ANALYSIS OF DOUBLE-WALL CYLINDRICAL SHELLS

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SUMMARY

The use of double wall shells in different industries has been increasing in recent years. One example is the drywell of nuclear reactors. Such shells consist of two concentric cylinders connected together by a number of gussets spaced at regular intervals. The analysis of these structures presents some problems because of their large sizes and the complexity of the loading conditions. Generally the different stages of contract and design require analysis techniques with different degrees of sophistication. For example, at the preliminary stage of design when different alternatives have to be examined a finite element analysis would be costly in terms of time and computer expenses. In addition, the large amount of data might make the choice of an optimum structure rather cumbersome. At the final stages, however, a more detailed analysis is needed which may warrant a finite element solution.

The different solution techniques described here were used for the various stages of design and were compared in terms of accuracy and efficiency. The first approximate solution was obtained by assuming an axisymmetric geometry. In the numerical example studied the number of gussets were varied from 44 to 88. Comparison with other methods of solution described below shows that the assumption of axi-symmetry is satisfactory only for a large number of gussets. For example decreasing the number of gussets from 88 to 44 changed the results from acceptable to erroneous. For an axi-symmetric loading condition, a segment extending halfway between two gussets was analyzed by the finite element method. In view of the large shell radii the shell segments as well as the gussets were modeled by a series of triangular plate elements. Even though such a small portion of the circumference for a segment equal to one fifth of the actual length was considered, a large number of elements had to be used to achieve the required accuracy. For the purpose of parametric studies at the preliminary stage of design an analysis using an anisotropic shell proved suitable. The solution to the eight order differential equation thus found could be obtained by a variational technique, finite difference or a numerical integration scheme. For a detailed stress analysis a solution was developed using Donnell's theory of cylindrical shells. The gussets were analyzed as plates. The different shell and plate panels were combined by a matrix analysis to yield the overall solution. Economy and efficiency could be achieved using the two latter methods for the design and the final analysis of the structure.

INTRODUCTION

Double wall shells are being used as structural components in different industries. One example is presented by containment vessels with a pressure suppression system and a dry well. The dry well houses the reactor vessel and, in turn, is enclosed in the containment vessel. Should a break occur in the steamline in the dry well, the steam would pass into the suppression pool. Since the drywell does not have to be completely leakproof, the solution may be provided by a reinforced or prestressed concrete shell without a steel liner. However, in such designs it must be shown that the crack size and thus the amount of leakage is not excessive. This is obviously a delicate undertaking. Another alternative is to use a leak-tight shell for the drywell such as a double-wall steel shell. A typical design is shown in Figure 1. It consists of two concentric cylindrical shells connected together by a number of radial gussets. The annulus space is filled with concrete and hence both radiation shield and load carrying members are provided. The reduction in the maintenance work and construction work makes this design competitive with the concrete alternatives, especially if the leakage is to be minimized or eliminated.

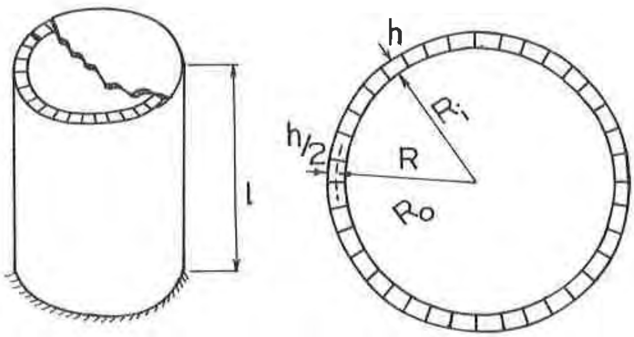

FIG. 1. SHELL GEOMETRY

The design of such a configuration presents certain problems both in terms of cost optimization and structural analysis. The large dimensions, complex geometry and intricate loading conditions make the use of the finite element method justifiable only at the final stage of the design. For preliminary investigations and parametric studies the solution technique must be more readily applicable. Accurate but more efficient methods are also desirable for other stages of design and analysis. In the present study, in addition to the available methods, some other formulations are discussed. The latter have proven to be efficient and reasonably accurate for the different stages of design.

