A LMFBR MODEL FOR COMBINED STUDIES
OF FLOW AND ACOUSTIC VIBRATION

P.G. BENTLEY, A.E. COLLINSON, D. FIRTH, A.F. TAYLOR
United Kingdom Atomic Energy Authority,
Risley Engineering and Materials Laboratory, Risley, Warrington WA3 6AT, United Kingdom

SUMMARY

The feasibility of a hydroelastic model which uses water in place of sodium is proposed as part of the programme of vibration studies for Liquid Metal Fast Breeder Reactors. Vibrations may be excited by local turbulence in the flow, acoustic energy generated by the circulating pump and transmitted through the fluid, or by sloshing wave motions at the free surface of the sodium pool. As a condition for modelling the three driving forces simultaneously, and hence ensuring that any interactive effects are represented, dimensionless groups are derived for amplitude and frequency in each case. It is shown that ideal modelling requires a geometrical scale of 0.392 and that frequency and flow velocity scales are also fixed. The effects of picking a smaller geometrical scale are examined and it is concluded that turbulence and acoustic mechanisms can be precisely simulated but that free surface waves are modelled with reduced accuracy.

Structural resonances are examined, both shell flexural and beam swaying modes and the problems of compensating for changed liquid loading discussed. Proposals are made for using steel and brass as modelling material for different components and for adjusting the vibration characteristics by thinning down shell structures. Vibration amplitudes are checked in representative cases and are shown to scale in the same ratio as geometrical scale if vibration damping remains unaltered.
1. Introduction

An elevation of a typical pool type sodium cooled fast reactor is shown in Figure 1. Sodium from the main outer pool is taken up by centrifugal pumps and passed via high pressure ducts to the core. On leaving the core hot sodium passes into the above core structure and thence to intermediate heat exchangers where the core heat is exchanged to a secondary sodium circuit, while the primary flow returns to the main outer pool. The driving head required to pass the primary flow through the heat exchangers is provided by a level difference between the free surfaces of the hot and cold pools. All the major reactor components are of stainless steel and are suspended from the reactor roof.

Economic incentives for compactness in the pool type reactor design result in complex flow paths with high turbulence levels which can excite components into vibration either directly or through sloshing wave motions of the free surface. The design can also potentially allow the acoustic energy generated in the pump to be easily translated to component vibration via acoustic modes in the fluid.

If vibration damage occurs after commissioning, remedial action is exceedingly difficult so that there is considerable incentive during the design/development stages to ensure that no significant vibration occurs. This paper summarises a study which has been carried out to determine whether a hydroelastic model using water can be built to reproduce realistically combined flow and acoustic induced vibration amplitudes.

This problem is examined in three parts. Firstly the driving forces are considered which are entirely a function of the fluid, and simple mathematical equations are derived which include the geometrical scale, velocity scale and frequency scale. These equations can be simultaneously satisfied and they determine the absolute values of the principal scales. For example the geometrical scale must be 0.392. Later in the paper the detailed effects of departing from these ideal values are considered.

Secondly the structure is examined to see how it should be designed to have the correct resonant frequencies. This is most important because some of the driving forces are sinusoidal or narrow band random so that maximum vibration is critically dependent on the coincidence of driving frequency and structural resonance frequency. Thirdly the vibration amplitudes at resonance are checked. There is no freedom to adjust the design at this point since all aspects are determined by the previous arguments.

2. Driving Forces

There are three distinguishable fluid driving forces.

2.1 Fluid Flow Forces

Components are induced to vibrate by the unsteady flow forces acting on them in a turbulent flow situation. The fluctuating fluid forces can broadly be characterised by a force coefficient defining the magnitude of the fluctuating force together with either a discrete frequency of oscillation or a mean frequency in a frequency range containing the fluctuating energy.

The force is proportional to the dynamic head \( (p_0 v^2) \) and the area of the component \( (L^2) \). The frequency is determined by the steady flow velocity and the size of the component. Non dimensionally these can be expressed as

\[
F \over p_0 v^2 L^2 = 0 \quad D
\]  

(1)
\[ \frac{f_a L}{V} = \text{Str} \]  

(2)

where \( V \) is the mean steady flow velocity, \( L \) is a characteristic dimension, \( \rho_o \) is fluid density, \( F \) is the magnitude of the fluctuating force, \( f_a \) is the frequency (e denoting 'eddies'), \( c_D \) is the non-dimensional drag coefficient for fluctuating forces, \( \text{Str} \) is the non-dimensional frequency (Strouhal number).

