PRACTICAL ASPECTS OF PROBABILISTIC STRUCTURAL RELIABILITY ANALYSES*

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SUMMARY

The purpose of the paper is to provide some practical perspective on the application of probabilistic structural reliability analyses to calculate the reliability of structural components and/or systems. Specifically, the discussion covers the most important considerations in applying structural reliability methodology, the limitations and implied development needs of structural reliability analysis state-of-the-art, and some practical guidance on the level of detail in applying the methods to engineering problems. Specific examples are presented in the full text to enhance this discussion.

Application of structural reliability methods begins with solid mechanics rather than statistics. First, a realistic criterion of structural failure must be chosen for the specific application. A conservative criterion may not be adequate as with conventional stress analysis. Furthermore, a careful distinction must be maintained between a realistic criterion of failure versus allowables as provided in design codes and standards. The difficulty is not to be underestimated since basic research has not yet yielded broadly accepted failure models for complex stress states.

The statistical elements of applying structural reliability methodology cover collection and reduction of basic data adequate to support the mechanical characterization as represented by both the stress/strain analysis and the failure model. These elements become quite sophisticated when considering direct, thermal, cyclic, and creep loadings perhaps further complicated by the presence of a flaw. Thus, state-of-the-art of solid mechanics places limitations on the precision of the structural reliability methodology. These limitations include incompletely developed models of failure for complex loadings and theoretical and practical limitations on the ability to analyze outside the elastic regime. A combined mathematics/data availability limitation exists in the sensitivity of structural reliability analyses to the shape of the probability density functions, especially for extreme values, the so-called “tails”. This sensitivity increases dramatically as the reliability of a given component becomes very high. A final practical concern is the ability of the structural analyst to predict before service the actual service conditions to be encountered. Readily available data, as the duty cycle supplied in the specification, do not provide thorough guidance on such things as the risk of human errors in fabrication or operation, or the likelihood and threat of remote natural catastrophes. The failure probability of highly reliable components is especially sensitive to such considerations.

In application of structural reliability methods, cost-effective use depends on suiting the level of detail of the analysis to the specific need. For example, trade-off studies are adequately supported by trends developed on a consistent basis rather than on the absolute magnitude of the results. As another example, highly conservative analyses on an approximate basis may adequately establish meeting reliability objectives. Finally, analytical sophistication is wasteful unless the quality of the input data warrants such refined treatment.

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1. Introduction

1.1 Background

Reliability assessments of critical, often non-replaceable, large mechanical system components typical of nuclear power plants are receiving more emphasis in recent years as evidenced by increased attention to such reliability-related techniques as failure modes and effects analyses, fault tree analyses, common cause failure analyses, and single point failure analyses. These techniques are ordinarily applied at the outset in a qualitative manner, tracing the causal sequences of potential component failure. The result of these analyses serve as the beginning point and framework for more sophisticated probabilistic structural reliability analyses which have the objective of calculating the probability of failure (that is, unreliability) of the system or component in question.

Probabilistic structural reliability analysis is a less developed technical activity than the types of conventional engineering analysis (e.g., thermo-hydraulic analysis, stress analysis, etc.) with which the profession is familiar. It is an unaccustomed amalgam of stress analysis and mathematical statistics. As with any maturing methodology, certain developmental activities suggest themselves. The objectives of this developmental work are:

* Improved validity in the guidance provided by structural reliability results;
* Increase cost-effectiveness in application of the methodology;
* Broader acceptance as another mode of engineering analysis to assist in equipment design decisions.

1.2 Objectives

The objective of this discussion is to provide some realistic perspective on structural reliability state of the art. This relatively new technique is being applied to increasingly complicated loading situations. Some examples are documented in References [1-5].

The purpose of this paper is to summarize conclusions drawn from experience with industrial applications of structural reliability analysis. Items covered include the more important considerations in applying structural reliability methodology, in interpreting the results, and in choosing the level of detail in applying the methods to engineering problems. Some specific examples are also presented.

