BOUNDS ON DYNAMIC PLASTIC DEFORMATION OF HEXAGONAL FRAMES

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SUMMARY

The response of thin walled subassembly ducts of hexagonal cross section to an internal pulse of arbitrary shape is studied. A typical reactor core configuration consists of an array of such constituents, each containing an array of fuel elements, whose failure may expose the duct to an internal pressure pulse, causing permanent deformation that can reach the order of the wall thickness and cause contact between adjacent elements.

Recently a number of papers have tackled the problem of estimating the permanent duct deformation, under various simplifying assumptions. The purpose of this study is to consider the possibility of application of bounding techniques developed in the dynamics of elastoplastic structures, which permit to bound from above total or permanent displacements under given dynamic actions, with a computational effort moderate with respect to a step-by-step evolutive dynamic analysis. The duct is conceived as a hexagonal frame and the effects of interaction between bending and membrane forces is accounted for, both in the plasticity condition and in its geometric consequences. In particular, attention is drawn on the following points:

(1) In the small deformation range (physically meaningful for short duration pulses), the response to different pulse shapes is compared, and some conclusion on the effect of the peak value, total impulse and duration is drawn.

(2) The possibility of applying second order theory when the permanent deformation is too large to be studied as in point (1) is considered. Thė underlying hypotheses and the conditions for the validity of this procedure are indicated.

(3) Conditions under which the pulse can be substituted by an initial velocity or by a step pulse of unlimited duration are also discussed. Both assumptions lead to far simpler computations.

Results are graphically illustrated and compared with those obtained by other authors under the assumption of rigid, perfectly plastic material behavior. Comparisons show good agreement in several cases. Also some of the conclusions concerning points (1) and (3) above turn out to be fairly similar, when applying bounding techniques and evolutive analysis.

1. Bounding Techniques

A discretized structure is considered as an assemblage of n constituents. Let the behavior of these constituents be elastic perfectly plastic, with their elastic domains defined by one or more (say y) inequalities of the type:

$$\phi_h\,(\underline{\sigma}^i) \leq 0 \quad , h=1,\ldots,y \quad ; \quad i=1,\ldots,n \tag{1}$$

where $\underline{\sigma}^i$ denotes the generalized stress vector of constituent i . Suppose that known external forces $\underline{F}(t)$ are applied at time t=0 on the structure, with initial conditions $\underline{u}_o$, $\dot{\underline{u}}_o$, and that these forces are of magnitude large enough to produce plastic deformations during the motion. It was shown in references | 1-2 | that suitable measures of plastic and total deflections developed over a given time interval (0,t) by an elastic-plastic structure are bounded from above by quantities that can be evaluated merely on the basis of a linear elastic dynamic analysis.

Bounds are obtained on the fictitiuos work done by conveniently introduced dummy loads $\eta\underline{R}$, $\underline{R}$ being a fixed vector and η a nonnegative scaling factor, for the actual total or plastic displacements of the structure. For the particular problem described, the bound on total displacements becomes:

$$\underline{R}^T\underline{u}(t) \leq U \equiv (1/2\eta)\left[(\underline{u}_o - \underline{u}_o^*)^T\,\underline{K}\,(\underline{u}_o - \underline{u}_o^*) + (\dot{\underline{u}}_o - \dot{\underline{u}}_o^*)^T\,\underline{I}\,(\dot{\underline{u}}_o - \dot{\underline{u}}_o^*) + \right.$$

$$\left. + \underline{\sigma}_s^T\,\underline{C}\,\underline{\sigma}_s\right] + (\eta/2)\,\underline{\sigma}_R^T\,\underline{C}\,\underline{\sigma}_R + \underline{R}^T\underline{u}^*(t) \tag{2}$$

while the bound on plastic displacement is:

$$\underline{R}^T\underline{u}^P(t) \leq U^P \equiv (1/2\eta)\left[(\underline{u}_o - \underline{u}_o^*)^T\,\underline{K}\,(\underline{u}_o - \underline{u}_o^*) + (\dot{\underline{u}}_o - \dot{\underline{u}}_o^*)^T\,\underline{I}\,(\dot{\underline{u}}_o - \dot{\underline{u}}_o^*) + \right.$$