Both drag and Strouhal numbers are expected to be independent of scale and fluid so they can be substantially the same in the reactor and model. However they are known to be dependent on Reynolds number in some circumstances. Reynolds numbers for the major reactor components of interest lie between \( 2 \times 10^6 \) and \( 2 \times 10^7 \) compared with model values of between \( 10^5 \) and \( 10^6 \) for realistic scales and velocities. Literature surveys have so far not revealed any strong Reynolds number dependence over this range for highly turbulent flow conditions. This is discussed by Beavers and Plunkett [1].

2.2 Acoustic Forces

Acoustic energy is generated by the pump impeller blades passing the non-rotating structure. The frequencies are the fundamental and harmonics of blade frequency so that in dimensionless form

\[ \frac{f_a}{n} = A \]  

(3)

where \( n \) is the pump rotational speed, \( f_a \) is the acoustic frequency, \( A \) is a non-dimensional frequency.

The pump source power depends on the Mach number given by

\[ \frac{V}{C_o} = M \]  

(4)

where \( C_o \) is velocity of sound in the fluid, \( M \) is dimensionless Mach number.

In general form the pump source power is the energy produced by relative motion of a solid body and a fluid which is, from Lighthill's work (see Richards and Mead [2])

\[ \frac{I}{\rho_o L^2 V^N M^n} = P \]  

(5)

where \( I \) is the source power, \( n \) is an index which depends on the mechanism of sound generation, \( P \) is a non-dimensional pump power.

Given identically scaled pumps, both the non-dimensional frequency \( A \) and the non-dimensional power \( P \) will be the same in the reactor and model. Since the index \( n \) is unknown it is desirable to keep Mach number \( M \) the same so that the true source power \( I \) changes in a predictable way with \( \rho_o \), \( V \) and \( L \).

Finally the transmission of acoustic energy through the fluid is determined by its acoustic resonant modes characterised by a non-dimensional wave number which is the same for model and reactor

\[ \frac{f_a L}{C_o} = N \]  

(6)

where \( N \) is the non-dimensional wave number.

2.3 Free Surface Effects

The model should operate with scaled liquid levels as gross departures from these will affect the flow patterns. This requires that Froude number remain constant between model and reactor if circuit resistances are also in scale. That is
\[ \frac{V^2}{g} = Fr \]  
(7)

where \( g \) is acceleration due to gravity, \( Fr \) is non-dimensional Froude number.

The gravity controlled surface oscillation (sloshing driving force) produced by turbulence in the pool will have a wave height amplitude proportional to the dynamic head and therefore the wave height will scale.

The sloshing frequency is related to the length scale and gravity acceleration (Rathbone [3]) by

\[ f_m \sqrt{\frac{h}{g}} = W \]  
(8)

where \( f_m \) is the sloshing wave frequency, \( W \) is the non-dimensional wave frequency.

3. The Ideal Scales

It is important that frequency is scaled in the same ratio for each of the driving forces. Once the frequency scale is chosen, the modal structural resonances will be fixed accordingly and cannot be adjusted in operation. Therefore

\[ \frac{f_m}{f_r} = \frac{f_m}{f_r} = \frac{f_m}{f_r} = \frac{f_m}{f_r} = \frac{f_m}{f_r} \]  
(9)

where \( m \) denotes model, \( r \) denotes reactor, \( f_m \) is structural resonant frequency.

