

$$\mathbf{T}_{MKN} = \mathbf{T}_{MK} \times \mathbf{T}_{MN}$$

The signs $s_{KLM}$, $s_{LMN}$ and $s_{MKN}$ are those of the first nonzero components of $\mathbf{T}_{KLM}$, $\mathbf{T}_{LMN}$ and $\mathbf{T}_{MKN}$ respectively. The resulting signs of the products of $s_{KLM}$ with each of the other sign variables $s_{IJN}$, where I and J may be any two of the set (K, L, M) is positive if N is on the same side of the line through corner nodes I and J as triangle KLM, and negative if it is on the opposite side. Using Table 1, the region of N is found. The corner node sets defining the three elements for regions II, IV or VI are listed in Table 2. The two pairs of potential corner node sets defining elements for regions I, III or V are listed in Table 3. For cases I, III or V, the areas of all potential elements are calculated. The
element pair having more nearly equal areas is used.

A total angle about the point occupied by elements and exterior region is maintained for each point during the element generation process. At the completion of each new set of elements these angles are updated, and any points with an angle of $2\pi$ radians filled are assigned to level 0.

Initially all angles on a connected set are zero. Whenever a boundary point is considered for use as an element corner point, the point’s angle is tested. If it is zero, the exterior angle of that point and all other points on the same boundary are calculated. Since the starting point on each connected set is on an exterior boundary, there is no ambiguity as to whether the boundary is interior or exterior. All other boundaries residing in the same connected set must be interior. Distinction between the exterior side and the image region side of the boundary is provided by totalling the angular changes in direction involved in encircling the perimeter along sequential boundary points. If positive angles are counter-clockwise on the image plane, a total angle of $+2\pi$ indicates counter-clockwise progression of sequential boundary points so that exterior region is to the right of the perimeter at all points. The opposite is true for a total angle of $-2\pi$.

The change of direction $\alpha_j$ at each point $J$ is found as follows:

1. Locate the previous adjacent boundary point, $I$, and subsequent adjacent boundary point, $K$, to $J$. Define vectors $P$ from $I$ to $J$ and $Q$ from $J$ to $K$.
2. Calculate the sign of the angle $\alpha_j$ through which $P$ must rotate to align with $Q$:
   \[ T = P \times Q \] (14)
   is normal to the image plane and parallel to $N$ calculated in eq. (1). The sign, $e$, of the scalar product of $T \cdot N$ is the sign of $\alpha_j$:
   \[ e = \frac{T \cdot N}{|T||N|} = \pm 1 \] (15)
3. Calculate the magnitude of $\alpha_j$:
   \[ |\alpha_j| = \cos^{-1} \left( \frac{T \cdot Q}{|P||Q|} \right) \] (16)
4. Then $\alpha_j = e|\alpha_j|$ (17)

Angle $\alpha_j$ is stored in association with point $J$. When $\alpha_j$ has been found for every point around the boundary, the sum $\Gamma$ is calculated as $\Gamma = \sum_{J=1}^{n} \alpha_j = 2\pi$ (18) where $n$ is the number of points on the loop. The sign of $\Gamma$, defined as $s$, is
   \[ s = \frac{\Gamma}{2\pi} = \pm 1 \] (19)

The exterior angle $\beta_j$ is calculated for each exterior boundary point from
   \[ \beta_j = s\pi + e|\alpha_j| \] (20)

as demonstrated by the four possible cases in Figure 5.

Each angle $\beta_j$ is stored in two separate arrays, one of which is maintained unchanged to determine in later algorithms where the exterior region lies with respect to a particular exterior boundary point. The other array is the one used to store the cumulative angle about each point occupied by the adjacent elements and exterior region. All entries in the latter array are stored as absolute values.

Any boundary points encountered during element generation on the connected set are on interior loop if they have zero cumulative angle in the latter array. When such a point is encountered and found to satisfy restrictions on element formation, all points on that boundary are processed analogously to the exterior boundary, except that, due to the exterior region now being located "inside" the boundary, the sign of $s$ in Eq. (20) is reversed.

At completion of the formation of each new element, each interior angle of the new
element is computed. For triangle KLM, the angle at corner point K is

$$\theta_K = \cos^{-1} \frac{\vec{p} \cdot \vec{q}}{|\vec{p}| |\vec{q}|}$$

(21)

where $\vec{p}$ is a vector from K to L and $\vec{q}$ is a vector from K to M. $\theta_K$ is added to the previous total occupied angle about K, and similar treatment is applied to angles at L and M.

At the completion of a connected set, levels 1 and 2 are empty. If points remain at level 3, a new connected set is begun by finding the level 3 point furthest from the origin of the image plane and assigning that point to level 1 as a new starting point. The generation of elements on the new connected set proceeds in the manner described above.

**ELEMENT GENERATION RESTRICTIONS**

The guidelines for element generation tend to consider all points in levels 2 and 3 as equally suitable candidates for new element formation providing only that the points are adjacent to point I at level 1. Frequently, however, consideration of the geometry and elements already formed about point I restricts the combination possibilities for other points in the region. Each potential corner point pairing or element corner point set is checked against the following list of restrictions before the pairing is permitted. Let points I and L be the pair of points being considered for use as corner points of a new element. Point I, as a level 1 point, is known to be in the connected set.