$$\left. + \underline{\sigma}_s^T\,\underline{C}\,\underline{\sigma}_s\right] \tag{3}$$

both inequalities hold provided that

$$\phi_h(\underline{\sigma}^{*i}(\tau) + \eta\,\underline{\sigma}_R^i + \underline{\sigma}_s^i) \leq 0 \quad , i=1,\ldots,n \; ; \; h=1,\ldots,y \; ;$$

$$\text{for all times } \tau \text{ in the interval } (0,t) \tag{4}$$

In the above expressions $\underline{u}(t)$ and $\underline{u}^P(t)$ denote the total and plastic displacement vectors at time t , $\underline{K}$ and $\underline{I}$ the stiffness and inertia matrices for the structure, $\underline{C}$ the flexibility matrix of the constituents, $\underline{\sigma}_R$ tha static, linear elastic stresses in the structure due to the dummy loads $\underline{R}$, and $\underline{\sigma}_s$ a self-stress state. Quantities with an asterisk ($\underline{u}^*(t)$, $\underline{\sigma}^*(t)$) refer to the dynamic response of the structure supposed linearly elastic to the actual load history $\underline{F}(t)$ and to suitably chosen initial conditions $\underline{u}_o^*$, $\dot{\underline{u}}_o^*$ (fictitious process) .

In the above expressions, plastic displacements are defined as the displacements that would develop if the actual plastic strain history was applied statically on the structure supposed elastic in absence of external loads (see ref. |1|).

Eqs. (2)-(3)-(4) have been obtained by particularizing to the case under consideration (discrete model, perfect plasticity) the more general expressions of ref. |1|. Bounds on maximum displacements are obtained by substituting in eq.(2) the scalar product $\underline{R}^T\underline{u}^*(t)$ with its maximum value for $t \geq 0$ and by requiring that eq. (4) is fulfilled for any time $\tau \geq 0$.

When bounds on a single displacement component are seeked, vector $\underline{R}$ is chosen so that only the corresponding component is different from zero. It was shown (see e.g. ref. |3|) that, when the yield conditions eq.(1) are piecewise linearized, the best bound can be obtained through the solution of a sequence of quadratic programming problems.

In ref.|3| some numerical experimentation was carried out with reference to frames. In particular two extreme load situations were considered.

(1) Impulsive loads:

$$\underline{F}(t) = \underline{0} \quad \forall \quad t \geq 0 \quad ; \underline{u}_o = \underline{0} \; , \; \dot{\underline{u}}_o = \underline{v}_o \qquad (5)$$

In this case eq.(4) is always fulfilled for η small enough, provided that the initial conditions for the fictitious process are chosen as zero (so that $\underline{\sigma}^*(t) = \underline{0}$, $\underline{u}^*(t) = \underline{0}$, $\forall \; t \geq 0$). Bounds turned out to be very close to the actual displacements.

(2) Periodic loads $\underline{F}(t+T) = \underline{F}(t)$, T being the period. In this case it can be shown on the basis of the dynamic shakedown theory that eq.(4) can always be fulfilled if the total plastic work is bounded (see, e.g., refs. |4|-|5|). The fictitious process to be assumed in this case coincides with the steady state portion of the structural response (ref.|3|).

The validity of bounds defined by eqs. (2)-(3)-(4) becomes questionable in the presence of actions that cease after a certain time; in particular it might happen that eq.(4) is violated, and hence no finite bound can be obtained, even if the actual plastic deformations are finite.

The above bounds are valid under the usual small displacements hypotheses. Geometric effects can be accounted for only in the spirit of second order theory, i.e. considering the increment (or decrement) of structural stiffness due to the presence of fixed membrane forces, but not the change in load carrying capacity due to shape modification.