This equation determines the frequency, length and velocity scales as follows. For the turbulence (eddy) and acoustic frequencies in equation (9) using equations (2) and (6)

\[ \frac{f_m}{f_r} = \frac{V_m/L_m}{V_r/L_r} = \frac{C_{om}/L_m}{C_{or}/L_r} \]  
(10)

that is, the model flow velocity is fixed independent of geometrical scale at

\[ \frac{V_m}{V_r} = \frac{C_{om}}{C_{or}} = 0.625 \]  
(11)

The frequency scale for turbulence (eddy) and acoustic effects is then

\[ \frac{f_m}{f_r} = \frac{C_{om}/L_r}{C_{or}/L_m} = 0.625 \frac{L_r}{L_m} \]  
(12)

3.1 Geometrical Scale

Including the free surface (wave sloshing) frequency requirement, the combined result of equations (8) and (12) is

\[ \frac{\sqrt{g/L_m}}{\sqrt{g/L_r}} = \frac{C_{om}/L_r}{C_{or}/L_m} \]  
(13)

That is, the geometrical scale must be fixed absolutely at a value

\[ \frac{L_m}{L_r} = \left( \frac{C_{om}}{C_{or}} \right)^2 = 0.392 \]  
(14)

4. Structural Resonances

The problems of modeling structural resonances (in principle) are independent of the geometrical scale. Once the frequency scale is fixed, some technique is required to adjust the resonances accordingly. There are basically two techniques, the first to use a compensating structural material (which turns out to be something like brass), the second to use stainless steel with the thickness scaled slightly differently from the overall geometrical scale. Structural resonances are conveniently divided into shell or beam and stretching modes.
4.1 Shell Modes

There are four dimensionless groups which can be derived from Bentley and Firth's [4] study of simple structural resonances:

a. The frequency scale from equation (12)

\[ K_1 = \frac{f L}{c_0} \]  

(15)

b. The restraining effect of structure on the liquid

\[ K_2 = \frac{\rho_s}{\rho_w} \left( \frac{C_o}{C_L} \right)^2 \]  

(16)

where \( h \) is the material thickness and \( C_L \) is the velocity of sound in the structure. (\( K_2 \) represents the modification to the frequency scale of acoustic resonances in ducts)

c. The structural shell modes in vacuo

\[ K_3 = \left[ \frac{\rho_s (1-\sigma^2)}{E} \right] \frac{f L^2}{h} \]  

(17)

where \( E \) is Young's Modulus, \( \rho_s \) is density of the structure, \( \sigma \) is Poisson's Ratio.

d. The effect of liquid loading on structural modes

\[ K_4 = \frac{\rho_s}{\rho_w} \]  

(18)

In these four equations, length and thickness scales are distinguished. It is convenient to define a thickness scale ratio

\[ \alpha = \frac{h_w}{h_L} \]  

(19)

which represents the departure from strict geometrical scaling.

First consider the solution of the four equations with \( \alpha = 1 \). A little manipulation shows that they can be simultaneously invariant for a structural modelling material having the properties

\[ \frac{\rho_{em}}{\rho_{sr}} = \frac{\rho_{cm}}{\rho_{cr}} = 1.165 \]  

(20)

\[ \frac{\sigma_{em}}{\sigma_{sr}} = 1 \] (may)  

(21)

\[ \frac{E_{em}}{E_{sr}} = \left( \frac{\rho_{cm}}{\rho_{cr}} \right)^2 \frac{\rho_{em}}{\rho_{sr}} = 0.456 \]  

(22)

That is a material which is slightly more dense and considerably less springy than stainless steel; brass is probably the nearest with an \( E \) ratio of 0.56.

An alternative is to choose stainless steel but allow \( \alpha \) to be non unity. In that case all the four equations cannot be satisfied perfectly. However the restraining effect of structure on the fluid (equation (16)) is often negligible and furthermore, in practice equations (17) and (18) can often be combined to get an expression for the liquid loaded structure.

\[ K_5 = K_3 K_4 \alpha^{\frac{1}{2}} = \left[ \frac{\rho_s (1-\sigma^2)}{E} \right] \frac{f L^2}{h^{3/2}} \left( \frac{\rho_o}{\rho_s} \right)^{\frac{1}{2}} \]  

(23)

(This is a valid combination when the mass of liquid loading predominates over the structural mass, a common situation in an LMRHR, particularly at low frequencies.) This results in \( \alpha = 0.73 \) for stainless steel.
4.2 Beam and Stretching Modes

A similar analysis for beam modes shows that the ideal modelling material has the same physical properties as for shell modes. If thinning of stainless steel is considered equation (17) becomes

\[
X_3 = -\left[ \frac{\rho_0 (1 - \alpha^2)}{E} \right]^{\frac{1}{3}} r L
\]

and combining this with \(X_4\) gives \(\alpha = 0.39\).