AXISYMMETRIC APPROXIMATION

The axisymmetric solution of general shells of revolution is well established [1-3]* and documented. As a preliminary step, the shell was analyzed using such an approximation. The annulus space was considered as an orthotropic layer. The technique used was based on the integration of the governing shell equations. The results are given in a later section and compared with the more suitable methods. Acceptable results are obtained only for a large number of gusset plates. The errors increase rapidly as the number of gussets decreases.

An improved solution technique may be obtained by formulating an anisotropic shell which includes the shear deformation of gussets.

ANISOTROPIC FORMULATION

The geometry of the shell cross section is shown in Figure 2 with R_i and R_0 representing the inner and outer radius and R the radius of the reference surface. The thickness

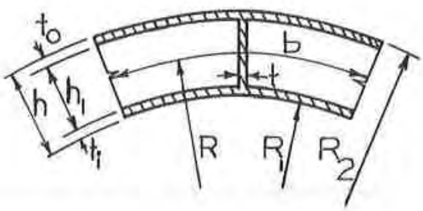

FIG. 2. SHELL SECTION

*Numbers in brackets refer to the entries in the list of References.

of the inner and outer shells are t_i and t_o and that of the gusset t. h and h_i are the total thickness and the thickness of the annulus space. The circumferential length between the two adjacent gussets along the reference surface is b. The arc lengths x and s in the axial and circumferential directions are used as the basic shell parameters, and the displacements in those directions are denoted by U and V. The radial displacement, W, is assumed to be constant through the thickness. The stress resultants are indicated by subscripts 1 and 2 referring to the x and s directions with a comma denoting differentiation. The positive direction of the variables is shown in Figure 3.

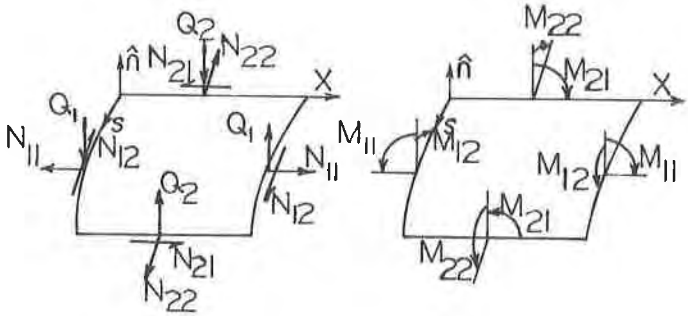

FIG. 3. STRESS RESULTANTS & COUPLES

The governing differential equations are obtained for arbitrary loading conditions using Donnell's [4] assumptions. Only radial pressure, P, is included in the formulation. However, if the other components are present, their inclusion is straight forward. The in-plane and bending rigidities in the circumferential direction are:

$$K = \frac{Et_1}{1 - \nu^2} \tag{1a}$$

$$D = \frac{E(h^3 - h_1{}^3)}{12(1 - \nu^2)} \tag{1b}$$

Where $t_1 = t_i + t_o$

E and ν are the modulus of elasticity and Poisson's ratio.

The rigidities in the axial direction are:

$$K_1 = E\left(\frac{t_1}{1 - \nu^2} + \frac{th_1}{b}\right) \tag{1c}$$

$$D_1 = \frac{E}{12}\left(\frac{h^3 - h_1{}^3}{1 - \nu^2} + \frac{th_1{}^3}{b}\right) \tag{1d}$$

The rigidity of gusset plates is assumed to be uniformly distributed in the axial direction.

