THEORETICAL AND EXPERIMENTAL STUDIES OF THE NONLINEAR TRANSIENT RESPONSES OF PLATES SUBJECTED TO FRAGMENT IMPACT

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SUMMARY

A numerical procedure designed to predict the elastic-plastic, large-deflection transient responses of initially-flat longeron-stiffened or unstiffened plates subjected to impact by an idealized rigid fragment is presented. Numerical predictions are compared with experimentally-determined transient strains for steel-sphere-impacted plates to assess the accuracy of the numerical scheme.

The assumed-displacement finite-element model is used to obtain the spatial properties of the plate structure. The governing matrix equations of motion are formulated by applying the Principle of Virtual Work and D’Alembert’s Principle with the effects of geometric and material nonlinearities included as equivalent nodal loads. The timewise solution of the governing equations of motion is accomplished by employing the Houbolt finite-difference operator, modified for the present analysis to achieve compatibility with the approximate impact-interaction procedure.

The motion of the plate structure and the idealized rigid spherical fragment are followed at each time step and an inspection is made to determine if the space occupied by the fragment overlaps that occupied by the plate at the current instant in time. If overlap is found, a collision is said to have occurred and all plate-element-nodes falling within a circle of radius \( L_{\text{eff}} \), where \( L_{\text{eff}} \) is approximated from the longitudinal wave speed in the plate material and the time-step size, centered at the point of collision are assumed to be affected by the collision. An approximate impact-interaction analysis is carried out to predict the post-impact velocities of the fragment and of the impact-affected nodes of the plate structure. This approximate impact-interaction analysis utilizes conservation of momentum and kinetic energy and includes the effects of friction and a possible elastic, inelastic, or intermediate type of collision.

A series of experiments was conducted in which initially-flat aluminum plates were subjected to impact by a 1.0-in diameter steel sphere having a known velocity prior to impact. Two types of plate structure were tested: doubly-clamped narrow plates of (nominally) 8.0-in length, 1.5-in width, and 0.1-in thickness, and fully-clamped square plates of 8.0-in side length and 0.06-in thickness. For the narrow plates, initial impact was designed to occur at the center of the plate; whereas, for the square plates both center and off-center initial-impact tests were carried out. In each test transient strain data, referenced to a well-defined time of initial impact, was sought at eight spanwise locations; additional permanent strain and permanent deformation data were also measured. Moderate to large permanent deformations were achieved by using sphere impact velocities in the range 2485 in/sec to 2796 in/sec for the narrow plates and 2125 in/sec to 2850 in/sec for the square plates.

A series of theoretical-experimental correlation studies has been carried out comparing numerical predictions with experimental data for both the narrow and square plates. For the plate tests in which center (normal) impact was achieved, symmetry arguments can be used and only one quarter of the plate need be modeled in the numerical analysis; typically, a nonuniform 2 by 11 mesh of rectangular plate elements for the narrow plates and a nonuniform 8 by 8 mesh of rectangular plate elements is used in the quarter-plate model. Good theoretical-experimental correlation is observed in comparisons of transient strain at several spanwise monitoring stations, as well as permanent strain and permanent deformation.
1. Introduction

The question of fragment containment is of great interest in a number of structural problems. In an aircraft engine or turbine generator, blades and/or disc sections may fail and must be safely contained within the engine/generator housing or be caused to be deflected in a "harmless direction". In nuclear reactors, essential systems must be protected against internally-generated fragments and the external structure must be capable of withstanding impact by objects such as automobiles, aircraft, and tornado-generated pipes, rods, and utility poles, for example.

The present paper addresses itself to a first necessary step in the prediction of fragment containment, that being the prediction of transient large-deflection, elastic-plastic responses of structures subjected to fragment impact. For present purposes initially-flat plates are considered and the attacking fragment is assumed to be rigid. The effects of possible inelastic collisions and friction are included in the impact-interaction procedure.

The assumed-displacement finite-element model is utilized to derive the governing matrix equation of motion for the plate structure. The effects of material and geometric nonlinearities appear as equivalent nodal forces on the structure. The timewise solution of the governing equations of motion for the structure and fragment is accomplished by employing a modified Houbolt finite-difference operator.