2. Formulation of the Problem

In this paper the above techniques are applied in order to estimate the maximum permanet displacements of hexagonal subassembly ducts subjected to internal pressure pulses. Results are compared with those obtained by Youngdahl (ref.|6|) considering the duct as a hexagonal frame and the material as rigid-perfectly plastic. The cross section of the duct is shown in fig. 1

and the following data are assumed:

$H = .14$ in (wall thickness)
$L = 2.48$ in (side length)
$\sigma_y = 30{,}000$ psi (yield stress)
$E = 29{,}000{,}000$ psi (Young's modulus)

$t_o \equiv L\sqrt{(\rho/\sigma_y)} = .3$ msec (time constant defining the material density ρ)

These data correspond to a FFTF reactor subassembly duct (ref.|6|). Bounds were obtained on the outward displacement u at a center of a side.

The frame was discretized into an assemblage of elastic finite elements of beam connected by rigid plastic joints. The joint yield condition is given by

$$\phi \equiv \frac{|M|}{M_o} + \left(\frac{N}{N_o}\right)^2 - 1 = 0 \qquad (6)$$

with

$$N_o \equiv \sigma_y H \quad ; \qquad M_o \equiv \sigma_y (H^2/4) \qquad (7)$$

and it is indicated with a dashed line in fig.2. N and M are the membrane forces and the bending moment (both per unit length) and the generalized stress vector $\underline{\sigma}^i$ for each joint i is defined as

$$\underline{\sigma}^{iT} \equiv \{M \quad N\}^i$$

The static collapse load P_y of the frame turns out to be

$$P_y = .0121\ \sigma_y = 363 \text{ psi} \qquad (8)$$

Twenty equally spaced joints were introduced on each side. Piecewise linearization of the yield condition eq.(6) was achieved through the six segments indicated by solid lines in fig.2.

3. Choice of the Fictitious Process

A rectangular pulse of peak value P_m and duration t_r was first considered:

$$\begin{aligned} P(t) &= P_m & t &\leq t_r \\ P(t) &= 0 & t &> t_r \\ \underline{u}_o &= \dot{\underline{u}}_o = \underline{0}\ . \end{aligned} \qquad (9)$$

Bounds on u and u^P were obtained making use of different fictitious processes (i.e. different choices of the initial conditions $\underline{u}_o^*$, $\dot{\underline{u}}_o^*$). The various alternatives are listed below.

(A) The fictitious process coincides with the actual linear elastic solution ($\underline{u}_o^* = \underline{u}_o = \underline{0}$, $\dot{\underline{u}}_o^* = \dot{\underline{u}}_o = \underline{0}$).

(B) Nonzero initial conditions are chosen for the fictitious process, in order to reduce the amplitude of the response. The conditions used are those that would produce zero motion for $t \geq t_r$ in an undamped simple oscillator.

Even if bounds obtained by means of procedure (B) are smaller, in the range of interest, than those reached through (A), both methods lead to finite bounds only for small values of P_m or for very short duration pulses, since only in these situations eq.(4) can be fulfilled.

More meaningful, even if not rigorous, results can be obtained by modifying the original pulse eq.(9). Two extreme situations were studied:

(C) The rectangular pulse is substituted with a step pulse of unlimited duration. Thus u_{max} is overestimated, but the approximation should be reasonable for long duration pulses.The initial conditions chosen for the fictitious process are those that would make the elastic response of a simple oscillator identical to the elastostatic solution under P_m.

In this way finite bounds can be obtained for values of P_m smaller than 1.06 P_y , value representing the dynamic shakedown load for the discretized frame (refs.|4|-|5|). The total plastic work is unbounded for larger P_m if the frame is subjected to a step pulse. Since the original pulse ceases after a certain time, actual displacements are finite, but the proposed technique fails to estimate them. However it was shown in ref.|6| that displacements produced by long duration pulses of high peak value are large, and hence can hardly be evaluated through methods which can account for geometrical effects only in the framework of second order theory.

Bounds on total (U) and plastic (U^P) displacements obtained with this method are plotted in fig.3, where the maximum "elastic" displacement u^{el} is also indicated. The latter is defined as the maximum displacement of the structure supposed to behave in a linearly elastic manner.

(D) The original pulse eq.(9) is considered as an impulse $I = P_m t_r$ applied at $t = 0$ on the structure at rest. Thus loading is of the type of the one described by eq.(5) and, as mentioned before,finite bounds can be reached for any intensity of I , since a zero fictitious process can always be assumed. Strictly speaking, the bounds so obtained are not necessarily upper bounds to the displacements due to the actual rectangular pulse, but the approximation is believed to be reasonable for short duration pulses.

