A THEORY ON THE VIBRATIONS OF A FUEL PIN MODEL IN PARALLEL TWO-PHASE FLOW

F. HARA

Department of Mechanical Engineering,
Tokyo University of Science, Shinjuku-ku, Tokyo, Japan

SUMMARY

This paper deals with a theoretical analysis of the vibrations induced by parallel two-phase (air + water) flow in a fuel pin model system and newly concludes that the mechanism of exciting the vibrations is mainly the so-called parametric excitation due to the quasi-periodic change of the two-phase fluid virtual mass, the centrifugal force and the Coriolis' force and also the resonance of the fundamental vibration to the force caused by resultant pressure difference of the two-phase flow.

The equation of motion of the fuel pin model in the parallel two-phase flow is formulated by taking account of elastic restoring force; inertia force acting on the fuel pin model and on the virtual mass of the two-phase fluid; centrifugal force; Coriolis' force; the momentum change of the two-phase flow and damping effects. The resultant pressure difference between both the surfaces of the fuel pin model is employed into this equation as an external force. The author represents the distributions of the two-phase flow virtual mass, the damping effect and the resultant pressure difference in terms of δ-function series. By substituting these distributions into the original equation and applying modal analysis techniques, the coupled ordinary differential equations with the time variant coefficients quasi-periodically changing are obtained. Employment of the first mode approximation and replacement of those time variant coefficients with the first term of their Fourier series expansion lead to the Mathieu type equation with a periodically forced term for the first mode vibration's amplitude X_1:

$$\ddot{X}_1 + 2\zeta\omega_0 \dot{X}_1 + \omega_0^2[1 - \varepsilon \cos(\omega_s t - \varphi)] X_1 = p_0 \sin \omega_s t.$$

Here ω_0 denotes the fundamental natural frequency of the fuel pin model system; ω_s means the dominant arrival frequency of water slugs in the two-phase flow; ε is a small parameter expressing the effect of the virtual mass; p_0 is the intensity of the pressure fluctuations and ζ is the damping coefficient. Instability analysis of this Mathieu type equation shows that the system becomes dynamically unstable when the ratio $\omega_0/\omega_s = 1/2, 2/2, 3/2, \ldots$ under certain conditions with regard to the parameters ε and ζ.

This paper can show the significant conclusion that there exist two main causes to the occurrence of an extraordinarily strong vibration of a fuel pin model in parallel two-phase flow; that is, the so-called parametric excitation due to the nearly periodic change of the virtual mass and the resonance of the fundamental vibration to the quasi-periodic external force resulting from the pressure fluctuations.

1. Introduction

At the first Conference on Structural Mechanics in Reactor Technology (Berlin, 1971), S.S.Chen and M.W.Wambsganss [1] showed that the vibration of fuel rods in parallel single-phase flow was excited by pressure fluctuations of the turbulent boundary layer. G.P.Gau, P. Grillo and G.Testa [2] described experimentally that the amplitude of the vibrations of fuel bundle was linearly dependent on the flow momentum and K.D.Appelt, J.Kadlec and W.Kruger [3] insisted on the relation between the vibration and the fluctuations of flow pressure. In their papers, it is shown that the vibrations of fuel rods in parallel flow are caused by flow pressure fluctuations, that is, are considered as forced vibrations. On the other hand, P.G. Avanzini [4] mentioned the self-sustained vibration of fuel rods in parallel flow. M.P.Paidoussis [5] also developed a theory for self-sustained oscillations taking into account the forces due to bowing of fuel rods as considered by E.P.Quinn [6] and also the velocity dependent drag forces.

This paper deals with a theoretical analysis of excitation mechanisms of the vibrations of a fuel pin model in parallel air + water two-phase flow. The equation of motion of a fuel pin model in the two-phase flow is formulated taking account of elastic restoring force; inertia forces acting on the fuel pin model and the virtual mass of the two-phase fluid; Coriolis' force; centrifugal force; damping effect and the pressure fluctuations as an external force. Representing the virtual mass distribution along the fuel pin in terms of δ-function series and applying modal analysis technique to the original partial differential equation, a system of coupled ordinary differential equations is obtained for vibration modes, in which periodically time-variant coefficients appear. The first mode approximation and the employment of the first terms of Fourier expansion of the coefficients lead to the Mathieu type equation with a periodically forced term. The instability analysis of this Mathieu type equation can show that the vibrations become unstable when the ratio of the lowest natural frequency of the fuel pin model to the dominant arrival frequency of water slugs is 1/2, 2/2, 3/2, 4/2, ---, and also become unstable when the natural frequency agrees with the dominant one of the pressure fluctuations.

This paper shows the significant conclusions that the vibrations of a fuel pin in the parallel two-phase flow are mainly caused by so-called parametric excitation due to the periodic change of the virtual mass of the two-phase fluid and also by the resonance of the fuel pin vibration system to the pressure fluctuations of the two-phase flow.