2. Discussion

2.1 Basic Principles/Concept of Inherent Unreliability

The basic principle of probabilistic structural reliability analysis is that the "overlap" of the statistical distributions for applied load and component strength (defined by a selected failure criterion) represents statistically possible situations of component strength being exceeded by the load. Therefore, the overlap is a measure of the probability that the component will fail. A representative sketch of this behavior is shown in Figure 1. In usual terminology the applied load condition is often referred to as the "stress" while the appropriate failure condition is referred to as the "strength". Thus,
the terminology "stress-strength overlap" or "stress-strength interference" is used. Details of the technique are available in References [6 and 7].

Computations of the unreliability are most readily performed if the two distributions, \( f(s) \) and \( f(\bar{s}) \), in Figure 1 are assumed to be normal. For normal distributions an exact closed form analytical solution exists to compute the probability of failure [7]. By introducing the standard-normalized variable \( Z \), with

\[
Z = \frac{\bar{s} - \bar{s}}{\sigma_s}
\]

(1)

a particular value of \( Z \) can be used to determine the probability of failure. This unique value is

\[
Z = \frac{\bar{s} - \bar{s}}{\sqrt{\sigma_s^2 + \sigma_s^2}}
\]

(2)

where \( \bar{s} \) and \( \bar{s} \) are the mean values of strength and stress, respectively, and \( \sigma_s \) and \( \sigma_s \) are the corresponding standard deviations. Tables are readily available over various intervals to determine the probability of failure once \( Z \) is determined. Failure, of course, corresponds to negative values of \( \tau \), i.e., \( s > \bar{s} \), and the failure probability corresponds to the area under the difference distribution, \( \tau \), below zero. The nature of the tables is such that as \( Z \) increases, the probability of failure decreases exponentially. While there is a calculational advantage for normal distributions, tractable means exist for working with any distribution for stress and strength utilizing numerical integration or gaming theory.

It is noted that in the formulation above, the distribution curves are not truncated but are allowed to extend from \(-\infty \) to \(+\infty \). The negative values of the distributions conflict in most cases with what is physically possible. In particular, a probability of failure exists for the no-load condition \( (s = \sigma_s = 0) \). However, for high reliability components, the customary case of interest, this discrepancy has no meaningful effect on the results. In those cases where this approximation is not acceptable, truncated distributions and another mode of computation is followed, as with non-normal distributions.

It has been found to be frequently useful to calculate what the authors will call the "inherent unreliability". The inherent unreliability relates to the fictitious situation of "probability of failure under no load". Specifically, the lowest unreliability possible, called the inherent unreliability, is associated with a value of \( Z \):

\[
Z = \frac{\bar{s}}{\sigma_s}
\]

(3)

Low probability failure events are characterized by values of \( Z \) which are, in general, close to that given by Eq. 3. That is, a low probability event is one in which

\[
\frac{\bar{s}}{\sigma_s} = \frac{\bar{s} - \bar{s}}{\sigma_s^2 + \sigma_s^2} \leq 0
\]

(4)
The concept of Inherent reliability can prove to be useful. A value of calculated unreliability which is close to the inherent unreliability implies a loading which represents almost no threat to the component's integrity, even considering the statistical uncertainties in the stress and strength. This kind of result can also be a signal that the wrong failure mode is being addressed, that other modes must be more likely.

An example is provided of sorting out significant failure modes. Should a preliminary analysis yield the following:

<table>
<thead>
<tr>
<th>Load</th>
<th>Unreliability</th>
<th>Inherent Unreliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal Fatigue</td>
<td>$1 \times 10^{-7}$</td>
<td>$1 \times 10^{-11}$</td>
</tr>
<tr>
<td>Creep</td>
<td>$5 \times 10^{-6}$</td>
<td>$1 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

one quickly sees that the structure under load will be more sensitive to changes in the fatigue loading than to comparable changes in creep loads (or time). Obviously if further evaluation is required, the fatigue situation should be given priority consideration.