When impact is determined to have occurred, a collision interaction analysis is performed. In this analysis the impact-induced velocity changes imparted to the fragment and a portion of the plate structure based on conservation of momentum and kinetic energy are calculated; hence, this analysis is termed the collision imparted velocity method (CIVM). This procedure has been used with success for two-dimensional (2-d) beam and ring impact analyses [1,2,3], and has been extended to 3-d structural deflections for the present plate impact analysis.

In order to assess the accuracy and adequacy of the numerical procedure, a series of experiments was conducted in which initially-flat 6061-T651 aluminum plates were subjected to impact by a 1.0-inch diameter steel sphere having a known velocity prior to impact. In each test, transient strain data, referenced to a well-defined time of initial impact, were sought for use in correlation with numerical predictions. Both narrow and square flat plates were tested. Results of theoretical-experimental correlation studies conducted on the narrow plates are presented in this paper.

2. Governing Equations of Motion and Timewise Solution

2.1 Governing Equations of Motion of the Plate Structure

In the present study, the assumed-displacement finite element model is utilized in conjunction with the Principle of Virtual Work and D'Alembert's Principle to derive the governing equations of motion of the structure which is permitted to undergo large, elastic-plastic, transient deformations. A Lagrangian description [4] is adopted so that all pertinent quantities are defined with respect to an initial undeformed configuration.

In the interest of conciseness, only the final form of the governing equations is presented; the reader may consult Ref. 5, for example, for a detailed derivation. The governing equation of motion for the complete assembled structure may be written in the following form:
\[
[M] \ddot{\{q\}} + [K] \{q\} = \{p^{ext}\} + \{p^{NL}\}
\]

where

- \{q\} and \{\ddot{q}\} are the global generalized displacement vector and acceleration vector, respectively.
- \([M]\) is the mass matrix of the assembled structure.
- \([K]\) is the linear elastic stiffness matrix of the assembled structure.
- \{\dot{p}^{ext}\} is the global generalized load vector corresponding to prescribed externally-applied forces.
- \{p^{NL}\} represents equivalent "generalized loads" arising from both large deflections and plastic strains.

For the present impact analysis, structural response results only from impact-induced nodal velocity changes; no externally-applied loads are present. Thus, the vector \{\dot{p}^{ext}\} in eq. (1) is eliminated and the following equations of motion are used:

\[
[M] \ddot{\{q\}} + [K] \{q\} = \{p^{NL}\}
\]  

(2)

Also it should be noted that the impact-interaction analysis (discussed in Section 3) is based on a lumped mass model and thus, for consistency, the mass matrix, \([M]\), used in eq. (2) is a diagonal lumped mass matrix.

In order to represent the nonlinear material behavior, the mechanical sublayer model [7] is utilized. In this model, the uniaxial stress-strain curve is approximated by \((n+1)\) piecewise linear segments. Each material point is then envisioned as consisting of \(n\) equally-strained "sublayers" of elastic, perfectly-plastic material with each sublayer having the same elastic modulus, \(E\), but an appropriately different yield stress, \(\sigma_y\). In the analysis, the stress state associated with each sublayer can be calculated and the stress state at the material point is then calculated as a weighted sum of the sublayer stress levels. Each sublayer may also be treated as being strain-rate dependent. In this case, the yield stress \(\sigma_y\) of a particular sublayer is calculated from the relation [7]:

\[
\sigma_y = \sigma_0 \left(1 + \frac{\dot{\varepsilon}}{D} \right) \frac{1}{P}
\]

(3)

where \(\dot{\varepsilon}\) is the strain rate, and \(D\) and \(P\) are material constants.

The plate element used in the present analysis is a 4-noded uniform-thickness rectangular element with six generalized displacements at each node: \(u, v, w, \theta = \partial w/\partial x, \psi = \partial w/\partial y\), and \(\chi = \partial^2 w/\partial x \partial y\), where \(u, v, \) and \(w\) are the translational displacements of the reference surface (plate midsurface) in the \(x, y, \) and \(z\) directions, respectively. The displacement behavior within each finite-element is represented by a bilinear interpolation in \(x \) and \(y\) for the inplane displacements \(u\) and \(v\) and a bicubic interpolation in \(x\) and \(y\) for the transverse displacement, \(w\). The resulting element can easily be shown to yield continuity of \(u, v, w, \) and \(\partial w/\partial n\) (normal derivative) along interelement boundaries, as required in the assumed-displacement model [5].
2.2 Governing Equations of Motion for the Fragment

