PROBABILITY EVALUATION FOR
DYNAMIC RESPONSE COMBINATIONS

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SUMMARY

Individual transient structural dynamic responses are required to be combined for extreme load nuclear power plant design. This combination is associated with individual events that are postulated to occur simultaneously.

The heuristic formula called the SRSS rule has been used for combining peak dynamic primary structural responses to obtain a "probable" response combination. The meaning of "probable" has not been quantitatively defined, and in addition, the SRSS rule has a problem of interpretation when the frequencies of the individual time history responses are different. When the individual responses to be combined have significantly different frequencies, and a cyclic repetition of the higher frequency response occurs, then a "probable" response combination rule may require the addition of the peak individual responses. The simple SRSS rule does not reflect this and can therefore be applied improperly. The conditions under which the SRSS rule is valid as well as what rules should be used when SRSS is not valid are subjects addressed in this paper.

It is shown that under certain conditions the SRSS rule takes on an invariant conditional probability level for the ideal case where a single degree of freedom system without damping is acted upon by two randomly phased impulse type force functions. Since the response of a single degree of freedom system to an impulse is a pure sinusoid at the single natural frequency, the question of differing frequency content does not enter this particular physical problem. Hence it is possible to quantitatively calibrate the SRSS rule for this type of application in probability terms. It was found that the conditional combined max and min response probability levels of the calibration problem associated with the \( \pm \text{SRSS} \) values was at a median or 50 percentile level. This fact was used to introduce a more general dynamics response combination rule designated the SRSSE (SRSS equivalent) rule. The SRSSE rule uses the same max and min conditional probability level which was found in the calibration of the SRSS rule and therefore always has a quantitative probability meaning. The so defined SRSSE rule produces the desired addition of peak responses when individual responses have significantly differing frequencies, and reduces to the SRSS rule when applied to the calibration problem. The SRSSE rule can be applied to any time history responses regardless of frequency content and for any number of transient time histories which are to be combined. The probabilistic definition of the SRSSE rule makes possible a simple design criteria by which to rationally classify extreme load response combinations for design of components under industry standards such as ASME III. The suggested design criteria consist of ensuring that the total probability (as distinguished from conditional) associated with the response combination is consistent with the various stress criteria of the appropriate industry standards.

The probability concepts are illustrated through an example in which the important influence of event dependence is also evaluated. The simplified example was chosen to illustrate the major ideas in a problem which can be solved in closed form.
1. Introduction

A question concerning the validity of the SRSS rule as applied to combining dynamic responses came up during the past three years from the U.S. Nuclear Regulatory Commission. The original question concerned the case where responses might have differing frequency content which could invalidate the SRSS rule. Since the SRSS rule is basically a heuristic way of combining peak responses which are randomly phased to produce a "probable" response combination, it was decided that further insight into its validity or possible limitations can come through a quantification of the probability aspects. Therefore this paper addresses the question of quantifying both the SRSS rule, and the general response which results when dynamic responses are combined with random phasing.

2. Problem Description

When two events A and B cause a single component to have peak dynamic responses $R_A$ and $R_B$ respectively, and if the two events can occur simultaneously with a significant probability, and we do not wish to conservatively add the peaks, then we need a design rule for combining the responses. In this discussion we shall assume that the dynamic system is linear so that superposition is valid. Consider the total design lifetime $t_d$ of the component, with both events and responses occurring as shown in Figure 1. The collinear response is that response occurring at the same position in the structure and in the same direction with the same units. In this figure we show that each response has as duration $t_A$ and $t_B$. In some cases the occurrence of the events A and B can be considered independent. This means that if $P(A)$ is the probability of A occurring during $t_d$ and $P(B)$ is the probability of B occurring during $t_d$, then the probability of both occurring during $t_d$ is $P(A,B)$.

$$P(A,B) = P(A) \cdot P(B) \quad (\text{If Independent})$$

Notice that the event combination A, B does not necessarily imply that the events occur simultaneously, it only implies that both events occurred during $t_d$. We shall designate the event that response durations $t_A$ and $t_B$ overlap each other so that the events are simultaneous as $A, B, \text{simul}$.