From [5] the equilibrium equations are:

$$N_{11,1} + N_{12,2} = 0 \tag{2a}$$

$$N_{12,1} + N_{22,2} = 0 \tag{2b}$$

$$M_{11,11} + 2M_{12,12} + M_{22,22} - \frac{N_{22}}{R} + P = 0 \tag{2c}$$

and the strain displacement relations at the reference surface are:

$$\varepsilon_{11} = U,_1$$

$$\varepsilon_{22} = V,_2 + \frac{W}{R}$$

$$2\varepsilon_{12} = V,_2 + U,_2$$

The relationships between rotations and displacements and curvatures and rotations are modified to include the shear deformation of the ribs:

$$\beta_1 = \frac{Q_1 \, b}{\kappa G t h_1} - W,_1 \tag{3a}$$

$$\beta_2 = \frac{V}{R} - W,_2 \tag{3b}$$

$$\kappa_{11} = \beta_{1,1} = -\left(\frac{Pb}{\kappa G t h_1} + W,_{11}\right) \tag{4a}$$

$$\kappa_{22} = \beta_{2,2} = - W,_{22} \tag{4b}$$

$$2\kappa_{12} = \beta_{1,2} + \beta_{2,1} = - 2W,_{12} \tag{4c}$$

Where G is the shear modulus, κ a shape factor and Q_1 shear force at a section along the axis of the shell. The constitutive equations:

$$N_{11} = K_1 \, \varepsilon_{11} + \nu K \, \varepsilon_{22} \tag{5a}$$

$$N_{22} = K \, (\varepsilon_{22} + \nu\varepsilon_{11}) \tag{5b}$$

$$N_{12} = \frac{1 - \nu}{2} K \, (2\varepsilon_{12}) \tag{5c}$$

$$M_{11} = D_1 \, \kappa_{11} + \nu D \kappa_{22} \tag{5d}$$

$$M_{22} = D \, (\kappa_{22} + \nu\kappa_{11}) \tag{5e}$$

$$M_{12} = \frac{1 - \nu}{2} D \, (2\kappa_{12}) \tag{5f}$$

In terms of displacements the stress resultants become:

$$N_{11} = K_1 U_1 + \nu K (V,_2 + \frac{W}{R}) \qquad (5a)^1$$

$$N_{22} = K (V,_2 + \frac{W}{R} + \nu U,_1) \qquad (5b)^1$$

$$N_{12} = \frac{1 - \nu}{2} K (V,_1 + U,_2) \qquad (5c)^1$$

$$M_{11} = -D_1 (W,_{11} + \frac{P}{\kappa G t h_1}) - DW,_{22} \qquad (5d)^1$$

$$M_{22} = -D \left[W,_{22} + \nu (W,_{11} + \frac{P}{\kappa G t h_1})\right] \qquad (5e)^1$$

$$M_{12} = -D (1 - \nu) W,_{12} \qquad (5f)^1$$

Substitution of the above relationships in the equilibrium equations results in:

$$U,_{11} + \frac{1 - \nu}{2} \frac{K}{K_1} U,_{22} + \frac{1 + \nu}{2} \frac{K}{K_1} V,_{12} + \frac{\nu}{R} \frac{K}{K_1} W,_1 = 0 \qquad (6a)$$

$$V,_{22} + \frac{1 - \nu}{2} V,_{11} + \frac{1 + \nu}{2} U,_{12} + \frac{W,_2}{R} = 0 \qquad (6b)$$

$$\nabla_1^4 W + \frac{K}{RD_1} (\nu U,_1 + V,_2 + \frac{W}{R}) = q \qquad (6c)$$

Where

$$\nabla_1^4 W = W,_{1111} + \frac{D}{D_1} (2W,_{1122} + W,_{2222}) \qquad (6d)$$

$$q = \frac{P}{D_1} - \frac{1}{\kappa G t h_1} (P,_{11} + \frac{\nu D}{D_1} P,_{22}) \qquad (6e)$$

by algebraic manipulation the above equations can be cast into three equations giving the values of U, V and W. The mixed derivatives of V are eliminated by first finding the second derivatives of eq. (6a) with respect to X and S and then substituting the values of $V,_{1112}$ and $V,_{1222}$ obtained from the latter equations into one obtained by finding the derivative of eq. (6b) with respect to X and S. This would yield:

$$\nabla_2^4 U = \frac{K}{RK_1} (W,_{122} - \nu W,_{111}) \qquad (7a)$$

with

$$\nabla_2^4 U = U,_{111} + \frac{2}{1 - \nu} (1 - \nu \frac{K}{K_1}) U,_{1122} + \frac{K}{K_1} U,_{2222} \qquad (7b)$$