The fragment is idealized as a rigid spherical fragment and thus the motion of the fragment may be defined in terms of the motion of its center of gravity. Translational and rotational motion are permitted. The effects of gravity are neglected, and it is assumed that no externally-applied forces act on the fragment. Thus, the equations of motion for the fragment may be written:

\[ m_e \dddot{x}_f = 0 \quad I_{e} \dddot{\theta}_x = 0 \]
\[ m_e \dddot{y}_f = 0 \quad I_{e} \dddot{\theta}_y = 0 \]
\[ m_e \dddot{z}_f = 0 \quad I_{e} \dddot{\theta}_z = 0 \]  \hspace{1cm} (4)

where \( m_e \) and \( I_e \) are the mass and mass moment of inertia of the fragment, respectively, \( \dddot{x}_f \), \( \dddot{y}_f \), and \( \dddot{z}_f \) are the translational acceleration components of the fragment's c.g. in the \( x \), \( y \), and \( z \) directions, and \( \dddot{\theta}_x \), \( \dddot{\theta}_y \), and \( \dddot{\theta}_z \) are the angular acceleration components of the fragment about the \( x \), \( y \), and \( z \) axes, respectively.

When plate/fragment impact occurs, an instantaneous velocity change is assumed to be imparted to the fragment. However, prior to the first impact and between subsequent impacts the velocity of the fragment is constant, as dictated by eqs. (4).

2.3 Timewise Solution of the Governing Equations of Motion

The timewise solution of the governing equations of motion for the assembled structure, eq. (2), is accomplished by employing the Houbolt finite-difference operator \[ 8 \]. The implicit Houbolt operator can be shown to be unconditionally stable for linear elastic analysis but is only conditionally stable for nonlinear analysis \[ 9 \]; thus, some care must be taken in the selection of a time-step size, \( \Delta t \), for the present analysis.

In the Houbolt operator, the acceleration vector \( \ddot{q}_m \) at time \( t = t_{m+1} \) is approximated by the following 4-point backward-difference formula

\[ \dddot{q}_m \approx \frac{1}{(\Delta t)^2} (2 q_{m+1} - 5 q_m + 4 q_{m-1} - q_{m-2}) \]  \hspace{1cm} (5)

which has truncation error \( O(\Delta t^2) \). A backward-difference formula for the velocity vector, \( \dot{q}_m \), which has the same truncation error, \( O(\Delta t^2) \), as eq. (5) is given by

\[ \ddot{q}_m \approx \frac{1}{2\Delta t} (3 q_{m+1} - 4 q_m + q_{m-1}) \]  \hspace{1cm} (6)

Generally, eq. (5) is substituted directly into the governing equations of motion to obtain a recurrence equation for the displacement vector, \( q_{m+1} \). However, for the present impact analysis, in which impact causes certain of the plate nodal velocities to be redefined, it is convenient to have the velocity vector appear in the final recurrence relation. To this end, the terms in eq. (5) are rearranged to give

\[ \dddot{q}_m \approx \frac{2}{(\Delta t)^2} (q_{m+1} - q_m) - \frac{1}{(\Delta t)^2} (3 q_m - 4 q_{m-1} - q_{m-2}) \]  \hspace{1cm} (7)

By comparison with eq. (6), it is seen that the expression in the second parentheses is the finite-difference approximation for \( \ddot{q}_m \) (times \( 2\Delta t \)), so that eq. (7) can be written as
\[
\ddot{q}_{m+1} = \frac{2}{(\Delta t)^2} (q_{m+1} - q_m) - \frac{2}{\Delta t} \frac{\dot{q}}{\dot{q}_{m+1}}
\]  
(8)

Equation (8) is substituted into the governing equations of motion for the assembled structure, eq. (2), to give the following recurrence relation (after rearranging):

\[
\left(\frac{2}{(\Delta t)^2} M + K\right) \ddot{q}_{m+1} = \ddot{q}_{m+1}^{NL} + \frac{2}{\Delta t} M (\dot{q}^{m+1} + \frac{1}{\Delta t} q^{m+1})
\]  
(9)

The displacement, \( q_m \), and velocities \( \dot{q}_m \), at time \( t_m \) are known. The vector of equivalent nodal forces, \( F_{m+1}^{NL} \), corresponding to geometric and material nonlinearities is, however, dependent on the displacements, \( q_{m+1} \), and some form of extrapolation and/or iteration is required to calculate \( F_{m+1}^{NL} \) in order to obtain the solution of eq. (9) for \( q_{m+1}^{NL} \). For the present analysis, a linear extrapolation is used to obtain \( F_{m+1}^{NL} \) with no iteration; the use of iteration will be studied in the near future.

