VISCOPLASTIC FINITE ELEMENT ANALYSIS BY UNCONDITIONALLY STABLE IMPLICIT METHODS

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SUMMARY

In this paper we present a new family of unconditionally stable implicit methods for quasi-static viscoplastic finite element analysis. The class of problems considered is governed by a constitutive equation of the form:

$$\dot{\varepsilon}_{ij} = c_{ijkl}(\varepsilon_{kl} - \varepsilon_{mp}^{sp}), \quad \varepsilon_{ij}^{sp} = \beta_{ij}(\sigma_{kl}),$$

where $\sigma$ is the Cauchy stress tensor, $c_{ijkl}$ are material constants, $\varepsilon$ is the infinitesimal strain tensor, and $\varepsilon^{sp}$ are the 'viscoplastic strain rates', given functions of the components of the stress tensor.

In several recent publications Zienkiewicz and Corneau have developed and studied an algorithm for quasi-static elasto/viscoplastic problems. The algorithm has been applied with success to a variety of engineering problems. Corneau has performed a valuable stability analysis and obtained the following results:

- The numerical solution procedure is equivalent to the step-by-step solution of a first-order system of ordinary differential equations. Hence, the stability of the procedure may be ascertained by applying techniques used to analyze discrete ordinary differential equation solvers.
- Time-step restrictions for a variety of constitutive models have been obtained for the Euler method (forward differences).

In addition, Corneau has alleged that several implicit methods suffer from the same time-step restriction as does the Euler method. He has thus concluded that implicit methods, offer no advantages than these circumstances over the simpler explicit method.

In the present paper we reconsider the application of implicit methods to quasi-static elasto/visco-plasticity. We propose a new one-parameter family of implicit algorithms. It is shown that, for appropriate values of the parameter, this family is unconditionally stable (i.e., there is no stability restriction on the size of the time step employed). Numerical calculations are described which support the theoretical stability analysis.

The time step restriction of the Zienkiewicz-Corneau algorithm is analogous to that for a forward difference solution of the heat equation, and thus is a stringent one in practice. For slowly varying loads, or when equilibrium response is of prime interest, stability requires that time steps be selected which are much smaller than those necessary for accuracy. On the other hand, the unconditionally stable algorithms presented do not suffer from this shortcoming and thus may be employed to economic advantage in many practical engineering problems.