In a similar manner, the equations obtained by finding the second derivatives of eq. (6b) give the values of $U,_{1112}$ and $U,_{1222}$. Substituting the latter values into the relationship found from eq. (6a) after taking its derivative with respect to S and X results in:

$$\nabla_2^4 V = \frac{1}{(1-\nu)R}\left[\nu(1+\nu)\frac{K}{K_1} - 2\right] W_{,112} - \frac{K}{RK_1} W_{,222}$$

Finally substituting X and S derivatives of eq. (7a) and (7b) in eq. (6c) after the latter is operated on by ∇_2^4 gives:

$$\nabla_1^4 \ \nabla_2^4 W + \frac{K}{R^2 D_1}\left(1 - \frac{\nu^2 K}{K_1}\right) W_{,1111} = \nabla_2^4 q \qquad (7c)$$

For a monocoque shell eqs. (7) reduce to Donnell's equations. Under axisymmetric behavior they simplify to:

$$W_{,1111} + 4\lambda^4 W = \frac{P}{D_1} - \frac{P_{,11}}{\kappa G t h_1} \qquad (7c)^1$$

with

$$4\lambda^4 = \frac{K}{R^2 D_1}\left(1 - \nu^2 \frac{K}{K_1}\right) \qquad (7d)$$

The solution of eqs. (7) yields the displacements and stresses. For the axisymmetric behavior the closed form solution of eq. $(7c)^1$ yields the values sought. For the general loading conditions eqs. (7) can be solved either in closed form or by a variational technique. Numerical solutions such as the finite difference method or numerical integration are also applicable.

THREE DIMENSIONAL SHELL SYSTEM

The structure can be conceived as a series of panels each consisting of an inner and outer cylindrical segment connected together by a gusset as shown in Figure 3. The analysis of a panel requires the solution of four cylindrical segments and a gusset. Once the solution of a typical panel is obtained the overall solution can be found by combining the panels. The cylindrical segments may be analyzed using Donnell's equations which are obtained from eqs. (7) by setting D_1 equal to D and K_1 equal to K and by neglecting the shear deformations. The gussets do not carry any loads except at their edges. In-plane forces resulting from shear in the shell segments and a moment and shear resulting from the moments at the adjacent shell elements are applied to the edges of the gussets. The latter moment and shear are small [6] and thus the gussets act as a deep beam. The general theory of thin-walled beams presented by Vlasov [7] may be used for the analysis of the gussets. The differential equation for the gusset is given in Reference [6] and the assemblage of the panels is done by matrix algebra.

FINITE ELEMENT SOLUTION

In addition to the solution technique just described, which may be viewed as a finite panel technique, the finite element method may also be applied. In this study plate elements were used to model the structure. A large number of elements were required, even for the axisymmetric behavior, to obtain acceptable results. For the nonsymmetric loading conditions, where a large portion of the structure had to be modeled, the solution was possible only by substructuring of the shell. Since the interaction between the inner and the outer shell could not be ignored, truss elements were used to model this interaction.

NUMERICAL RESULTS

Several numerical examples were studied in order to establish the limitations of the different methods discussed. As a first step, the applicability of a shell of revolution was examined. The results of such an analysis assuming orthotropic behavior for the annular space is plotted in Figures 4 and 5, where the radial deformation and meridional stresses are compared to values obtained by finite element method. In this example the radial gussets were placed at every 8^0 around the curcumference of the shell. For a