We will illustrate the important situation where the events A and B should not be assumed independent. An example of this is the case where seismic response is postulated to cause or trigger a loss of coolant pipe rupture response. The procedures required to compute dependent probabilities require individual consideration of each case. An example analysis of simultaneous seismic and pipe rupture event probability computation is contained in reference 3. The application of dependent event analysis to an example problem solution is contained in Section 4.

3. Meaning of SRSS and SRSSE

Before embarking on a sample problem solution, we shall digress to an analysis of sinusoidal response combinations in order that a quantitative probability meaning can be associated with the SRSS rule. Dynamic linear systems respond principally as time history sinusoids or combinations of sinusoids. Therefore we will begin by consideration of two equal frequency idealized sinusoids corresponding to events A and B of Figure 1. The frequencies are chosen as equal here because it is the only situation where the SRSS rule leads to an invariant probability level. We will assume that the phase relationship between the peaks
and $R_B$ is unknown and therefore can be described by a random variable.

In order to better fix ideas, it is helpful to consider a real physical situation where the response combination problem is clearly defined. Such a problem is illustrated in Figure 2, where a single degree of freedom system (without damping) is acted upon by two pairs of impulse forcing functions $F_1$ and $F_2$ which are delayed from each other by a random quantity $\tau$. The responses will be purely sinusoidal as illustrated in Figure 2(d). If the random phasing is assumed uniform from 0 to $\pi$ as shown in the probability density function of Figure 2(c), then all possible combinations of maximum response will be considered on an equally weighted basis. This contrived physical problem should be thought of as merely a calibration problem so that a probability meaning can be associated with the SRSS rule. If such a problem is not defined, the SRSS value will in general have no unique probability meaning. The x time scales of Figure 2 are dimensionless.

From a structural or component design viewpoint we are interested in both the maximum or minimum values of the sum $R_1 + R_2$. In this discussion we are focusing first on the summed maximum response because the calibration problem provides some complication relative to the minimum summed response if $|R_A - R_B| < R_A$. It is shown in reference 3 that the PDF of the maximum sum $R_1 + R_2$ is given by

$$f_{\text{max}}(R_{AB}) = \frac{2R_{AB}}{\pi \left[ 2R_{AB}^2 (R_A^2 + R_B^2) - (R_A^2 - R_B^2)^2 - R_{AB}^4 \right]^{1/2}}$$

where $R_{AB}$ is the summed random variable $\left[ \Sigma (R_1 + R_2) \right]$ maximum.

Equation 2 is plotted as shown in Figure 3.

This PDF is non-normal and has two singular points. This fact is important when integrating $f_{\text{max}}(R_{AB})$ to obtain probabilities for $R_{AB}$ exceeding certain values. It can be shown that the total area under $f_{\text{max}}(R_{AB})$ is 1.0 as it must be for a valid PDF.

We wish to determine the probability associated with the +SRSS value. It is shown in Reference 3 that

$$\int_{-\infty}^{\infty} \frac{(R_A + R_B)}{\sqrt{R_A^2 + R_B^2}} f_{\text{max}} \, dR_{AB} = 1/2$$

This means that the +SRSS value corresponds to a 50% conditional probability that the maximum sum $R_1 + R_2 > \sqrt{R_A^2 + R_B^2}$. The term conditional is inserted here to emphasize that this probability associated with SRSS is conditional that the events A and B have occurred simultaneously.

In a similar manner, and by redefining the calibration problem, the PDF for a minimum sum can be shown to provide a 50% conditional probability that the minimum sum $R_1 + R_2 < -\sqrt{R_A^2 + R_B^2}$.

From this discussion we have established a quantitative meaning to the term 'probable' conditional SRSS response combination when applied to the ideal case of two equal frequency sinusoids with random phasing.

