CONFIDENCE BOUNDS FOR FAILURE PROBABILITY IN MECHANICAL SYSTEMS

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SUMMARY

The purpose of this paper is to outline a procedure for obtaining confidence bounds to mechanical system reliability based on test data. An important problem facing the nuclear reactor designer is the demonstration of design reliability through a test program. To fabricate and test a complete reactor or a mechanical subsystem is in most cases too expensive and/or time consuming. Several tests must be planned to evaluate all environmental factors. One approach which can be used is over stress testing that is planned to produce failures. The failure data can be used to establish the system reliability within a given confidence bound.

The primary control rod system of a nuclear reactor will be used to illustrate a method for the use of over stress test data to confirm reliability. Engineering analysis of current data indicate that the irradiation and thermal gradients within the control rod system will cause swelling and bowing. This swelling and bowing is significant to system reliability because the control rod system has an inner duct which must move axially through a bowed outer duct for reliable performance. Because the clearance between the ducts is limited by design considerations, swelling and bowing could result in reduced clearances between the ducts and lead to additional drag forces. The drag forces for various bow configurations can be measured during a test program and a distribution of drag forces developed for each bow configuration. The force available to move the inner duct is known from the design specifications. To demonstrate acceptable scram reliability the designer must define a confidence bound to the failure probability determined by the overlap between the available insertion force and the drag force measured during the tests. For a fixed bow condition, several measurements of the drag force will be made. Thereafter, each measurement will be converted to a corresponding "rod insertion time" by appropriate analysis. The drag force measurements will be repeated for different bow conditions, and corresponding insertion times will be calculated. If the insertion time exceeds a given value, the design requirements are exceeded (in effect equivalent to failure). The objective for the analysis is to obtain estimates of (a) the probability of failure in each different bow condition under which the tests are made, and (b) the overall probability of failure, taking into account the frequencies with which different tested bow conditions are expected to occur. Since we would like to reflect that these estimates are based on a limited number of observations and are subject to sampling fluctuation, we would like to compute an upper bound to the probability of failure at a specified confidence level.

This paper outlines procedures for obtaining confidence bounds to failure probability corresponding to cases (a) and (b) above. To obtain these formulas, it is assumed that the rod insertion times are normally distributed. The formulas for case (a) are well-known and use the non-central \( t \) distribution for obtaining the \( K \)-factor in computing tolerance limits. The formulas for case (b) were derived using the asymptotic properties of the "maximum likelihood" estimates. Numerical examples are given demonstrating the application of these formulas.