2. The Equation of Motion

The system under consideration consists of a flexible flat strip of rectangular cross section, immersed in an air + water two-phase fluid flowing with uniform velocity U parallel to the x-axis, which concides with the position of rest of the strip axis. The strip is considered to be simply fixed at both ends. Let us assume the following at the formulation of the equation of lateral small motion of the strip:

(1) There is no axial tension due to viscous forces on the surface of the flat strip, but
(2) the viscousity of the two-phase fluid is taken account of as a damping effect on the vibration system, and
(3) the pressure fluctuations of the two-phase flow are considered as an external force.

When the strip is given a small lateral displacement $y(x,t)$ from the straight position, the resultant relative velocity between the strip and the fluid is given by

$$v(x,t) = \frac{\partial y}{\partial t} + U \frac{\partial y}{\partial x} . \qquad (1)$$

It is supposed that this flow has momentum m x v per unit length of the strip, where m is the virtual mass of the two-phase fluid. Then the rate of change of this momentum, which is equal to ($\partial/\partial t$ + U $\partial/\partial x$) (m x v), gives rise to an equal and opposite lateral force on the strip. Therefore the equation of motion in the lateral direction may be written as

$$EI \frac{\partial^4 y}{\partial x^4} + \left\{ \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right\} \left\{ m \left(\frac{\partial y}{\partial t} + U \frac{\partial y}{\partial x} \right) \right\} + M \frac{\partial^2 y}{\partial t^2}$$

$$(K_o + c) \frac{\partial y}{\partial t} = p , \qquad (2)$$

where
- EI : stiffness;
- m : virtual mass of the two-phase fluid per unit length;
- M : mass of the strip per unit length;
- K_o : damping coefficient due to mechanical friction at both ends of the strip;
- c : damping coefficient due to viscousity of the two-phase fluid; and
- p : external force due to pressure fluctuations of the two-phase flow per unit length of the strip.

As both the ends are simply supported, the eigen-mode functions are easily obtained as

$$\sin \frac{i\pi x}{l} , \quad (i = 1, 2, 3, ---),$$

in which l is the length of the strip. The lateral displacement y may be expanded in the following form

$$y (x,t) = \sum_{i=1} Y_i(t) \sin \frac{i\pi x}{l} . \qquad (3)$$

Substituting eq.(3) into eq.(2) and applying so-called modal analysis method [7], a system of ordinary differential equations is obtained

$$EI \left(\frac{j\pi}{l} \right)^4 Y_j + M \ddot{Y}_j + \sum_{i=1} m_{ji} \ddot{Y}_i + 2U \sum_{i=1} \left(\frac{i\pi}{l} \right) m_{j\bar{i}} \dot{Y}_i - U^2 \sum_{i=1} \left(\frac{i\pi}{l} \right)^2 x$$

$$x \, m_{ji} Y_i + K_o \dot{Y}_j + \sum_{i=1} c_{ji} \dot{Y}_i + \sum_{i=1} m^{(\cdot)}_{ji} \dot{Y}_i + U \sum_{i=1} \left(\frac{i\pi}{l} \right) m^{(\cdot)}_{j\bar{i}} Y_i +$$

$$+ U \sum_{i=1} m^{(\prime)}_{ji} \dot{Y}_i + U^2 \sum_{i=1} \left(\frac{i\pi}{l} \right) m^{(\prime)}_{j\bar{i}} Y_i = p_j . \qquad (4)$$

Here

$$m_{ji} = \frac{2}{l} \int_0^l \sin \frac{j\pi x}{l} m(x,t) \sin \frac{i\pi x}{l} dx ;$$

$$m_{j\bar{i}} = \frac{2}{l} \int_0^l \sin \frac{j\pi x}{l} m(x,t) \cos \frac{i\pi x}{l} dx ;$$

$$m^{(\cdot)}_{ji} = \frac{2}{l} \int_0^l \sin \frac{j\pi x}{l} \frac{\partial m(x,t)}{\partial t} \sin \frac{i\pi x}{l} dx ;$$

$$m_{ji}^{(\cdot)} = \frac{2}{l}\int_0^l \sin\frac{j\pi x}{l}\,\frac{\partial m(x,t)}{\partial t}\,\cos\frac{i\pi x}{l}\,dx\ ;$$

$$m_{ji}^{(\prime)} = \frac{2}{l}\int_0^l \sin\frac{j\pi x}{l}\,\frac{\partial m(x,t)}{\partial x}\,\sin\frac{i\pi x}{l}\,dx\ ; \qquad \text{---------- (5)}$$

$$m_{j\bar{i}}^{(\prime)} = \frac{2}{l}\int_0^l \sin\frac{j\pi x}{l}\,\frac{\partial m(x,t)}{\partial x}\,\cos\frac{i\pi x}{l}\,dx\ ;$$

$$c_{ji} = \frac{2}{l}\int_0^l \sin\frac{j\pi x}{l}\,c(x,t)\,\sin\frac{i\pi x}{l}\,dx\ ;$$

$$p_j = \frac{2}{l}\int_0^l \sin\frac{j\pi x}{l}\,p(x,t)\,dx\ .$$

As the first mode of vibrations is most important from the view point of engineering, only the first mode in eq.(4) is here employed for further investigation on excitation mechanisms of the vibrations of the strip in parallel two-phase flow. Then