For stretching modes which could occur in the primary tank or the high pressure ducts, the ideal structural material is again similar to brass and \(\alpha\) again turns out to be 0.39 for stainless steel.

5. Frequency Calculation Examples

Table 1 shows calculated frequency ratios for a 0.3 scale model. For shell modes in the primary tank and ducts there is little difference between steel with \(\alpha = 0.73\) and brass \(\alpha = 1.0\). For bending and stretching modes brass is much better with errors of less than 10%.

6. Structural Response

The displacement amplitude should ideally remain in scale with the model to reflect any interaction effects. No free variables remain to adjust the response so that amplitudes can only be evaluated.

6.1 Response of Shell Modes to Acoustics

Displacement amplitude is determined by the acoustic source intensity, the response of the structure and its damping. Coupling between liquid and structure, which is part of its response characteristic, forms a difficult subject which is only just beginning to be understood. However the details of this problem can be avoided and some estimates made as follows. It seems that by correct geometric modelling of the fluid circuit and shell components there will be the same energy distribution in the model and reactor. That is, the power density at any point will be proportional to the energy provided by the pumps.

For shell vibrations where liquid loading is important, most of the energy resides in the fluid through its motion adjacent to the shell wall and is given by Firth [5]

\[
E = \rho_o V^2 r^2 \delta^2
\]

where \(\delta\) is the shell vibration amplitude.

The component loss factor determines how this energy is dissipated and therefore gives the power loss which must be made up by pump energy

\[
P = \rho_0 V^2 \delta^2 \eta
\]

where \(\eta\) is component loss factor per cycle.

If the pump power given by equation (5) remains in the same proportion to the power dissipated than from equations (5) and (26) the amplitude scaling ratio \(\beta\) is given by

\[
\beta = \alpha L^2 \left( \frac{V}{L} \right)^2 \left( \frac{E}{r} \right)^{\frac{1}{2}} \left( \frac{f L}{E r^3} \right)^{\frac{1}{2}} = \left( \frac{V}{L} \right)^2 \left( \frac{f L}{E r^3} \right)^{\frac{1}{2}} \chi^{1/2}
\]

Both Strouhal number (the second term in the right hand side of this equation) and Mach number are invariant from the requirement to keep frequency scales the same. Hence the result is, apart from \(\eta\), the ideal value of \(\beta = 1.0\).
In the other extreme case where all the energy is in the structure, a similar analysis can be made from Firth [5] to give

$$\beta = \left( \frac{\rho_{\text{cm}}}{\rho_{\text{or}}} \frac{P_{\text{cm}}}{P_{\text{or}}} \right)^{1/2}$$

(28)

making $\beta = 1.25$ for modelled steel shells and $1.04$ for brass.

In most cases of practical importance the situation will be $\beta \approx 1.0$ and the departure from ideal scaling is always less than $25\%$.

6.2 Response of a tube in Beam Mode to Fluid Flow

The displacement amplitude is determined by the turbulent driving force, the response of the structure and its damping. In general the amplitude can be written

$$S = D + \frac{\text{Driving Force (P)}}{\text{Stiffness (S)}}$$

(29)

where $D$ is the dynamic magnifier.

For a beam the stiffness is

$$S = E I \left( \frac{1}{L} \right)^3 C_n$$

(30)

where $I$ is the moment of inertia ($\pi h^4/4$ for a tube) and $C_n$ is an eigenvalue for the mode $n$.

Combining equation (30), (29) and (1) gives for $C_D'$ and $C_n$ invariant

$$\beta = \left( \frac{D}{D'} \frac{E}{E'} \frac{\rho_{\text{cm}}}{\rho_{\text{or}}} \right) \left( \frac{V}{V'} \right)^2 \frac{1}{\sigma}$$

(31)

Thus when $D = D', E = E', \rho = \rho$, and with equations (18) and (24) $\beta = 1$. If brass is used with $\sigma = 1$, then $\beta$ will be in error by $20\%$ due to the difference between E brass and E for the ideal material.