For the more general case with individual time history responses which are not single frequency sinusoids, and where the dynamic system is a multiple degree of freedom system, we can define a logical extension to the SRSS combination rule where we utilize the same conditional probability levels which occurred in the case of two equal frequency sinusoids. If
for example, we have N events which have time history responses \( R_1, R_2, \ldots, R_N \), and random phasing (it is not necessary that the PDF of the phasing be uniform as shown in Figure 2(c) if the problem indicates other distributions) we can perform the same type probability analysis to determine the maximum and minimum sums \( R_1 + R_2 + \ldots + R_N = \sigma_{R_N} \). From a component structural design viewpoint, we are interested in the maximum and minimum probability distributions. The max. and min. PDF's of \( \sigma_{R_N} \) will have some arbitrary shapes which can be computed and will look somewhat like Figure 4.

There is no reason to expect symmetry of the max. and min. distributions for totally arbitrary time histories. We can designate points b and e of Figure 4 as the equivalent tSRSS points, but call them SRSSSE, max and SRSSSE, min points. We can then define points b and e as follows:

\[
\begin{align*}
\int_b^c (\sigma_{R_N}) \, d (\sigma_{R_N}) &= \frac{1}{2} \\
\int_d^e (\sigma_{R_N}) \, d (\sigma_{R_N}) &= \frac{1}{2}
\end{align*}
\]

The exceedance probability is 0.5 which is the same exceedance probability as the SRSS value had for the calibration sinusoids. Thus we have defined a new SRSSSE response combination rule which can handle any number of arbitrary time history responses with random phasing, and which is equivalent to the SRSS rule for two ideal sinusoids. The computation of points b and e of Figure 4 generally will require a numerical procedure. Two computer programs, DYNSUM and STAGAR, have been developed for that purpose.

The total response probabilities associated with SRSSSE values as opposed to the conditional response probabilities can be computed as

\[
\begin{align*}
P(\text{max of } \sigma_{R_N} > \text{SRSSSE, max}) &= (0.5) \, P(A_1, A_2, \ldots, A_N, \text{ simul}) \\
P(\text{min of } \sigma_{R_N} < \text{SRSSSE, min}) &= (0.5) \, P(A_1, A_2, \ldots, A_N, \text{ simul})
\end{align*}
\]

where \( P(A_1, A_2, \ldots, A_N, \text{ simul}) \) is the probability that all N events occur during the design lifetime and that they all occur simultaneously.

The probabilities evaluated according to equations (6) and (7) should be sufficiently small so as to be compatible with applicable design criteria (such as upset, emergency, or fault of ASME III) when the SRSSSE rule is used. This result is very important and can provide the basis for a criteria which assures safe use of SRSS or SRSSSE rule for response combination.

Since detailed probability standards have not yet been adopted by the nuclear industry for all situations, the following Table 1 is a suggested criteria for reactor event classification.

This criteria basically says that if events are classified according to the suggested table, the corresponding margins of ASME III will be assumed to adequately account for the exceedance probability associated with the use of the SRSSSE rule. More work is needed to verify that this specific table is appropriate. Such work is currently being carried out.
4. Probability of Events and Responses - An Example

As can be observed from equations (6) and (7), there is a difference between the probability of an event and the probability of a response which is associated with the event. To clarify concepts, it is helpful to discuss a specific numerical example.