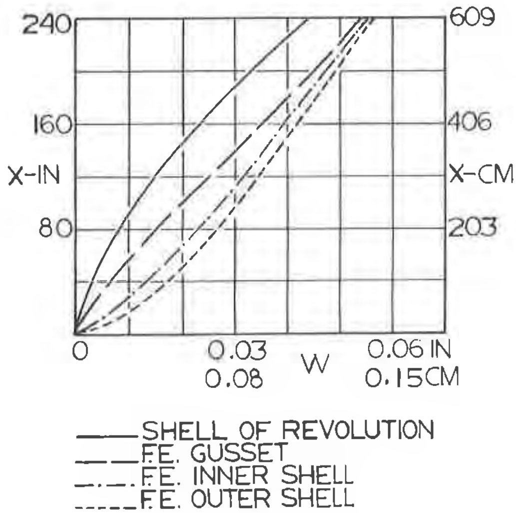

FIG. 4. RADIAL DEFORMATION

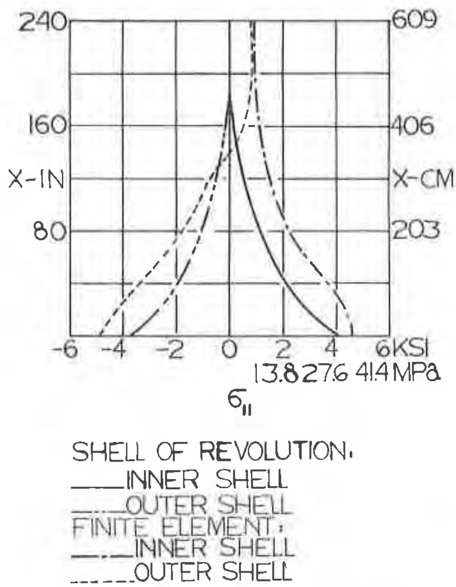

FIG. 5. MERIDIONAL STRESS

smaller number of gussets the error was much too large to be overlooked, even for preliminary design. In the finite element and the three dimensional shell system when the interaction between the inner and outer shells was ignored a more flexible shell was resulted. In order to examine the extent of this error, in the finite element method the inner and outer shells were connected together by a number of bars. The results are shown in Figure 6. It can be seen that the true solution lies somewhere in between the shell of revolution and the finite element model. Comarison between the results obtained by an anisotropic shell formulation, outlined in the previous section, with both shell of revolution and the finite element results is made in Figure 7. The anisotropic shell results fall between the other two solutions and are closer to the expected values. The plot also shows results of anisotropic shells in which the shear deformations in the gussets were included. This shear deformation is significant and may not be ignored in a final analysis.

CONCLUSION

Different methods of analysis were discussed and compared for a double wall cylindrical shell. Each of the methods have some merits for a given stage of design and under certain conditions. The anisotropic shell formulation can yield good results for a wide range of configurations. The shear deformation and the interaction between the inner and outer shell are significant in general and cannot be ignored.

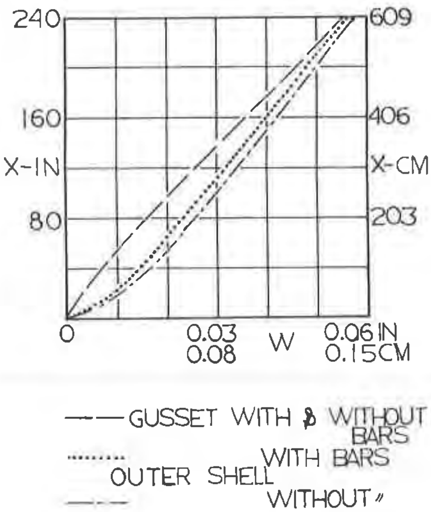

FIG. 6. INTERACTION BETWEEN INNER & OUTER SHELLS

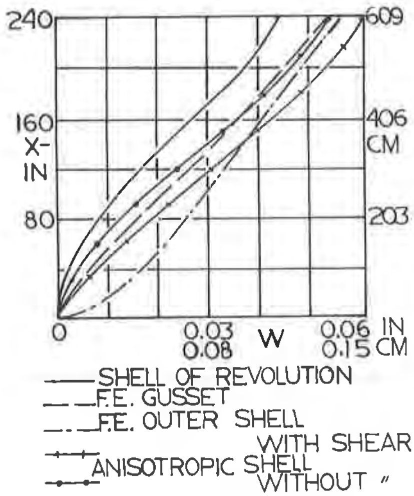

FIG. 7. ANISOTROPIC SHELL

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