$$(M + m_{11})\,\ddot{Y}_1 + \left(K_o + c_{11} + \frac{2U\pi}{l}\,m_{1\bar{1}} + m_{11}^{(\cdot)} + U\,m_{11}^{(\prime)}\right)\dot{Y}_1$$

$$\left[\,EI\left(\frac{\pi}{l}\right)^4 - U^2\left(\frac{\pi}{l}\right)^2 m_{11} + \frac{U\pi}{l}\left\{m_{1\bar{1}}^{(\cdot)} + U\,m_{1\bar{1}}^{(\prime)}\right\}\right] Y_1 = p_1\ . \qquad \text{------- (6)}$$

3. Mathematical Representation of Virtual Mass, Viscous Damping and Pressure Fluctuations

The space-time distribution of the virtual mass of the two-phase fluid is illustrated in Fig.2 for the case of slug flow. In this illustration, the virtual mass due to a water slug is assumed to be shruncken at a certain point corresponding to the position of the water slug, although in reality it distributes over a certain range of the strip. Denoting the virtual mass and the arrival period of water slugs by m_0 and T_s , respectively, the distribution is mathematically written as

$$m(x,t) = m_0\sum_{k=1}^{N-1}\delta\{x - U(k-1)T_s - Ut\}$$

$$+\ m_0\,\delta\{x - U(N-1)T_s - Ut\}\ h\left(\frac{t}{\beta T_s}\right), \qquad \text{---------- (7)}$$

where $N = [\,l / U T_s\,] + 1,$

$\beta = l / U T_s + 1 - N,$

$$h(s) = \begin{cases} 0 & (s < 0)\ , \\ 1 & (0 \leq s \leq 1)\ , \\ 0 & (s > 1)\ . \end{cases} \qquad \text{---------- (8)}$$

The damping effect is considered to be mainly caused by water slugs of the two-phase flow, then its distribution along the strip may be similarily expressed in the form

$$c(x,t) = c_0\sum_{k=1}^{N-1}\delta\{x - U(k-1)T_s - Ut\}$$

$$+\ c_0\,\delta\{x - U(N-1)T_s - Ut\}\ h\left(\frac{t}{\beta T_s}\right), \qquad \text{---------- (9)}$$

where c_0 is the damping coefficient due to a single water slug. Similarily the pressure fluctuations of the two-phase flow might be mathematically described as follows:

$$p(x,t) = \sum_{k=1}^{N-1} p_k \delta\{x - U(k-1)T_p - Ut\} + p_N \delta\{x - U(N-1)T_p - Ut\}\, h\left(\frac{t}{\beta' T_p}\right) . \quad \text{------------ (10)}$$

Here p_k is the resultant pressure difference acting on unit length of the strip, which is caused by the k-th travelling water slug, and T_p is a dominant period of the pressure fluctuation.

Substituting eqs.(7), (9) and (10) into eq.(5), the following relations are obtained for the case $i = j = 1$,

$$\left.\begin{aligned} m_{11} &= \frac{m_o}{l}\{N - 1 + Q(t)\}, \\ m_{1\bar{1}} &= \frac{m_o}{l}\{R(t)\}, \\ c_{11} &= \frac{c_0}{l}\{N - 1 + Q(t)\}, \\ U^{-1} m_{11}^{(\cdot)} + m_{11}^{(\prime)} &= U^{-1} m_{1\bar{1}}^{(\cdot)} + m_{1\bar{1}}^{(\prime)} = 0, \end{aligned}\right\} \quad (k-1)T_s \le t \le k T_s \quad \text{------------ (11)}$$

$$p_1 = \frac{2}{l} S(t), \quad (k-1)T_p \le t \le k T_p, \quad k = 1, 2, 3, \cdots . \quad \text{------------ (12)}$$

Here

$$\begin{aligned} Q(t) &= -\sum_{k=1}^{N-1} \cos\frac{2\pi U\{(k-1)T_s - t\}}{l} + \left[1 - \cos\frac{2\pi U\{(N-1)T_s - t\}}{l}\right] h, \\ R(t) &= \sum_{k=1}^{N-1} \sin\frac{2\pi U\{(k-1)T_s - t\}}{l} + \sin\frac{2\pi U\{(N-1)T_s - t\}}{l}\, h, \\ S(t) &= \sum_{k=1}^{N-1} p_k \sin\frac{\pi U\{(k-1)T_p - t\}}{l} + p_N \sin\frac{\pi U\{(N-1)T_p - t\}}{l}\, h . \end{aligned} \quad (13)$$

Thus a concrete form of eq.(6) has been obtained for the first vibration mode as

$$\left[M + \frac{m_o\{N - 1 + Q(t)\}}{l}\right]\ddot{Y}_1 + \left[K_o + \frac{m_o R(t) + c_0\{N - 1 + Q(t)\}}{l}\right]\dot{Y}_1 + \left(\frac{\pi}{l}\right)^2\left[EI\left(\frac{\pi}{l}\right)^2 - \frac{m_o U^2\{N - 1 + Q(t)\}}{l}\right] Y_1 = \frac{2}{l} S(t) . \quad \text{---- (14)}$$