The calculation of the impact-induced response of the plate structure is performed in the following manner. Based on the values of \( q_{m} \) and \( q_{m}' \) at time \( t_{m}' \), and the extrapolated value of \( F_{m+1}^{NL} \), eq. (9) is solved for the trial displacement vector \( q_{m+1}' \) and the trial position of the fragment is also obtained. Based on these trial locations of the plate and fragment, and the geometry of the plate and fragment, a check is made (see Section 3) to see if any plate/fragment overlap is present. If no overlap is found, then no collision has occurred between times \( t_m \) and \( t_{m+1}' \) and the trial displacements of the plate and fragment are assumed to be correct.

If plate/fragment overlap is found, a plate/fragment collision is said to have occurred between times \( t_m \) and \( t_{m+1}' \). Because of the complex geometry of the deformed plate, the exact instant of impact (between \( t_m \) and \( t_{m+1}' \)) cannot be easily determined, and the impact is assumed to occur at the beginning of the present cycle (i.e., at time \( t_m \)). An impact-interaction analysis (described in Section 3) is then performed in which the plate nodal velocities, \( \dot{q}_{m} \), and fragment velocities at time \( t_m \) are modified to correspond to their "post-impact" values. Then eq. (9) is again solved for the trial displacements, \( q_{m+1}' \) at time \( t_{m+1}' \), here \( \dot{q}_{m}' \) now corresponds to the modified post-impact velocity vector, but \( q_{m} \) and \( F_{m+1}^{NL} \) are unchanged. Based on the new trial displacements of the plate (and the fragment --- since the fragment velocity is also modified by the impact-interaction calculation), the inspection for plate/fragment overlap is again performed. If no overlap is found, the trial displacements at time \( t_{m+1}' \) are now assumed to be correct. If overlap is found, the impact-interaction analysis is performed again, resulting in another modification of \( \dot{q}_{m}' \). This process is repeated until no overlap is found at time \( t_{m+1}' \), the resulting displacements are then taken as the correct displacements.

Following the calculation of the correct displacement vector, \( q_{m+1}' \), and fragment position, the velocity vector, \( \dot{q}_{m+1}' \) at time \( t_{m+1}' \) is calculated from eq. (6), and the equivalent load vector, \( F_{m+1}^{NL} \), is calculated (by extrapolation). Then the trial solution for the next time step can be obtained by solving eq. (9). This process is repeated for as many time cycles as desired. It should be noted that the matrices \( M \) and \( K \) in eq. (9) remain constant throughout the solution process. The matrix sum \( \left[ M + 2(\Delta t)^2 (M + K) \right] \) is formed prior to the first time step and factored; the solution of eq. (9) at each cycle involves only a back-substitution operation using the factored matrix and the known right-hand-side of Eq. (9).
3. Impact Inspection and Interaction Analysis

The accurate prediction of structural response to fragment impact is dependent on the ability to detect when and where plate/fragment impact occurs (inspection procedure), and to estimate the instantaneous effects of each impact on the plate and fragment (interaction analysis). The subsequent response of the plate and fragment (after instantaneous impact effects are imposed) is accommodated in the solution of the governing equations of motion, as described in Section 2.

In principle, the impact inspection is based on a knowledge of the current plate node locations and plate thickness, and the fragment radius and current location. In practice, inspection for overlap of the deformed plate and spherical fragment geometries is best carried out at the element level, processing each element (or a plausibly selected group of elements) to find the maximum penetration (overlap of geometries). For each element, the current coordinates of the four element nodes are known, and a general curved surface could be defined. However, even at this level, inspection for overlap of the fragment and element geometries would be difficult and could require excessive computation time.