7. Damping

A study has been carried out on the various damping mechanisms and it has been concluded that damping is unlikely to remain constant between model and reactor due to material and environment differences so that interpretation will depend on a feed back of information from full scale components in air, water and sodium. However general indications are that model amplitudes will be too large when scaled to the reactor.

8. Scales less than the ideal of 0.392

The predominant factor that determines scale is the requirement to keep sloshing frequency and amplitude in scale. It can be shown that scale reduction to 0.3 would produce an ideal amplitude error of $20\%$ and a frequency error of $10\%$. As the modelling accuracies of other aspects previously examined are of this magnitude, it would seem reasonable to allow the scale to decrease towards 0.3. Further scale reductions result in rapidly increasing errors.

9. Discussion

Eddy and acoustic exciting frequencies are simultaneously simulated by equality of Mach numbers in model and reactor, thus fixing the flow velocity, independent of scale chosen. The scale is then fixed by a Froude similarity which then also brings free surface sloshing effects on to the same timescale. The ideal scale is 0.392. There appear to be no problems of pump simulation but errors could be introduced by Reynolds number effects on eddy frequencies although from work to date these do not seem significant.

Structural resonances can be set independently to the same frequency scale, independent of linear scale by the use of a modelling material, the relevant physical properties of which
are determined by the driving force frequency simulation. The nearest available material is brass, which for exact geometric scaling could introduce errors in resonant frequency scale of about 10%. As an alternative, where particular modes are expected, reactor materials can be used with modified wall thickness where liquid loading predominates; if this is practically more convenient, eg shell modes.

Amplitude response is shown to be within about 20% of the scaled value using brass or steel with adjusted wall thickness if damping is constant. A study of damping has indicated that this could be different between model and reactor, so that feedback of information from prototype damping measurements in air, water or sodium will be required for ultimate interpretation of reactor amplitudes.

A linear scale reduction down to 0.3 would produce errors in sloshing frequencies and amplitudes of about 10% and 20% respectively. These errors are considered acceptable as both eddy and sloshing driving forces are likely to spread over a frequency band of about 20%.

An additional problem not discussed in the paper is that of interaction of component vibration through the roof structure.

The authors' judgement is that a model can be built in water at a scale between 0.3 and 0.4 to represent adequately flow and acoustic induced vibration of major reactor components.

10. Conclusions

A model can be built to represent adequately both flow and acoustic vibration including free surface effects at a scale between 0.3 and 0.4.

Structural resonances can be adequately modelled, brass being near to the ideal material but stainless steel can be used for large components where bending and stretching modes are unimportant. Frequency scaling is in line with eddy and acoustic frequencies to within about 10%.

Amplitude response is adequately modelled except for damping, where uncertainties have to be allowed for by measurement on full size components.

References


### TABLE 1

**Examples of Computed Frequencies and Frequency Ratios for Idealized Components**

Geometrical Length Scale = 0.3. Frequency Scale for Driving Forces \((f_m/f_r) = 2.08\).

<table>
<thead>
<tr>
<th>Component</th>
<th>Mode</th>
<th>Steel</th>
<th>Brass</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(\alpha)</td>
<td>(f_r) (Hz)</td>
</tr>
<tr>
<td>Primary Tank</td>
<td>Shell</td>
<td>0.73</td>
<td>112</td>
</tr>
<tr>
<td></td>
<td>Bending</td>
<td>0.73</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>Stretching</td>
<td>0.73</td>
<td>7.5</td>
</tr>
<tr>
<td>Control Rod Shroud</td>
<td>Bending</td>
<td>0.26</td>
<td>14.9</td>
</tr>
<tr>
<td>High Pressure Duct</td>
<td>Ovalling</td>
<td>0.73</td>
<td>47.2</td>
</tr>
<tr>
<td></td>
<td>Bending</td>
<td>0.73</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>Acoustic</td>
<td>0.73</td>
<td>123</td>
</tr>
<tr>
<td>Sodium Pool</td>
<td>Acoustic</td>
<td>-</td>
<td>152</td>
</tr>
</tbody>
</table>

* Note: \(\alpha\) is adjusted to allow for the difference between \(E\) values of brass and the ideal material. The frequency error is due to the mass of the tube.

+ This is an idealized situation where liquid loading predominates over tube mass.
Fig 1 Pool Type Sodium Cooled Fast Reactor (Elevation)