2.2 Solid Mechanics Considerations

The total analysis under discussion is based on sound application of solid mechanics principles. The application of solid mechanics methodology to probabilistic structural reliability analysis includes failure modeling and stress analysis. The failure model will be discussed first.

2.2.1 Failure Criterion Selection

Not to be overlooked is the fact that structural reliability analysis is directed toward the probability of failure of a component, not the probability of exceeding a legislated loading limit(s). That is to say, the analyst does not have the luxury of assessing against packaged limits provided to him in a mandatory code or structural standard. The suggested need in this regard is increased emphasis in the classroom and laboratory on theories of failure. What are ordinarily presented as "theories of failure" for ductile materials are "theories of yielding". Moderate permanent deformation represents failure in only a minority of components, that is, those situations where a function or other interface requirement requires rigid retention of as-built dimensions. True failure criteria must be established beyond simply exceeding the yield stress.

In consideration of failure models several practical things should be kept in mind. First, the available stress analysis most often available is the stress report oriented toward meeting code requirements (e.g., in the United States, Section III of the ASME Code for Class 1 Nuclear Components). A practical failure model approach would utilize these results to the greatest extent possible. For example, if fatigue results for high temperature operation are under consideration, then effective strain range may be a judicious choice for a failure criterion since the ASME Code analysis employs strain range/cycles to failure data in calculating fatigue life fractions. Furthermore, statistical convenience derives from the choice of strain range for a specified number of cycles as a failure criterion. The number of cycles in an equipment specification represents a discrete upper limit in most
cases and it is difficult to find agreement on the statistics to be used for such a number. Quite often the number of specified cycles falls within the range of the available experimental results thereby avoiding the need to extrapolate.

Generally speaking, a failure criterion based on strain rather than stress is preferable for reasonably ductile materials. A stress criterion is more appropriate to the evaluation of brittle materials by such methods as linear elastic fracture mechanics. While more advanced concepts of strain energy appear to have great potential (e.g., the J-integral approach [8] or the more general equivalent energy approach [9], both of which are applicable to flawed structures only), research along these lines is limited and a more basic approach is generally required. Within the authors' experience, the effective strain,

$$c_{\text{eff}} = \sqrt{\frac{2}{3} \left( (\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_1 - \varepsilon_3)^2 \right)}$$

(5)

where $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$, are principal strains, is the most plausible choice. For example, the analyses discussed in Reference [4] largely employ strain-based criteria.

The failure criterion selection is a subject too broad for comprehensive treatment in this discussion. In summary, the structural reliability analyst must circumspectly select a criterion of actual failure, taking into account the material, function, and service/environment of the component being analyzed. A broader discussion of failure criteria selection for numerous loading conditions can be found in Reference [10].

2.2.2 Stress Analysis

The other element in the solid mechanics realm is the deterministic stress analysis necessary to support the probabilistic analysis. Most commonly, the results of an elastic stress analysis are available. In most engineering applications, the elastic analysis is sufficient to satisfy Code requirements, especially in the case of thermal stress conditions. However, structural reliability analysis with its emphasis on actual approach to failure requires true stresses and strains, results not available from an elastic analysis for cases of significant non-linear response.

Lacking elastic-plastic results, practical considerations require that reasonably simple procedures be developed for determining elastic-plastic strains from elastically calculated stresses for both the strain-induced and load-induced situations.

Procedures for determining inelastic strain correction factors have been developed for thermal loadings [11]. Essentially, elastic and elastic-plastic computer runs were made for problems of this class. The elastically calculated effective stress was divided by the yield stress and plotted against the ratio of the elastic-plastic effective strain to the effective elastic strain. The ratio of actual to elastically calculated strain is called the inelastic strain correction factor, $K_e$ (see Figure 2).

Having determined $K_e$ for a given situation, it is then multiplied by the elastically calculated effective strain. This strain which may be calculated by Hooke's Law or, as an
alternative by

$$c_{\text{eff}} = \frac{2(1 + \nu)}{3E} s_{\text{eff}}$$

where $\nu$ is Poisson's ratio and $E$ is the modulus of elasticity.