For $P_m = .8\ P_y$, the bounds on total and plastic displacements obtained through the four previous assumptions are indicated by means of solid and dashed lines, respectively, in fig.4, as function of the pulse duration. The dash-dot line illustates the maximum elastic displacement. No damping was considered. Presence of damping will in general reduce the amplitude of the fictitious process response; hence, it will slightly improve the results obtained as in points (A) and (B) but not those concerning points (C) and (D). In fact, in the last two cases the fictitious response is independent of damping.

The increase in frame stiffness caused by the presence of tension in the members was also considered, in the spirit of second order theory. However bounds decrease only by a negligible amount (never more than about 5%).

4. Comparison of Bounds with Rigid-Plastic Solutions

Bounds on plastic displacements reached as indicated in point (B) above are now compared to the results obtained in ref.|6| under the assumption of rigid-plastic material behavior. This comparison is to be intended as purely qualitative, due to the different physical meaning of the two quantities: the actual values of plastic displacements, as defined in Section 1, are different from the values of the displacements developed in a rigid-plastic structure, above all when of the same order or smaller than the elastic displacements. In the same spirit, bounds on total displacements are compared to the sum of elastic and rigid-plastic results.

Bounds on plastic displacements turn out to be a very simple function of the total impulse, precisely:

$$U_I^P = 3.34\ H\ (P_m t_r / P_y t_o)^2 \qquad (10)$$

This value is plotted, in nondimensional form, as line 3 in fig.5. Line 1 (u^{RP}) reproduces the rigid-plastic results illustrated in fig.19 of ref.|6|. It appears that for $P_m/P_y \geq 2$, eq.(10) overestimates the rigid-plastic displacements by an amount ranging from 70% to 150%.

A smaller estimate can be obtained by assuming that the elastic motion of the frame is according to its first mode. The frame is thus substituted with a simple oscillator whose stiffness is given by the ratio between the load and the elastostatic displacement at the side center and whose mass can be obtained on the basis of the first natural frequency of the frame ($\omega = 2\pi/.17$). Under this assumption the following value is obtained:

$$U_{II}^P = 2.078\ H\ (P_m t_r / P_y t_o)^2 \qquad (11)$$

plotted as line 2 in fig. 5. It can be seen that, for $P_m/P_y \geq 2$, eq.(11) overestimates u^{RP} by less than 30%. In fig.5 are also plotted, with dashed lines, the "errors" ε, defined as

$$\varepsilon \equiv (U^P - u^{RP}) / u^{RP} \qquad (12)$$

The difference in meaning between "plastic" and "rigid-plastic" displacements is apparent for values of P_m of the order of P_y. In fact under the rigid-plastic hypothesis, deformations cannot be produced unless the applied pulse is at least as large as the static collapse load, while, in the elastic-plastic case, inertia effects can produce yielding also for $P_m < P_y$. (On the other hand, eqs.(10) and (11) give nonzero plastic displacements also for values of P_m/P_y for which the actual motion is still in the elastic range). This difference makes the "error" defined by eq.(12) unbounded for $P_m \leq P_y$.

Clearly the plots in fig.5 are meaningful only for t_r small enough for the response to be in the small deformation range, since line 1 was obtained under this hypothesis and the procedure of point (D) is questionable for long duration pulses.

Results concerning a rectangular pulse of peak value $P_m = 2\ P_y$ and various durations are illustrated in fig.6. Lines 1 and 2 show the linear elastic (u^{el}) and rigid-plastic (u^{RP}) displacements (the latter from ref.|6|), respectively. Line 2 exhibits a plateau for $u^{RP} = 2.22$ H, due to the fact that the hexagon is distorted into a shape that can statically withstand the applied load; hence the displacement is in the small deformation range only for $t_r \leq .4\ t_o$. Lines 3 and 4 (dashed) show plastic bounds given by eqs.(11) and (10) respectively. Line 5 indicates the bound on total displacement u obtained through eqs. (2)-(4) and the hypothesis (D) in Section 3 . The "errors" are also shown: the one concerning plastic displacements (line 6) is defined by eq.(12), while the "error" on total displacements (line 7) is taken as

$$\varepsilon_t \equiv (\ U - (u^{el} + u^{RP}))\ /\ (\ u^{el} + u^{RP}) \qquad (13)$$

Bounds seem reasonably useful when the response is in the small deformation range.