To alleviate these difficulties, an approximate plate geometry is used in the inspection procedure. An element is subdivided into two-triangular subregions: element node numbers 1, 2, and 3 define Subregion 1, and element nodes 1, 3, and 4 define Subregion 2. Within each subregion, the current coordinates of the three nodes serve to define a plane. An inspection is then performed to determine if the spherical fragment overlaps this plane. If overlap is detected, a second check is made to see if the point of maximum overlap is within the triangular subregion. If so, then a plate/fragment overlap is identified.

This inspection procedure is performed for the two subregions of each element surrounding the node which is closest to the centroid of the fragment. The largest of all overlaps and the corresponding element number and subregion are identified.

The process called the collision-imparted velocity method (CIVM), previously developed for the 2-d ring/fragment impact interaction analysis of Refs. 1 through 3, has been extended for the present 3-d impact-interaction analysis. In this procedure, energy and momentum considerations are employed to predict the instantaneous collision-induced velocities which are imparted to the fragment and to the impact-affected zone of the plate (i.e. the fraction of the plate that responds to fragment impact "instantaneously" with momentum changes). Included in the analysis are the effects of possible inelastic collision and friction between the fragment and plate surfaces.

The impact-affected zone of the plate is defined as that region of the plate falling within a circle of radius \( L_{eff} \) centered at the point of maximum plate/fragment overlap. The parameter \( L_{eff} \) is estimated from the speed of a longitudinal elastic wave in the material and the computational time increment size \( \Delta t \) and is given by

\[
L_{eff} = \left( \frac{E}{\rho} \right)^{1/2} \Delta t \tag{10}
\]

where \( E \) and \( \rho \) are the Young's modulus and mass density, respectively, of the plate material; alternate selections for \( L_{eff} \) could be used if desired. The impact-affected nodes are defined as those plate nodes which fall within the impact-affected zone; it is assumed in the following that \( k \) nodes are impact-affected.
In the impact-interaction, the equations are written in an orthogonal \( \vec{x}-\vec{y}-\vec{N} \) system where \( \vec{N} \) is the direction normal to the plane of the triangular subregion where the maximum penetration is detected, and \( \vec{x}-\vec{y} \) fall in the plane of the triangular subregion; pre-impact velocities of the fragment and impact-affected plate nodes are transformed into this coordinate system. The notation \( v_{1x}', v_{1y}', v_{1N}' \) and \( v_i \) is used to denote the pre-impact translational velocities in the \( \vec{x}, \vec{y}, \) and \( \vec{N} \) directions at impact-affected node \( i \), and \( v_{ix}', v_{iy}', v_{iN}' \), \( \omega_{ix}', \omega_{iy}' \), and \( \omega_{iN}' \) denote the pre-impact translational and rotational velocity components of the fragment c.g.

Denoting by a prime, the post-impact translational and/or rotational velocity components, the following impulse-momentum relations may be written:

**Normal Direction: Translation**

\[
\begin{align*}
\text{m}_f [v_{fN}' - v_{fN}] &= -\vec{p}_N \\
\text{m}_l [v_{lN}' - v_{lN}] &= \alpha_l \vec{p}_N \\
\vdots \\
\text{m}_k [v_{KN}' - v_{KN}] &= \alpha_k \vec{p}_N
\end{align*}
\]

**\( \vec{x} \)-Direction: Translation**

\[
\begin{align*}
\text{m}_f [v_{fx}' - v_{fx}] &= -\vec{p}_x \\
\text{m}_l [v_{lx}' - v_{lx}] &= \alpha_l \vec{p}_x \\
\vdots \\
\text{m}_k [v_{kx}' - v_{kx}] &= \alpha_k \vec{p}_x
\end{align*}
\]

**\( \vec{y} \)-Direction: Translation**

\[
\begin{align*}
\text{m}_f [v_{fy}' - v_{fy}] &= -\vec{p}_y \\
\text{m}_l [v_{ly}' - v_{ly}] &= \alpha_l \vec{p}_y \\
\vdots \\
\text{m}_k [v_{ky}' - v_{ky}] &= \alpha_k \vec{p}_y
\end{align*}
\]

**Rotational**

\[
\begin{align*}
\text{I}_f [\omega_{fy}' - \omega_{fy}] &= -r_i \vec{r}_x \\
\text{I}_f [\omega_{fx}' - \omega_{fx}] &= r_i \vec{r}_y
\end{align*}
\]