The conversion of load-controlled (versus thermally induced) elastically calculated stresses which exceed yield to true effective elastic-plastic strains is not a well-established procedure. Some guidance is provided in Code Case 1592-7 [12].

However, an applicable study is currently in progress. Using finite element procedures (ANSYS [13]), a rectangular beam was subjected to incremental pure bending loads resulting in elastic-plastic strains up to 1%. The material properties were minimum values for 304 stainless steel at 800°F [14]. In addition to pure bending, combinations of tensile loads superimposed on bending loads were analyzed. A summary plot of elastic-plastic stresses as a function of elastically calculated stresses for the beam is given in Figure 3. From this figure elastically calculated stresses can be approximately converted to elastic-plastic stresses for similar loading conditions. In turn the elastic-plastic stresses can be converted to total true elastic-plastic effective strains from the true stress-true strain curve. The need for more work in this area of providing improved ability to interpret elastically calculated stress analysis results is recognized outside the structural reliability context.

2.3 Statistically Related Needs

2.3.1 Uncertainty Definition for Stress and Strength

The uncertainties which influence the statistical distribution of the "stress" in the stress-strength overlap calculation are modeling uncertainties, software (computer program interval) uncertainties, and input property uncertainties.

The model uncertainty is related to the construction of the calculational model by the engineer. The description of the physical situation as represented by the engineering drawings, the expected loads (including intercomponent reactions), and the environment, is converted into an analytic model. The uncertainties in engineering modeling are estimated at 5% [15], interpreted as a single standard deviation.

Formal interest in quantifying software (computer program) uncertainties is a relatively recent development. Published information is outside the structural engineering framework (e.g., References [16-17]). Progress with software uncertainty definition should be monitored for production of practicably usable results.

Since the analytical techniques have traditionally been chosen conservatively and material properties chosen conservatively or in such a way as to maximize the strain, it seems reasonable to take the deterministically calculated results as mean values of stress (in a generalized sense). Determination of the uncertainty in the mean for general applications is problematic. However rational approaches to simplified estimates can be developed. For
example, a nominal thermal stress due to a step thermal gradient in a cylindrical shell away from a discontinuity may be given by

$$\frac{E\alpha}{1 - \nu} (\Delta T)$$  \hspace{1cm} (7)

where $E$ is Young's modulus, $\alpha$ is the coefficient of thermal expansion, $\nu$ is Poisson's ratio and $\Delta T$ is a calculated minimum to average wall temperature difference (step change). If $\Delta T$ is taken as a discrete number, then the standard deviation in the calculation is the standard deviation of the product $E \alpha/(1-\nu)$. If $\sigma(\int)$ denotes the standard deviation of $\int$, and assuming

$$\frac{E\alpha}{1 - \nu} \approx E_0(1 + \nu)$$  \hspace{1cm} (8)

then the formulation by Reethof [18] for products may be applied directly. It follows that

$$\frac{C \cdot E_0(1 + \nu)}{E_0(1 + \nu)} \approx \frac{C}{E} + \frac{\sigma}{E} + \sigma_\nu$$  \hspace{1cm} (9)

By examining the available data the value for $\frac{C \cdot E_0(1 + \nu)}{E_0(1 + \nu)}$ may well approach 25% of the mean. Thus values for the standard deviation of this fraction of the mean are not unreasonably conservative. Such large percentages, whether estimated or directly based on data, significantly influence the unreliability only when the calculated mean stress is a significant fraction of the strength. The procedure suggested by Reethof can be refined if appropriate by the application of Monte Carlo methods to propagate (combine) the individual input parameter uncertainties.

A final suggestion is offered for estimating the uncertainties on the results of finite element analysis. If a relationship like Eq. [7] can be written which is characteristic of the situation being computed by a complex finite element model, the process described in the preceding paragraph can be followed to derive an estimated coefficient of variation (ratio of standard deviation to the mean) for the finite element result. The result of interest is customarily the maximum calculated stress or strain, the so-called failure-governing stress, from the finite element analysis and this is chosen to represent the mean values. The uncertainty then is a function of the directly calculated mean and a rationally estimated coefficient of variation.