*Restriction 1*: Vector $\vec{r}$ from I to L may not cross any boundary or previously defined element side. An array of all pairs of adjacent points connected by boundary segments or element sides is maintained for this check. Initially this array contains all boundary segments, but element sides are added as they are defined. For a given pair of points J and K defining the ends of one of the "barriers", three regions are defined in which point L may be located, as indicated by $L_1$, $L_2$, $L_3$ in Figure 6. Referring to Figure 6, the following vectors and angles are defined:

$$\vec{p} = \text{vector from I to J} \ \ {\text{where K is closer}}$$

$$\vec{q} = \text{vector from I to K} \ \ {\text{than J to point I}}$$

$$\beta = \text{angle JIK}$$

$$\gamma = \text{angle JIL with positive sense in direction of} \ \ \vec{r}$$

$$\vec{r} = \frac{\vec{p} \times \vec{q}}{|\vec{p} \times \vec{q}|}$$

(22)

$$\hat{\beta}_{JKI} = \frac{\vec{p} \times \vec{r}}{|\vec{p} \times \vec{r}|}$$

(23)

$$\hat{\beta}_{JIL} = \frac{\vec{p} \times \vec{q}}{|\vec{p} \times \vec{q}|}$$

(24)

Region 1 includes all points not within angle $\beta$, and region 2 includes points within angle $\beta$, but between point I and the barrier. Region 3 includes all points within angle $\beta$, but on the opposite side of the barrier from I. Use of the pair I and L is prohibited if L is in region 3. L lies within region 3 if and only if both of the following criteria are met: a) $0 < \gamma < \beta$ (not met by region 1) b) $|\vec{r}| > d$ (not met by region 2)

where $d$ is the intercept distance from point I to the barrier in the direction of $\vec{r}$.

Angle $\beta$ is always chosen to be positive. If the sign of $\gamma$ by eq. (22) is negative, L is in region 1 and no further calculations are necessary for the barrier being considered. If $\gamma$ is positive, the magnitude is calculated as

$$\gamma = \cos^{-1} \frac{\vec{r} \cdot \vec{r}}{|\vec{r}|}$$

(25)
which is compared to the value of $\beta$,

$$\beta = \cos^{-1} \left( \frac{\mathbf{p} \cdot \mathbf{q}}{|\mathbf{p}| |\mathbf{q}|} \right)$$  \hspace{1cm} (26)

If $\gamma > \beta$, L is in region 1. Values of $\gamma \leq \beta$ require calculation of $d$:

$$d = \frac{1}{|\mathbf{p}|} \cos \gamma + \frac{\sin \gamma}{\sin \beta} \left( \frac{|\mathbf{q}|^2 - \cos \beta}{|\mathbf{q}|} \right)$$  \hspace{1cm} (27)

Pair IL is disallowed if $d < |\mathbf{p}|$

**Restriction 2:** Points I and L may not be connected if exterior region lies between them. This Restriction is necessary only if I and L are both boundary points, since Restriction 1 covers all other cases. Let J and K be the previous and subsequent adjacent boundary points respectively to point I. The following variables are defined as indicated in Figure 7:

- $\beta_I = \text{Angle JIK through exterior region}$
- $\gamma = \text{Angle JIL through exterior region}$

Angle $\beta_I$ is the exterior angle calculated and stored for point I in the Guidelines. Angle $\gamma$ is calculated in the same manner, except that point L replaces point K in the procedure. Point L is in one of two regions in Figure 9. For shaded region $2\alpha yz\beta$ within the connected set, pairing of I and L is permitted. Region $\alpha yz\beta$ is outside the connected set, and pairing of I and L is forbidden.

**Restriction 3:** A set of points $(I, J, L)$ is not permitted to define an element if the element area is zero.

**Restriction 4:** A line between adjacent boundary points I and L may not be common to more than one element. A list of all elements common to point I is assembled. If any of these elements are also common to point L, further pairing of I and L is prevented.

**Restriction 5:** Two elements with common nodes I and L may not lie on the same side of line IL. This Restriction prevents formation of overlapping elements. If a previously defined element, IKL, is common to line IL, a second element, IJL, is permitted only if $s$ is negative in the equation $s = \mathbf{v}_{I JL} \cdot \mathbf{v}_{I KL}$

**COMPUTATION OF MEMBRANE FORCES AND BENDING MOMENTS**

When all connected sets on the surface have been covered with elements, a set of local axes is defined. The local x-axis is oriented normal to the (average) plane of the original surface. This normal direction $\mathbf{n}_s$ is obtained by using original surface coordinates to calculate the vector product $\mathbf{v}_{KLM}$ of vectors along any two sides of an element and adding the $\mathbf{v}_{KLM}$ each multiplied by the sign of $\mathbf{n} \cdot \mathbf{v}_{KLM}$ from every element. Local y and z axes are selected on the average surface plane perpendicular to $\mathbf{n}_s$.