Finally the effects of different pulse shapes on bounds were studied. In ref.|6| it was suggested that any pulse can be substituted by an equivalent rectangular pulse of peak value P_e and duration t_e whose impulse is defined as

$$I = P_e t_e = \int_{t_y}^{t_f} P(t)\ dt \qquad (14)$$

where t_y is the time at which plastic deformations first occur and t_f can be approximately computed as

$$t_f \doteq t_y + (\ I/P_y) \qquad (15)$$

Fig.7 illustrates the results concerning the exponential decay pulse:

$$P(t) = P_m \exp\ (\ -t/t_r) \qquad \text{for } t \geq 0$$

$$P_m = 2\ P_y \quad ; \quad t_r = .01\ t_o$$

whose peak value and total impulse are equal to the ones given by a rectangular pulse, eq.(9), defined by the same P_m and t_r . Solid lines show the pulse and the rigid plastic displacement (from ref.|6|) as function of time. The maximum value turns out to be $u^{RP}_{max} = 1.88\ H \cdot 10^{-4}$. The bound U^P_{II} on the final plastic displacement (eq.(11)) obtained using an impulse $I = P_m t_r$ is shown by the dash-dot line, and it is more than twice u^{RP}_{max} . The dashed line shows the result obtained with the equivalent rectangular pulse defined by eqs. (14)-(15) : the "error", eq.(12), is 41.5%. It was assumed $t_y = 0$. It seems that also in the present context any pulse shape can be replaced by an equivalent rectangular pulse defined as in ref.|6|.

5. Conclusions

On the basis of the previous discussion the following conclusions can be drawn.

(i) Meaningful results cannot be obtained by formally applying the bounding techniques illustrated in Section 1 to the actual pulse shape.

(ii) For long duration rectangular pulses it is possible to substitute the original excitation with a step pulse of unlimited duration, provided that the peak value is smaller than the shakedown load of the structure. This technique yield unlimited bounds for high peak values, for which, however, the actual response is in the large deformation range.

(iii) For short duration pulses, good estimates can be easily obtained by substituting the original pulse with an equivalent impulsive excitation. For nonrectangular pulses, the equivalent excitation can be defined, as in ref.|6|, through eqs.(14)-(15).

(iv) Long duration pulses of high peak value cause displacements of the order of H . The proposed technique fails in this circumstance, since it can account for geometrical effects only within the framework of second order theory. The increase in frame stiffness due to tensile membrane forces makes the bounds decrease only slightly and in no case it is able to represent the substantial increase in global stiffness due to the change of shape.

(v) Finally it can be mentioned that the impulsive technique proposed in point (D) of Section 3 is very simple and requires, even for complex structures, a moderate computational effort. This fact suggests that some shortcomings inherent to the drastic idealization of the duct as a plane frame could be easily overcome and more realistic models used.

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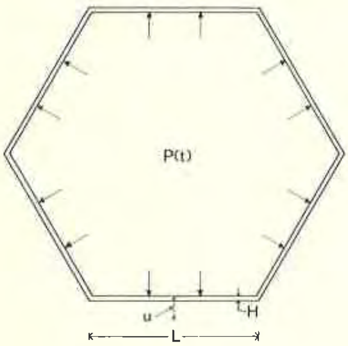