The definition of uncertainty in the "strength" is principally a matter of statistical data collection. Progress in this area would be furthered by consistent publication of both a mean and standard deviation (and sample size) for all properties data, recovery of previously collected raw data which may not have been originally statistically reduced, and injection of statistical considerations in future properties test planning. The preceding is the first step in improving statistical properties availability. The next level of refinement is a more precise definition of the test specimens (such items as heat treatment and chemistry). At this same level is the assessment of potentially important lot-to-lot differences for a given material. These same concerns are clearly pertinent to the aspects of "stress" uncertainties which depend on material properties.
2.4 Statistical Methodology

The subjects discussed in this section relate primarily to highly reliable components, presumed to be the components of prime interest.

The probabilistic approach does require further development in the treatment of the overlap of the extreme values of the probability density function—the so-called "tails". The sensitivity of the reliability calculations to the behavior in the tails increases dramatically as the reliability of a component becomes very high. The analyst is often forced to choose between sophisticated time-consuming methods which may be difficult to justify and simple engineering procedures which may be difficult to defend. Wirsching [19] has compared some of the more typical distributions which are summarized in Figure 4. It is seen that the assumption of normality is reasonably conservative for engineering application. Nonetheless, precise definition of the extreme portions of the stress and strength statistical distributions is essential if absolute values of unreliability are to be generated.

Even with precision in the definition of tail shapes, calculation of the overlap can become costly for non-normal distributions. For unreliabilities, say of the order of $10^{-6}$, even simpler problems require an unwieldy number of Monte Carlo trials to develop meaningful results. Numerical integration nomograms or tables for overlap combinations of other (non-normal) statistical distributions would be useful if creatively formulated. Importance sampling has been used for structural reliability analyses [2]. However, more examples of practical applications are needed to equip the working engineer with a better sense of utilizing this concept and appreciating its limitations.

Finally, there is the matter of experimental verification of the methodology. To calculate a probability of failure, the assumption is made that the overlap of the stress and strength is not zero. If is a typical engineering expectation that such a non-zero overlap can be demonstrated by test or by service. For large components with a critical safety function, one such failure occurrence already suggests totally too great an unreliability. For high reliability components, the analysis must be defended in a piecemeal basis (valid basic principles, adequate properties data, etc.) rather than on the basis of experimental verification of the total solution. Thorough-going application of the methodology to some carefully selected case histories of actual equipment service or testing would be a welcome addition to the technical literature.

2.5 Anticipating Service Conditions

The primary and obvious source of guidance for a component's service loads is the equipment specification duty cycle description. Some alternatives to blind reliance on the equipment specification are proposed. A pessimistic aspect of blanket acceptance of specified loads is that many duty cycle events are implied to have a likelihood of occurrence of unity. Assigning reasonable probabilities to the less likely specified duty cycle events would support more realistic structural reliability predictions. On the other hand, exclusive use of specification loads can be overly optimistic. While remote natural catastrophes can be
the major threat, relatively speaking, to highly reliable components, emphasis here is on other potential loads not described in the specification. Reference is made primarily to interaction loads with adjacent components. An effective source of such loads is the Failure Modes and Effects Analysis on the component being analysed and on the interfacing components.

Another point under the broad category of anticipated service conditions is the matter of human error. Discussing human errors under this topic is done for convenience, recognizing that human error risk begins in the design process and extends through all phases of the hardware through operation over the plant lifetime. Coping with human error remains an elusive problem, not only in all aspects of quantitative and qualitative reliability activities but also in conventional engineering practice. The current capability of treating human error probability is summarized in Reference [20]. The task is scarcely complete; creative and persistent effort to improve prediction capability is an outstanding need.

