A CURVED FINITE ELEMENT FOR GENERAL THIN SHELL STRUCTURES

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SUMMARY

This work describes the development of a curved quadrilateral shell finite element which demonstrates very good convergence properties. A general description is used in deriving the element so that it may be applied to any thin shell problem. The element is shown to be very efficient. It has a total of 36 degrees-of-freedom with 9 at each of the corners of the element.

There are several distinct advantages that the element offers for practical applications. Most of the shell elements that have been presented in the past are limited to problems in which the coordinates on the shell surface are orthogonal. The element that is described in the paper is derived using a general description so that it may be applied to any thin shell problem including those in which the shell coordinates are not orthogonal. The degrees-of-freedom at each of the four nodes are the three Cartesian displacements and their first derivatives with respect to the two surface coordinates. The imposition of boundary conditions is simplified since each of the degrees-of-freedom can be associated with a quantity which has a simple physical meaning. Many shell elements that have been developed in the past include in their degrees-of-freedom, second derivatives of the displacement functions and the correct boundary conditions for these elements are often difficult to determine. Further, the degrees-of-freedom for the element can be transformed easily since they include only first order derivatives. This is important for the correct imposition of boundary conditions along skewed boundaries. The stiffness matrix and the consistent loading vector for distributed loading are determined numerically using a minimum number of integration points which minimizes computing time and thus provides efficient solutions to problems.

During the course of the derivation of the element, the strain displacement relationships are derived in a very simple manner consistent with Love's first approximation for thin shells. In the past, the membrane part of the strain is generally accepted to be derived from the difference in the First Fundamental Form for the deformed and the initial shell surface. The bending strain comes from the differences in the Second Fundamental Form for the two surfaces but recently there has been some argument that the mixed components of this tensor should be used rather than the covariant components as is used in the generally accepted Koiter-Sanders shell theory. The derivation in the paper starts from basic principles and should help to shed some light on the proper form for the bending strain.

Two primary contributions are presented in this work. The first is the presentation of a procedure for the development of a general quadrilateral shell element. The second is the simple derivation of the bending strain for the thin shells which apparently has not been presented previously.