Fig. 1 . Duct Cross Section

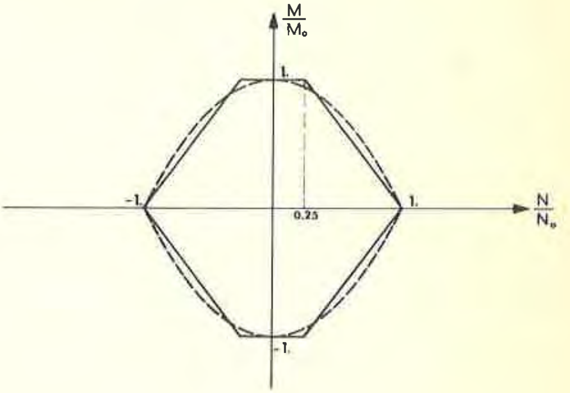

Fig.2 . Yield Condition

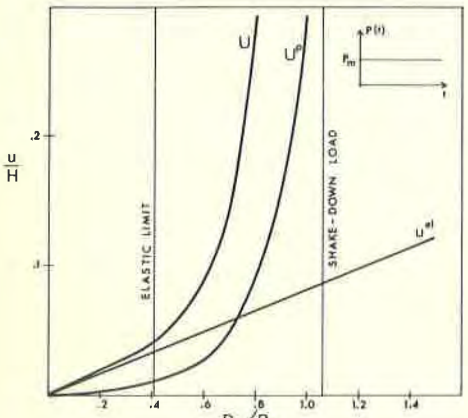

Fig.3 Bounds for Step Pulse

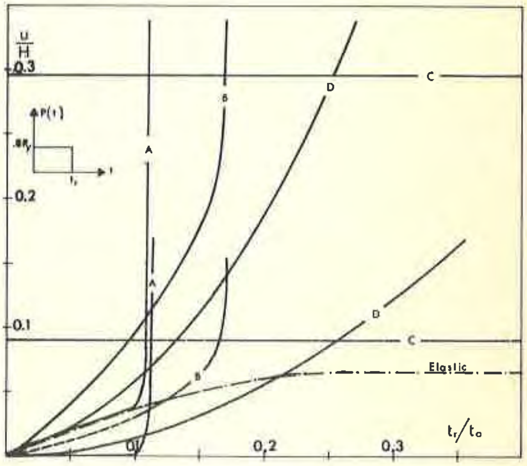

Fig.4 . Comparison of the Various Proposed Techniques for Rectangular Pulses

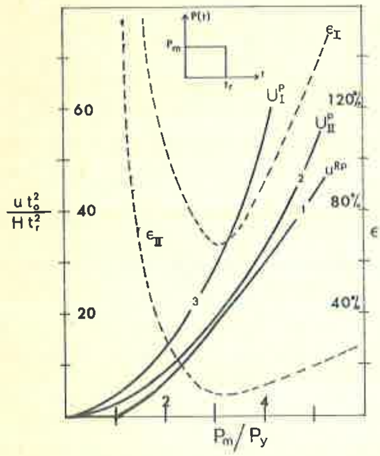

Fig.5 . Comparison of Bounds and Rigid-Plastic Solutions

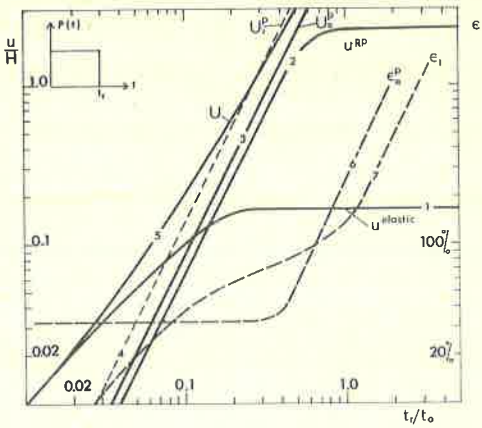

Fig.6 . Bounds as Function of Rectangular Pulse Duration

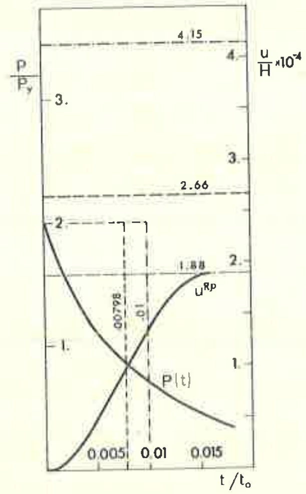

Fig.7 . Influence of Pulse Shape.