2.6 Cost-Effective Application

Probabilistic analyses historically have not been uniformly planned for optimum expenditure of engineering resources. Simple rules for intelligent planning of probabilistic work defy formulation; good planning depends on developing a perspective by experience with the methodology. However, the two general guides can be indicated, namely, the end use of the analysis and the quality of the input data.

Example end uses of structural reliability analyses are in trade-off studies (assessing the relative reliabilities of alternative design concepts), sensitivity studies (assessing the relative influence of individual input parameters), preliminary assessment (corresponding to preliminary engineering analyses in the early stages of a design), and specific later assessments. In trade-off studies and sensitivity analyses, the need is more for a reasonably consistent treatment than for precision. In fact, at the current stage of development of probabilistic structural reliability analysis, these seem to be the most fruitful applications. For preliminary assessments, gross modeling is usually adequate, so long as the elements of the analysis are all defensible as conservative. As with other engineering analysis performed in support of preliminary design, an economical and timely indication of adequacy is the prime objective. Specific later assessments call for more detail. Present limitations on probabilistic methodology prevent, in the authors' judgment, reliability predictions which can be interpreted literally. Hand in glove with this constraint, it is unrealistic to set numerical reliability requirements on power industry mechanical components generally. However, design objectives related to reliability can be useful. Assessment of a component's approach to (or departure from) that objective and assessment of a component's reliability growth can be usefully supported by moderately detailed probabilistic analyses (usually in conjunction with qualitative reliability analysis, deterministic analysis, and test results).

The wastefulness of probabilistic analyses whose level of complication far exceeds the quality of the input data is obvious. The potential for complexity in structural reliability
analysis, which can encompass the most refined techniques in use in modern stress analysis overlaid with the intricacies of mathematical statistics, is virtually limitless. Disciplined planning of the probabilistic work is essential to conserving limited engineering resources and promoting respect of the new technology within the design engineering function.

2.7 Interpretation of Results

The ultimate objective of structural reliability and, for that matter, all engineering analysis is to support design decisions. Prudent utilization of probabilistic results requires, first, an overall perspective on the capabilities and limitations of the methodology and, second, an understanding of the specific nature of a given probabilistic analysis. One purpose of this report is to contribute to that general perspective. Providing perspective on the specific analysis is the responsibility of the analyst, principally by means of a clear presentation of his assumptions and input.

The need for perspective in interpreting analytic results is not peculiar to probabilistic work but applies likewise to deterministic mainline design analysis. For example, few structural analyses include the effect of component residual stresses. This is because data limitations regarding residual stress magnitude (and distribution) and practical analysis limitations regarding incorporation of residual stress results impede incorporation of these loads. Non-linear structural effects generally are handled on a best effort basis, precise analysis being blocked out again by a combination of basic test data needs and practical computational constraints. Nevertheless, design decisions are influenced by these results as interpreted by knowledgeable cognizant design staff.

3.0 Conclusions

This discussion of some practical considerations regarding structural reliability analysis has been intended to:

* Indicate areas of developmental needs;
* Guide intelligent application; and
* Assist with proper interpretation and utilization in the design decision-making process.

Discussions of this nature tend to deal primarily with the limitations of a technique rather than with its capability. The capability is an important one, simply stated as a mode of analysis which accounts explicitly for uncertainties in the parameters affecting component structural adequacy. This capability is imperfect, as are the capabilities of more conventional engineering analysis methods, more so because it is a relatively new technique. However, structural reliability methodology warrants development because of its capability to integrate uncertainties in the elements of the analysis in a manner consistent with established statistical analysis principles. The fact that the methodology seeks to model physical situations by representing the statistical quality which is characteristic of natural phenomena in its own best justification.
References


Figure 1. Stress-Strength Overlap Schematic

Figure 2. Inelastic Strain Correction Factor For Cylinders Under Thermal Loading

Figure 3. Elastically Calculated Versus Actual Stresses, Beam in Bending and Tension.

Figure 4. Lower Tail Probabilities For Common Statistical Models