4. Mathieu Equation with a Forced Term

As the functions Q(t), R(t) and S(t) are generally periodic, their Fourier expansion is here employed in order to develop the theoretical analysis to further extent. That is,

$$\left.\begin{aligned} Q(t) &= q_o + q_1 \cos\frac{2\pi t}{T_s}, \\ R(t) &= r_1 \sin\frac{2\pi t}{T_s}, \\ S(t) &= s_1 \sin\frac{2\pi t}{T_p} . \end{aligned}\right\} \quad \text{------------ (15)}$$

Here it should be noted that the following is assumed with respect to the constant term s_0 in the Fourier expansion of S(t);

$$s_0 = 0 \qquad \text{------} \quad (16)$$

because S(t) is essentially the fluctuations of pressure difference between both the surfaces. The coefficients appeared in eq.(15) are defined as

$$\begin{aligned} q_o &= \frac{1}{T_s}\int_0^{T_s} Q(t)\,dt = \beta , \\ q_1 &= \frac{2}{T_s}\int_0^{T_s} Q(t)\cos\frac{2\pi t}{T_s}\,dt , \\ r_1 &= \frac{2}{T_s}\int_0^{T_s} R(t)\sin\frac{2\pi t}{T_s}\,dt , \\ s_1 &= \frac{2}{T_p}\int_0^{T_s} S(t)\sin\frac{2\pi t}{T_p}\,dt . \end{aligned} \qquad \text{------} \quad (17)$$

Substituting eq.(15) into eq.(11), the equation of motion of the first vibration mode amplitude, namely eq.(14) can be written as follows:

$$(1 + \epsilon_1\cos\omega_s t)\,\ddot{Y}_1 + \left(\frac{2\pi U}{l}\epsilon_2\sin\omega_s t + \epsilon_3\cos\omega_s t + K\right)\dot{Y}_1$$
$$\omega_0^2\left\{1 - \frac{U^2\pi^2\epsilon_1}{(\omega_0\, l)^2}\cos\omega_s t\right\} Y_1 = p_1\sin\omega_p t , \qquad \text{------} \quad (18)$$

where the new parameters introduced in eq.(18) are defined as

$$\begin{aligned} \epsilon_1 &= \frac{m_o}{l}\, q_1 \Big/ \left\{M + \frac{m_o}{l}(N - 1 + \beta)\right\} , \\ \epsilon_2 &= \frac{m_o}{l}\, r_1 \Big/ \left\{M + \frac{m_o}{l}(N - 1 + \beta)\right\} , \\ \epsilon_3 &= \frac{c_o}{l}\, q_1 \Big/ \left\{M + \frac{m_o}{l}(N - 1 + \beta)\right\} , \\ K &= \left\{K_o + \frac{c_o}{l}(N - 1 + \beta)\right\} \Big/ \left\{M + \frac{m_o}{l}(N - 1 + \beta)\right\} , \\ p_1 &= \frac{2s_1}{l} \Big/ \left\{M + \frac{m_o}{l}(N - 1 + \beta)\right\} , \\ \omega_0^2 &= \left(\frac{\pi}{l}\right)^2\left[EI\left(\frac{\pi}{l}\right)^2 - \frac{m_o}{l}U^2(N - 1 + \beta)\right] \Big/ \left\{M + \frac{m_o}{l}(N - 1 + \beta)\right\} , \\ \omega_s &= 2\pi/T_s , \\ \omega_p &= 2\pi/T_p . \end{aligned} \qquad \text{---} \quad (19)$$

ϵ_1 being a small parameter, both the sides of eq.(18) are allowed to be divided by $(1 + \epsilon_1 \cos\omega_s t)$. Then

$$\ddot{Y}_1 + \left\{K + \frac{2\pi U}{l}\epsilon_2\sin\omega_s t + (\epsilon_3 - K\epsilon_1)\cos\omega_s t\right\}\dot{Y}_1 +$$

$$+ \omega_0^2 [1 - \epsilon_1 \{1 + (\frac{U\pi}{\omega_0 l})^2\} \cos \omega_s t] Y_1 = p_1 \sin \omega_p t . \quad (20)$$

An attention should be here paid to that only the first order terms are employed. In eq.(20), the contribution of the damping terms K and ϵ_3 to time varying coefficients is considered to be smaller. After neglecting these terms in eq.(20) and adopting the well known transformation of the variable Y_1 into X_1

$$Y_1 = \exp - \int^t \frac{\pi U}{l} \epsilon_2 \sin \omega_s \tau \, d\tau \, X_1 , \quad (21)$$

eq.(20) can be rewritten in rather simpler form

$$\ddot{X}_1 + 2\omega_0 \xi \dot{X}_1 + \omega_0^2 (1 - \epsilon_0 \cos \omega_s t) X_1 = p_1 \sin \omega_p t , \quad (22)$$

where $\xi = K / 2\omega_0$,

$$\epsilon_0 = \epsilon_1 (1 + (\frac{U\pi}{\omega_0 l})^2) + \frac{2\pi U}{l} \frac{\omega_s}{\omega_0^2} \epsilon_2 . \quad (23)$$