In eqs. (11) through (14), \( \vec{p}_x, \vec{p}_y, \) and \( \vec{p}_N \) are the (unknown) total impulses in the \( \vec{x}, \vec{y}, \) and \( \vec{N} \) directions, respectively, imparted to the impact-affected nodes, and, in an antiparallel fashion, to the fragment; \( \text{m}_l \) is the lumped mass at node \( i \), \( \text{m}_f \), \( \text{I}_f \), and \( r_f \) are the fragment mass, mass moment of inertia about its c.g., and radius, respectively.

The \( \alpha_i \) term in eqs. (11) through (13) is the proportionality constant which determines the portion of the total impulse imparted to node \( i \). For present purposes, a cone-shaped distribution is assumed in the impact-affected plate region, and \( \alpha_i \) is given by
\[ \alpha_i = C \left( 1 - \frac{|e_i|}{L_{\text{eff}}} \right) \] 

where \( |e_i| \) is the distance from impact-affected node \( i \) to the point of plate/fragment collision, and \( C \) is a normalizing factor chosen so that the sum of all \( \alpha_i \)'s is equal to unity (i.e., so that the sum of the impulses imparted to the nodes is equal to the total impulse imparted to the fragment). Thus \( C \) is given by

\[ C = \frac{1}{\frac{1}{k} \sum_{i=1}^{k} \left( 1 - \frac{|e_i|}{L_{\text{eff}}} \right)} \] 

Once the total imparted impulses \( \vec{P}_X, \vec{P}_Y, \) and \( \vec{P}_N \) have been determined, the post impact velocities of the impact-affected nodes and fragment can be calculated from eqs. (11) through (14) and transformed back to the x-y-z coordinate system. The calculation of \( \vec{P}_X, \vec{P}_Y, \) and \( \vec{P}_N \) at the instant of termination of the impact is obtained by extension of a graphical technique suggested by Goldsmith [10]; details of this graphical solution technique may be found in Ref. 11. This graphical solution procedure yields a series of expressions for the total imparted impulses. The resulting post-impact velocities calculated from eqs. (11) through (14) and transformed back to the x-y-z system are then used in the global solution process described in Subsection 2.3.

4. Experimental Studies

In order to assess the accuracy and adequacy of the transient impact analysis scheme, a series of experiments was conducted in which initially-flat aluminum plates were subjected to impact by a 1.0 in-diameter steel sphere having a known velocity prior to impact. Transient strain, permanent strain, and permanent deformation data were sought to allow for correlation with predictions from the numerical procedure.

Two types of plate structures were considered. In the first series of experiments [12], narrow 6061-T651 aluminum plates of (nominally) 8.0-in length, 1.5-in width, and 0.10-in thickness, clamped at both ends, were subjected to impact at the midspan. Transient strain data were sought at 8 spanwise locations along the centerline at both the upper and lower surface of the plate.

Sphere pre-impact velocities in the range 2485 in/sec to about 2800 in/sec were found to produce moderate to large permanent deformations in the plates; rupture of the plate was observed for sphere velocities above about 2870 in/sec. These tests were intended primarily to verify the 2-d beam/ring impact codes CIVM-JET 4B [2] and CIVM-JET 5B [3], but may also be used as a preliminary verification of the 3-d plate impact program. It should be noted that except in the near vicinity of the location of initial impact, the narrow-plate (or beam) specimen exhibited essentially 2-d deflections; for those 2-d regions, the use of 2-d impact/response codes would appear to offer a reasonable approximation. However, a significantly better modeling of this impact/response problem is provided by the use of plate-type elements which accommodate 3-d structural deformations.

In the second series of experiments [13], fully-clamped square 6061-T651 aluminum plates of (nominally) 8.0-in side length and 0.06-in uniform thickness were subjected to
steel-sphere impact. Similar specimens with integral rectangular-cross-section stiffeners were also tested. These experimental deformation, transient strain, and permanent strain results are given in Ref. 13. Here also the pre-impact sphere velocities ranged from those required to produce moderate permanent deflections to that which produced panel rupture.

5. Comparison of Predictions with Experiment

Preliminary results of theoretical-experimental comparisons for steel-sphere-impact against the cited flat plate specimens will be discussed. Further studies of this type are in progress.