Stress components at every point on the surface are transformed to the local coordinate system. A set of stress components representing each surface element is calculated as the mean of corner point components. For example

$$\sigma_{x,KLM} = \left( \sigma_{x,K} + \sigma_{x,L} + \sigma_{x,M} \right) / 3$$  \hspace{1cm} (29)

$$\tau_{xy,KLM} = \left( \tau_{xy,K} + \tau_{xy,L} + \tau_{xy,M} \right) / 3$$  \hspace{1cm} (30)

$$\tau_{xz,KLM} = \left( \tau_{xz,K} + \tau_{xz,L} + \tau_{xz,M} \right) / 3$$  \hspace{1cm} (31)

represent the stresses of element KLM which contribute to normal and shear stresses acting on the average surface plane.

The forces produced by the stresses acting on an element are calculated as the product of the actual element area multiplied by the stress component associated with force direction being considered. For each force direction the sum of all element forces is calcula-
ted and divided by the total surface area to produce the membrane stress.

\[ \sigma_{\text{membrane}} = \frac{\Sigma_{\text{KLM}} \lambda_{\text{KLM}}}{\Sigma_{\text{KLM}}} \]  

(32)

Membrane stresses are subtracted from the associated element stress components. The total bending moments about the local axes are then

\[ M_y, \text{ bending} = \Sigma (\sigma_{x, \text{KLM}} - \sigma_{\text{membrane}}) \lambda_{\text{KLM}} R_{y, \text{KLM}} \]  

(33)

\[ M_z, \text{ bending} = \Sigma (\sigma_{x, \text{KLM}} - \sigma_{\text{membrane}}) \lambda_{\text{KLM}} R_{z, \text{KLM}} \]  

(34)

\[ M_x, \text{ bending} = \Sigma (\tau_{xy, \text{KLM}} - \tau_{\text{membrane}}) \lambda_{\text{KLM}} R_{z, \text{KLM}} \]  

(35)

where \( R \) is the moment arm from the element centroid to the subscript local axis. Division of the total bending moments by the surface section modulus produces the bending stresses about local axes.

After selecting the code categories which apply to the structural region and orientation of the surface, the stress classification is completed by comparing the membrane and bending stresses to the corresponding code limits.

SAMPLE RESULTS

Figure 8 is an example of surface element generation by Babcock & Wilcox Company program SURCL, incorporating the strategy and equations presented above. The complexity of surfaces which can be processed is demonstrated. Surfaces selected for stress classification on a segment of a circular cylinder are shown in Figure 9. Membrane and bending stress values having errors of less than 2% were calculated for a variety of stress distributions on each the tangential and longitudinal surfaces.

CONCLUSIONS

A method has been developed for calculating membrane and bending stresses within three-dimensional finite element stress models. Results are highly accurate, and can be used directly in the classification of stresses and comparison with code limits such as ASME Section III. In the form of a post-processing computer program, the method is economical and easy to use. Required input from the analyst consists of a basic description of surfaces over which the classifications are desired, and a pair of points defining the direction of a local axis on the surface. Any surface capable of being projected onto a plane can be selected for stress classification. The method satifies a vital need for a tool to permit application of code stress limits directly to results of three-dimensional finite element analyses.

ACKNOWLEDGEMENTS

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REFERENCES


### TABLE 1 Region of Point N

<table>
<thead>
<tr>
<th>Region</th>
<th>$S_{KLM}$</th>
<th>$S_{KLN}$</th>
<th>$S_{LMN}$</th>
<th>$S_{KMN}$</th>
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### TABLE 2 Element-Defining Corner Points

<table>
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<th>Element 2</th>
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### TABLE 3 Pairs of Element-Defining Corner Points

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<th>Region</th>
<th>Pair 1</th>
<th>Element 1</th>
<th>Element 2</th>
<th>Pair 2</th>
<th>Element 1</th>
<th>Element 2</th>
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<td></td>
<td>KLM</td>
<td>MNK</td>
<td></td>
<td>LMN</td>
<td>NKL</td>
</tr>
<tr>
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<td>LNM</td>
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</tr>
<tr>
<td>V</td>
<td></td>
<td>KLM</td>
<td>NKL</td>
<td></td>
<td>MNK</td>
<td>LNM</td>
</tr>
</tbody>
</table>
FIG. 1 Relation of Terminology to Example Surface

FIG. 2 Projection of Selected Surface onto a Plane
FIG. 3 General Orientation Cases
for Point Set (K, L, M, N)

FIG. 4 Convention for Regions Surrounding
Triangle KLM

FIG. 5 Boundary Point Exterior Angle Cases
FIG. 6 General Configuration of Barrier

FIG. 7 Angular Definition of Exterior Region
NUMBERING GIVES
THE ORDER IN WHICH
ELEMENTS ARE GENERATED

FIG. 8 Example Surface with Numerically Generated Elements

SLICE FROM CIRCULAR CYLINDER

LONGITUDINAL SURFACE

TANGENTIAL SURFACE

FIG. 9 Example Stress Classification Surfaces