When the following parameters are introduced,

$$a = 4 (\frac{\omega_0}{\omega_s})^2 , \qquad q = 2 (\frac{\omega_0}{\omega_s})^2 \epsilon_0 ,$$
$$\Omega^2 = 4 (\frac{\omega_p}{\omega_s})^2 , \qquad 2\tau = \omega_s t , \quad (24)$$

eq.(22) can be transformed into a canonical form

$$\ddot{X}_1 + 2\sqrt{a}\, \xi \dot{X}_1 + (a - 2 q \cos 2\tau) X_1 = p_1 \sin \Omega\tau . \quad (25)$$

Here it is worth noting that a means principally the ratio of the fundamental frequency of the fuel pin model to the dominant frequency of void-signals, q denotes its variation due to the two-phase flow, and p_1 is an external force caused by the pressure fluctuations.

5. Stability Analysis

The linearity of eq.(22) leads to that the general solution can consist of its fundamental solutions and a particular one. Then it is necessary to investigate the instability conditions of these solutions in order to make clear the excitation mechanisms of the flow induced vibrations of a fuel pin model.

[1] Instability Conditions for the Fundamental Solutions

The instability of the fundamental solutions of Mathieu equation with a viscous damping term has been already studied [8] and the result is shown in Fig.3. In this figure the instability boundaries are shown in terms of the relation between q and a $(1 - \xi^2)$ when a damping coefficient ξ is given.

[2] Instability Conditions for a Particular Solution

At a study on the instability of a particular solution of eq.(22), the following two cases are investigated; (1) one at which $\xi \ll 1$ and $\omega_p = \omega_s$ and (2) the other is at the case when $\epsilon_0 \ll 1$.

For the first case (1), eq.(22) can be approximately written as

$$\ddot{x}_1 + \omega_0^2 (1 - \epsilon_0 \cos \omega_s t) x_1 = p_1 \sin \omega_s t . \quad \text{------} \quad (26)$$

Denoting a particular solution of eq.(26) by $x_1^{(1)}$ and expanding it in terms of $\sin j \omega_s t$, ($j = 1, 2, ---$) as

$$x_1^{(1)} = u_1 \sin \omega_s t + u_2 \sin 2 \omega_s t + u_3 \sin 3 \omega_s t + --- , \quad \text{------} \quad (27)$$

a system of algebric equations with regard to the coefficients u_1, u_2, u_3, --- ,and so on is obtained through a simple procedure of balancing the coefficients for each $\sin j \omega_s t$ when eq.(27) is substituted into eq.(26):

$$\begin{aligned} &(\omega_0^2 - \omega_s^2) u_1 - \frac{\omega_0^2}{2} \epsilon_0 u_2 = p_1 , \\ &- \frac{\omega_0^2}{2} \epsilon_0 u_1 + (\omega_0^2 - 4\omega_s^2) u_2 - \frac{\omega_0^2}{2} \epsilon_0 u_3 = 0, \\ &- \frac{\omega_0^2}{2} \epsilon_0 u_2 + (\omega_0^2 - 9\omega_s^2) u_3 = 0, \end{aligned} \quad \text{------} \quad (28)$$

for the first three coefficients. The condition for some of u_1, u_2, and u_3 to become infinite is the existence of ω_s and ϵ_0 satisfying

$$\begin{vmatrix} \omega_0^2 - \omega_s^2 & - \frac{\omega_0^2}{2} \epsilon_0 & 0 \\ - \frac{\omega_0^2}{2} \epsilon_0 & \omega_0^2 - 4\omega_s^2 & - \frac{\omega_0^2}{2} \epsilon_0 \\ 0 & - \frac{\omega_0^2}{2} \epsilon_0 & \omega_0^2 - 9\omega_s^2 \end{vmatrix} = 0 . \quad \text{------} \quad (29)$$

The approximate solutions of eq.(29) can be easily obtained to the cases when $\omega_s \doteqdot \omega_0$, $\omega_0 / 2$, and $\omega_0 / 3$.

1) When $\omega_s \doteqdot \omega_0$, eq.(29) yields a characteristic solution

$$\omega_s = \omega_0 (1 + \frac{1}{24} \epsilon_0^2) , \quad \text{------} \quad (30)$$

2) when $\omega_s \doteqdot 2^{-1}\omega_0$, it is easily verified that the solution of eq.(29) is

$$\omega_s = \frac{1}{2} \omega_0 (1 + \frac{1}{3} \epsilon_0^2) , \quad \text{------} \quad (31)$$

3) and similarily when $\omega_s \doteqdot \frac{1}{3} \omega_0$,

$$\omega_s = \frac{1}{3} \omega_0 (1 - \frac{2}{9} \epsilon_0^2) . \quad \text{------} \quad (32)$$

Consequently a particular solution of eq.(26) becomes unstable when the relation among ω_0, ω_s and ϵ_0 expressed in eqs.(30), (31), or (32) is held.