Suppose we wish to consider the case where event A is a seismic event which produces a peak response $R_A$ on a particular component. We will assume for the purposes of this example that a LOCA event (Loss of Coolant Accident) B can be triggered by the seismic event (implying dependence) and the collinear peak response on the component due to $B$ is $R_B$. In this example we will assume that there is no uncertainty on $R_A$ or $R_B$ if events A or B occur. Even though the probability of this combined event may be small, our intention is to illustrate a reasonable way to evaluate the combined response probability where dependence is important. We will further assume that the response of each is exactly like the calibration sinusoids as illustrated in Figure 2 with equal frequencies and an unknown phase shift $\tau$. This assumption is made only so that the problem can be illustrated in closed form, it is not essential to the procedure. The following arbitrary values will be used:

\[
P(A) = 0.1 \quad \text{seismic SSE event} \quad (8)
\]

\[
P(B) = 0.2 \quad \text{LOCA event} \quad (9)
\]

These values are not intended to be realistic, they are chosen for the sake of numerical simplicity. Actual values would be at least an order of magnitude smaller. Also let:

\[
R_A = 4 \quad (10)
\]

\[
R_B = 3
\]

Since these events are dependent, we will use the specific formulas of reference 3 to find the probability of their simultaneous occurrence.

\[
P(A, B, \text{Simul}) = \frac{P(A) \cdot P(B)}{P(A) + 2.4} \quad (11)
\]

The development of equation (11) from reference 3 illustrated that it is reasonable to assume that the phase shift distribution between events A and B is uniform. We shall now examine the probability levels associated with any response or response combination. In particular we are interested in the probability associated with the maximum and minimum values of the SRSS response $\pm \sqrt{R_A^2 + R_B^2} = \pm 5$. The Venn diagram of Figure 5 will be helpful.

First we will compute the areas (probability) of each of the mutually exclusive sub regions of Figure 5. From equation (11) we have

\[
P(A, B, \text{simul}) = .008 \quad (12)
\]

From the sum of equations (16) and (17) of reference 3 we obtain $P(A, B, \text{not simul})$

\[
P(A, B, \text{not simul}) = \frac{2 \cdot P(A, B, \text{simul}) + P(A, B, \text{simul})}{1.25} = 0.0192 \quad (13)
\]

The total region of $P(A, B)$ is the sum of (12) and (13)

\[
P(A, B) = .0272 \quad (14)
\]
We can now construct the total probability of having a maximum response at exactly $R_{AB} = 4$.

This is the area

$$P(R_{AB} = 4) = P(A) - P(A, B, simul)$$

$$P(R_{AB} = 4) = 0.1 - 0.008 = 0.092$$  \hspace{1cm} (15)$$

The total probability of having a maximum response at exactly $R_{AB} = 3$ is

$$P(R_{AB} = 3) = P(B) - P(A, B) = 0.1728$$  \hspace{1cm} (16)$$

By the use of the distribution from equation (2) (Section 3) we can find the conditional probability that the combined response lies anywhere in the region between $R_A - R_B = 1$ to $R_A + R_B = 7$. Table 2 was constructed in this way.

The last column of Table 2 is the total probability of $R_{AB}$ falling in the stated region due to simultaneous occurrence only. In order to complete the probabilistic evaluation of all possible maximum responses, we must add to the results of Table 2 those cases where the response is zero, exactly 3, and exactly 4. This computation is carried out in Table 3, and the results can be presented in either one of two ways, a complimentary cumulative distribution function (CCDF), or a cumulative distribution function (CDF).

In Figure 6 the total complimentary cumulative function is shown. We can see by a study of the results shown in Figure 6 that the response probabilities must be distinguished from the associated event probabilities. We can also see in clear perspective the probabilistic significance associated with the SRSS value and the importance of identifying the difference between the conditional response probability (0.5) and the total response probability (0.004).

In this particular illustrative example we have assumed that there are no responses with high probability and therefore there is a large probability (0.7272) that the maximum response is exactly zero. No conceptual complication arises if there had been highly probable responses to be combined with the lower probability responses, this would simply change the shape of Figure 6.

The acceptability of SRSS values (or its equivalent SRSSE values) depends on the acceptability of the total probabilities such as shown in Figure 6 and its compatibility with the code stress criteria such as upset, emergency, or fault.