In particular, for these preliminary studies to assess the accuracy and adequacy of the numerical solution scheme, it was desired to minimize the required computing. Accordingly, the narrow plate impact case was selected since a reasonable finite element modeling of the structure will require only a relatively small number of elements (and corresponding nodal degrees of freedom), thus allowing for a relatively economical preliminary verification of the plate-sphere impact analysis procedure.

Discussed here is plate specimen CB-18 for which impact was found to occur within approximately 0.06-in of the plate center. The fragment pre-impact velocity for this case was 2794 in/sec in the global z direction which is normal to the surface of the plate; a schematic of the model showing global coordinate directions is given in Fig. 1. In this test, transient relative elongation data were measured successfully along the x axis at locations \( x = \pm 0.6\)-in (upper surface), 1.2-in (upper surface), \( x = -1.5\)-in (upper surface), and \( x = 1.5\)-in (upper and lower surface).

In the finite-element solution, it is assumed for computational convenience and thrift that impact occurs at the center of the plate. With this approximation, the transient deformation of the plate will be symmetric with respect to both the x and y axes. Hence, in the finite-element modeling, one-quarter of the plate is modeled by a 2 x 11 mesh of unequal-size rectangular plate elements as shown in Fig. 2. Initial impact is prescribed to occur at node 1 (x = 0, y = 0) of element 1 and subsequent impacts will occur within element 1. It should be noted that a special doubly-symmetric impact interaction analysis has been formulated and implemented in the breadboard plate-impact computer program.

In the results to be presented, a time step size of \( \Delta t = 1.0 \mu \text{sec} \) was used and no strain-rate effects are included. The locations \( x = 0.6\)-in, 1.2-in, and 1.5-in, where experimentally-determined transient relative elongation data are available, fall at nodes 7, 13, and 16, respectively, in the finite-element mesh. The theoretical relative elongation predictions are then obtained from nodal averaging.

Shown in Figs. 3 through 5 are the theoretical predictions and experimental results obtained for the relative elongation, \( R_x \), in the x direction as a function of time after initial impact (TAII) at nodes 7, 13, and 16, respectively. Reasonable agreement between theoretical and experimental results is seen particularly at locations \( x = 1.2\)-in and \( x = 1.5\)-in.

During the solution process the fragment velocity is modified as a result of each impact. Shown in Fig. 6 is the velocity of the fragment as a function of TAII. As shown, the analysis predicts that the fragment begins to rebound at approximately 640 \( \mu \text{sec} \) after initial impact. Also, the fragment velocity is constant (indicating no further impacts) after approximately 800 \( \mu \text{sec} \) after initial impact.
Permanent deformation data have been obtained for specimen CB-18. However, computer predictions have been carried out only to a value of TAI = 900 μsec; meaningful comparisons of theoretical vs. experimental permanent deformation data would require carrying out the calculations to larger TAI. It is useful to note, however, that the theoretical predictions of displacement up to TAI = 900 μsec are in qualitative agreement with the experimental observation that general 3-d deformation is prominent but is restricted roughly to a region with approximately 0.8-in of the center of the plate.

6. Summary Comments

The preliminary comparisons shown here between measured transient response of a steel-sphere-impacted narrow flat aluminum plate and finite-element predictions carried out in small time increments by the use of a modified Houbolt operator indicate encouraging agreement. However, various aspects of the prediction scheme will require evaluation before one can employ this method confidently to obtain accurate results. These factors include: various finite element sizes and modelings, the use of iteration rather than extrapolation to determine the \( F_{\text{NL}} \) terms, appropriate selections for time increment size to minimizing computation without encountering computational instabilities, etc.

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References


Fig. 1 Schematic of 6061-T651 Aluminum Narrow Plate Model Subjected to Impact by a One-inch Diameter Steel Sphere

Fig. 2 Finite-Element Array Used to Represent One Quarter of Narrow Plate Specimen CB-18
Fig. 3 Comparison of Predicted and Measured Transient Relative Ejections at Various Spacing Stations x along y for Steel-Sphere-Impacted Narrow-Plate Specimen CB-18
Fig. 4 Predicted Steel Sphere Velocity as a Function of Time After Initial Impact.