For the second case (2), eq.(22) can be also approximated as

$$\ddot{x}_1 + 2 \omega_0 \xi \dot{x}_1 + \omega_0^2 x_1 = p_1 \sin \omega_p t . \quad \text{------} \quad (33)$$

As well known, the resonance to the external force $p_1 \sin \omega_p t$ occurs when

$$\omega_p = \omega_0 (1 - \xi^2) . \qquad (34)$$

Therefore the solution of eq.(33) obtains a large amplitude when the condition described by eq.(34) is satisfied.

6. Discussions

The results of theoretical analysis of the vibrations of a fuel pin model in parallel two-phase flow show that the ratio ω_0 / ω_s and a parameter ϵ_0 play an important role in the excitation mechanisms. The quantitative evaluation of these parameters and other ones such as ξ, ω_p, and p_1 is done as follows by using the physical data obtained by the experiments [9]:

(1) The lowest natural frequency ω_0 and the dominant one of the void-signals of the two-phase flow ω_s are evaluated as a peak frequency of power spectral densities of vibrational strain signals and the void-signals, respectively.

(2) The parameters necessary to evaluate ϵ_0 are shown in Table 1 for the cases of experiments A, B, --- , R. As the parameters N and β are specified by the relation

$$N - 1 + \beta = l \omega_0 U / 2\pi , \qquad (35)$$

the coefficients of Fourier expansion of Q(t) and R(t) arenumerically calculated. On the other hand, the virtual mass of a water slug m_o is estimated by the relation

$$m_o = \frac{[EI (\pi / l)^4 - \omega_o^2 \{ M + m_o / l (N - 1 + \beta) \}]}{(N - 1 + \beta) / l \{ \omega_o^2 + U^2 (\pi / l)^2 \}} , \qquad (36)$$

which is easily derived from eq.(19). By using these quantities, ϵ_1, ϵ_2 and ϵ_0 are calculated through the relations (19) and (23).

(3) By using the wave forms of free vibrations of the fuel pin model in air and in still water, mechanical damping coeffiecient K_o and viscous one per unit length of the pin c are obtained. Then c_0 is estimated as c x (length of a water slug). Finally ξ is calculated from eqs.(19) and (23).

(4) The dominant frequency of differential pressure fluctuations ω_p and its strength are evaluated as the steepest peak frequency and its power, respectively.

The results are shown in Tables 1 and 2. Fig.4 shows the experiment points A, B, ----, R in terms of point (a, |q|) on the instability diagram of Mathieu equation. A keen attention should be paid to the points A, B, C, and Q. These points are located near the unstable regions. Fig.5 shows the experimental relation between the vibration strength defined as the ratio of the variance of vibrational strains to that of pressure fluctuations and the frequency ratio ω_p / ω_0. This figure describes mainly the resonance feature of the two-phase flow induced vibrations in a fuel pin model but there exist several points out of this feature. It should be again noted that these outsided points are located near the unstable regions in Fig.4.

7. Conclusions

This paper can obtain the following significant conclusions about the excitation mechanisms of the external two-phase flow induced vibrations in a fuel pin model:

(1) The equation of motion of the fuel pin model in parallel air + water two-phase flow was formulated by taking account of elastic restoring force; inertia forces acting on the fuel pin model and the virtual mass of the two-phase flow; Coriolis' force; centrifugal force; damping effect due to mechanical friction at both the ends of the fuel pin and viscousity of the two-phase flow; momentum change of the two-phase flow and differential pressure fluctuations as an external force, and it was transformed into the Mathieu type equation with a term of external force.

(2) The water slugs travelling arround the pin with a certain interval plays an important role in the equation of motion of the pin as periodically time varying coefficients and the pressure fluctuations of the two-phase flow as a periodic external term.

(3) Instability analysis of this Mathieu type equation with a forced term verified the experimental results and explained the excitation mechanisms of the vibrations of the pin in parallel two-phase flow being as follows; when ω_0 / ω_s and $\omega_0 / \omega_p = 1$, the parametric excitation and the resonance are main causes to our two-phase flow induced vibrations and $\omega_0 / \omega_s = 1/2, 3/2, 4/2, ---$, the parametric excitation is the dominant cause to the vibrations.

8. Acknowledgements

The author would like to express his appreciations to Prof.H.SHIBATA of Institute of Industrial Science, University of Tokyo for his valuable suggestions. He also thanks Research Assistant T.YAMASHITA of Department of Mechanical Engineering, Tokyo University of Science and Messrs K.UEHARA and H.MASAKI who were students of his laboratory for their assistance to numerical calculations.