5. Conclusion

The SRSS rule for combining randomly phased dynamic responses has no consistent probability meaning except under the restrictive conditions used in the so called calibration problem. The calibration problem establishes a 50% conditional response probability level with the SRSS value. A new rule is introduced called the SRSSE rule which corrects this deficiency. The SRSSE rule also ensures that when slowly varying dynamic responses are combined with repetitions of higher frequency response that the peak responses will be effectively added. The question of whether the SRSS or SRSSE values are appropriate in a given situation can be answered by requiring that the associated total response exceedance probability be sufficiently small so as to be compatible with margins in the design codes.
6. **Acknowledgment**

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**References**


**TABLE I**

<table>
<thead>
<tr>
<th>Probability of Event</th>
<th>ASME III Code Stress Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.0 &gt; P(A_1, A_2, \ldots A_N, \text{simul}) &gt; 10^{-1}$</td>
<td>normal or upset</td>
</tr>
<tr>
<td>$10^{-1} &gt; P(A_1, A_2, \ldots A_N, \text{simul}) &gt; 10^{-3}$</td>
<td>emergency</td>
</tr>
<tr>
<td>$10^{-3} &gt; P(A_1, A_2, \ldots A_N, \text{simul}) &gt; 10^{-6}$</td>
<td>fault</td>
</tr>
</tbody>
</table>

**TABLE II**

| a | b | $P(a \leq R_{AB} \leq b | A, B \text{simul})$ | $\times .008$ |
|---|---|----------------------|----------------|
| 1 | 7 | 1.0000               | .00800         |
| 2 | 7 | 0.8392               | .00671         |
| 3 | 7 | 0.7323               | .00586         |
| 4 | 7 | 0.6224               | .00498         |
| 5 | 7 | 0.5000               | .00400         |
| 6 | 7 | 0.3484               | .00279         |
| 7 | 7 | 0.0000               | .00000         |
### TABLE III
Total Response Probability Computation

<table>
<thead>
<tr>
<th></th>
<th>R_{AB}</th>
<th>R_{B^3}</th>
<th>R_{A^4}</th>
<th>P(R_{max} &gt; R)</th>
<th>P(R_{max} \leq R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(R_{max} &gt; 0)</td>
<td>.00800</td>
<td>.1728</td>
<td>.092</td>
<td>.27280</td>
<td>.72720</td>
</tr>
<tr>
<td>P(R_{max} &gt; 1)</td>
<td>.00800</td>
<td>.1728</td>
<td>.092</td>
<td>.27280</td>
<td>.72720</td>
</tr>
<tr>
<td>P(R_{max} &gt; 2)</td>
<td>.00671</td>
<td>.1728</td>
<td>.092</td>
<td>.27151</td>
<td>.72849</td>
</tr>
<tr>
<td>P(R_{max} &gt; 3)</td>
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<td>.1728</td>
<td>.092</td>
<td>.27066</td>
<td>.72934</td>
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<tr>
<td>P(R_{max} &gt; 4)</td>
<td>.00498</td>
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<td>.092</td>
<td>.09786</td>
<td>.90214</td>
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<tr>
<td>P(R_{max} &gt; 5)</td>
<td>.00400</td>
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<td>0</td>
<td>.09698</td>
<td>.90302</td>
</tr>
<tr>
<td>P(R_{max} &gt; 6)</td>
<td>.00279</td>
<td>0</td>
<td>0</td>
<td>.00498</td>
<td>.99502</td>
</tr>
<tr>
<td>P(R_{max} &gt; 7)</td>
<td>.00000</td>
<td>0</td>
<td>0</td>
<td>.00000</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

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![Figure 1](image-url)

**Figure 1**
Figure 2 Calibration Problem for SRSS
Figure 3  Probability Density Function for Maximum Response

Figure 4  Probability Density Functions for Generalized Response Sums

This portion of the area P(A, B) corresponds to states 1 and 2 of ref. 3, Appendix B, i.e., seismic event occurs and LOCA event occurs, but not simultaneously P(A, B, not simul)

Figure 5
Figure 6  Total Complimentary Cumulative Function for Maximum Combined Response