9. References

[1] S.S.Chen and M.W.Wambsganss, "Parallel Flow Induced Vibrations of Fuel Rods", Preprints of 1st Intern. Conf. on SMiRT, E3/1, Berlin, Sept., 1971

[2] G.P.Gau, P.Grillo and G.Testa, "Fuel Rod Vibrations Induced by Coolant Flow",ibid.,E3/3, Berlin, Sept., 1971

[3] K.D.Appelt, J.Kadlec and W.Kruger, "Schwingungsverhalten von Schnellbruter Brennelementen" ibid., E3/4, Berlin, Sept., 1971

[4] P.G.Avanzini, "Self-Sustained Vibrations of Rods in Parallel Flow", ibid., E3/5, Berlin, Sept., 1971

[5] M.P.Paidoussis, "Dynamics of Flexible Slender Cylinder in Axial Flow, Part I", J. of Fluid Mech., Vol.26, p.717, 1966

[6] E.P.Quinn, "Vibration of Fuel Rods in Parallel Flow", GEAP-4059, July, 1962

[7] F.Hara, "A Theory on the Two-Phase Flow Induced Vibrations in Piping System", Preprints of 2nd Intern. Conf. on SMiRT, F5/1, Berlin, Sept., 1973

[8] S.Kunii, Y.Sawaragi and S.Takahashi, "Determination of the Critical Diagram of Instability in the Damped Quasi-Harmonic Vibration", Trans. of JSME, Vol.17, No.64, p.9, 1951

Table 1. Physical quantities necessary to evaluation ϵ_1, ϵ_2, ϵ_3 and the parameters ϵ_0 and ξ for experiments A, B, --- , and R

Exp	Q_w (l/m)	Q_a (l/m)	N	β	m_0[kg]	C_0[kg/s]	q_1	γ_1	ϵ_1	ϵ_2	ϵ_3 [s⁻¹]	ϵ_0	ξ
A	15	20	3	.9586	.0261	1.705	-.0104	.0546	-.0012	.0061	-.0762	-.0008	.0729
B	15	40	2	.7043	.0291	.3300	-.1591	.2281	-.0226	.0323	-.2554	-.0197	.0635
C	15	50	2	.3961	.0349	.3949	.2044	.5542	.0349	.0945	.3947	.0510	.0633
D	15	60	2	.2866	.0377	.4269	.4721	.7198	.0871	.1328	.9865	.1118	.0629
F	25	40	3	.3641	.0515	.5834	.0505	-.0143	.0094	-.0027	.1061	.0086	0875
G	24	60	2	.4156	.0771	.8006	.1645	.5212	.0495	.1570	.5146	.0995	0815
I	30	40	3	.5283	.0242	.2740	-.0092	.0987	-.0010	.0110	-.0116	0014	0678
L	34	40	4	.9902	.0222	1.836	-.0013	.0994	-.0001	.0090	-.0097	0037	0771
M	34	60	3	.2135	.0265	.3003	.0777	-.0745	.0096	-.0092	.0116	.0068	0673
N	38	50	4	.5425	.0381	.4317	-.0066	.3009	-.0009	.0394	.1221	.0291	0923
P	41	40	5	.1419	.0449	.3416	.0145	.0394	.0019	.0052	.0145	.0101	.1052
Q	41	50	4	.1092	.0281	.3181	.0227	.0056	.0026	.0006	.0298	.0031	.0951
R	41	60	3	.3352	.0356	.4030	.0596	-.0330	.0089	-.0049	.1005	.0071	.0762

Table 2. Numerical values of ω_0, ω_s, ω_p, a , q , Ω and p_1 for experiments A, B, --- , and R
(ω_0, ω_s and ω_p are in unit of Hz.)

			ω_p				Ω		P_1 N·10⁻²	
Exp	ω_0	ω_s	1st	2nd	a	q	1st	2nd	1st	2nd
A	8.0	5.0	3.8		10.24	-0.004	0.94		1.26	
B	8.5	5.3	1.8	6.0	10.49	-0.103	0.67	2.99	1.96	1.93
C	8.5	5.5	2.0	6.5	9.55	0.244	0.73	2.36	2.03	1.91
D	8.5	5.3	2.5	7.0	10.49	0.587	0.95	2.67	2.87	2.00
F	7.3	8.3	6.0		3.10	-0.013	1.66		1.89	
G	7.5	6.8	6.5	9.8	4.94	0.245	1.93	2.89	2.46	2.40
I	8.3	9.0	8.8	6.0	3.36	0.002	1.94	1.33	2.00	1.39
L	7.8	14.3	8.3		1.17	0.002	2.13		1.60	
M	8.3	11.0	8.0	11.5	2.25	0.008	1.45	2.09	2.59	2.60
N	7.0	16.3	8.5		0.74	0.011	1.05		1.69	
P	6.5	14.0	16.5		0.85	0.010	5.08		1.35	
Q	6.5	12.3	11.8		1.12	0.002	1.76		1.42	
R	7.8	12.8	13.0	8.5	1.48	0.005	2.08	1.33	2.15	1.93

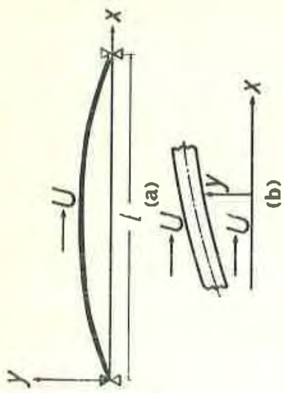

Fig.1 Illustration of the vibration system of a fuel pin model in parallel air+water two-phase flow

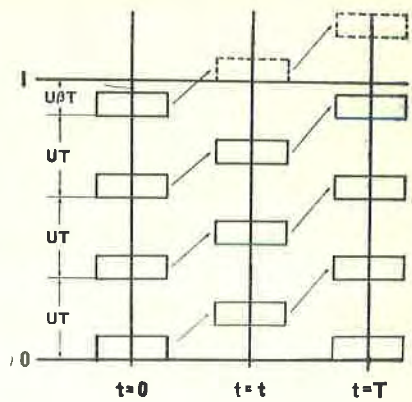

Fig.2 Illustration of space-time distribution of virtual mass of the two-phase flow along a pin

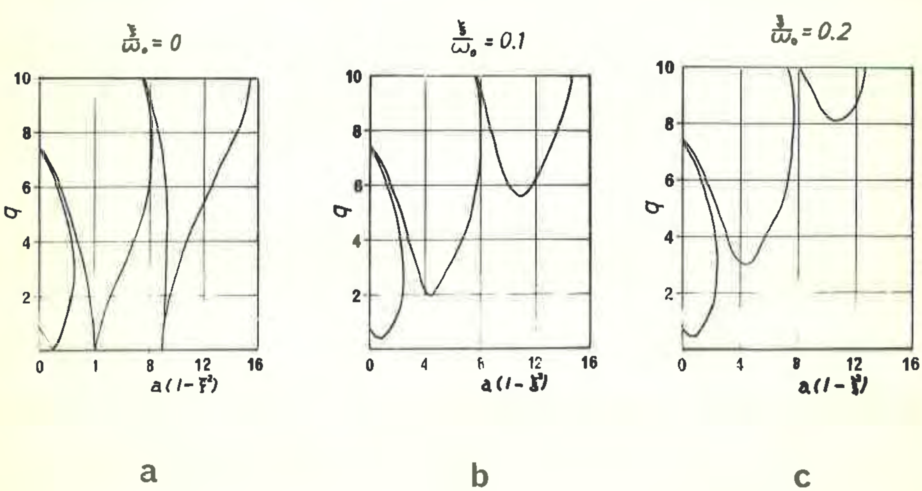

Fig.3 Instability diagrams of the damped Mathieu equation with respect to q and $a(1-\zeta^2)$, (a) $\zeta/\omega_o = 0$, (b) $\zeta/\omega_o = 0.1$, (c) $\zeta/\omega_o = 0.2$

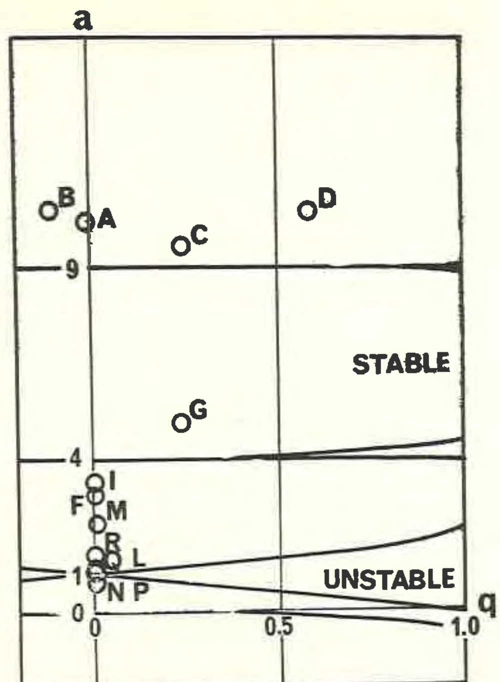

Fig.4 Location of experiment points A, B, --- , and R on the instability diagram of Mathieu equation

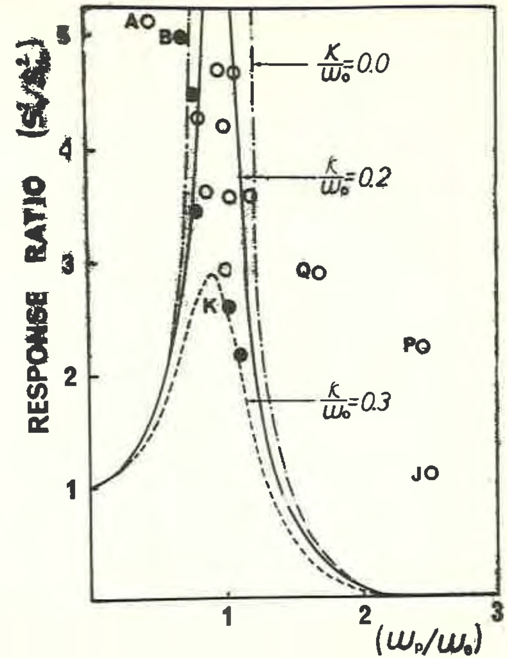

Fig.5 Relation between response ratio of the vibrations to ω_p / ω_0