ANALYTICAL AND EXPERIMENTAL STUDY OF TWO CONCENTRIC CYLINDERS COUPLED BY A FLUID GAP

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SUMMARY

A breeder reactor vessel is a substantial steel cylinder which is partially protected from the nuclear reaction temperature by a "thermal liner"—a relatively thin concentric shell separated from the vessel by a narrow fluid-filled gap. This paper describes an experimental and analytical study of the vibration of a model of such a shell used in the U.S. Atomic Energy Commission's Fast Test Reactor.

The analytical work consists of two parts, the first uses a free vibration solution of the shell in vacuum as a basis for extrapolating vibration behavior for the same shell with a fluid gap. The free vibration analysis is carried out both by the finite element method and by a Rayleigh Ritz formulation. The extrapolation is based on coefficients computed analytically for a structure of the same geometry but simple boundaries. The second part consists of a direct finite element solution of the actual problem—the shell and the fluid coupling it to the rigid outer container. All the finite element computations were carried out using the NASTRAN program. By allowing the boundary condition at the bottom to vary over a range of values in the finite element solution, it was concluded that the degree of flexure restraint provided at this boundary did not affect results significantly. This was not true for the radial flexibility of this boundary.

The experimental setup consisted of a steel sheet rolled and welded into a cylinder, free at the top edge and at the bottom soldered to a disc which in turn was bolted to a heavy base plate. This setup was intended to provide a fixed condition at the bottom of the cylinder. The fluid gap was provided by using a thick concrete shell as the outer cylinder. A series of these cylinders was used to provide several sizes of annular gap. The case of the steel shell alone, without fluid, was also considered. The steel cylinder was vibrated by an electromagnetic exciter using both harmonic loading and random loading functions. This exciter was mounted on a relatively rigid support on the inside of the cylinder, near the top edge of the shell.

In general, correspondence of experimental and analytical results is within acceptable limits; however, several vibration modes corresponding to solutions with low circumferential wave numbers were not detected experimentally. Response analysis performed to compare the response amplitude at various modes indicates that the intensity at the modes in question is very low. The feasibility of using coupled fluid-elastic finite element analysis to solve vibration problems has been demonstrated; however, the computer time costs involved prohibit the extensive application of this technique at present.

1. Introduction

From a structural point of view a liquid coolant type nuclear reactor consists of a heavy steel vessel containing the core and related mechanical components, filled with a hot fluid. To some extent this vessel is protected from the severe environment of the core by a shielding structure, the thermal liner, which is usually a relatively thin steel cylinder concentric with the reactor vessel and separated from it by a gap filled with the coolant fluid. This arrangement leads to a potential vibration problem if the fundamental frequency, or one of the higher natural vibration frequencies, of this liner system is close to the frequency of some vibration source present in the reactor vessel. The natural vibration frequency of the liner shell vibrating in a vacuum is readily calculated by generally available techniques [1,2]; however, it is felt that the influence of the fluid cannot be ignored since it may reduce the fundamental frequency by a factor of two to five and lower it into a range in which strong vibration sources may be present. Some natural frequency data for the case of a cylindrical shell filled with liquid has been presented by Arya, et al. [3]; but, the case of a cylindrical shell coupled to a concentric shell through a thin fluid gap is only covered for simply supported conditions at both top and bottom - see Chen and Rosenberg [4] and Krajcinovic [5]. The study described here was undertaken to provide information for the shell rigidly clamped at its base and free at the top since this is a better description of the conditions encountered in typical reactor designs.

The dimensions of shell considered in this report were selected to model the liner used in the Fast Flux Test Facility reactor designed for the U. S. Atomic Energy Commission. The scale factor is approximately 1/14, giving nominal dimensions of 20.5 inches height, 17 inches diameter, and 1/16 inch thickness. The behavior of prototype coolant liquid, sodium, is modeled by water which has density, compressibility, and viscosity properties that are adequately representative. In identifying mode shapes, this report will use the conventional designations: n to signify the number of complete waves in the circumferential direction (n=0 - axisymmetric, n=1 - translation, n=2 - "ovalling," etc.); and, m to signify the modal characteristic in the longitudinal direction (m=1 - simple cantilever beam type mode, m=2 - a mode with one nodal circle, etc.).

2. Experimental Setup

The experimental model was fabricated by rolling a 0.058 inch thick, 20.125 inches wide steel plate, seam welding, and soldering to a 1/2 inch brass base plate which in turn was securely bolted to a 6 x 17 x 38 inch steel block. Relatively rigid concrete outer cylinders were formed by casting around the cylindrical model utilizing a hard durometer neoprane spacer to obtain the nominal annular gap desired. The concrete was water proofed and attached to the steel base as shown in Fig. 1.

The average outside diameter of the fabricated shell (17.08 inches) was determined by taking the average of 36 measurements. The maximum diametral deviation was less than 0.5%. The shell wall thickness (0.058 inch) was more uniform. The annular gap size (0.151, 0.253, 0.538, 1.03, and 2.94 inches) was determined by measuring the volume of water necessary to fill the annular region between the concrete shell and the steel shell to various levels. For any of the gap sizes used the maximum deviation from the average value was about 10%. The geometric accuracy of the model exceeds the uniformity expected in the prototype. Steel density (0.27 lb/in.) and elastic modulus 28 x 10^6 psi) were determined from test strips of the steel plate used in fabricating the shell by weighing and by a cantilevered beam frequency test.

A single exciter coil (280 turns of #16 enamel wire) was situated close to the inside shell wall (with an approximate 3/16 inch air space) providing a magnetic force over approximately 25 square inches of the shell. Current was provided by a 200 watt AV010 amplifier (McIntosh Model MI-200AB). Both a sinusoidal and a wide-band random current signal, controlled by signal generators (Hewlett Packard's Model 203A variable phase function generator and Model HOI-3722A noise generator) were utilized during testing. The motion of the top of the test cylinder was monitored by miniature piezoelectric accelerometers (Endevco Picomin. No. 22) cemented to the inside of the cylinder every 30°, with 0° defined to be opposite the center of exciter coil. In addition, three accelerometers were cemented at equal spaces along the 0° longitudinal line on the cylinder and a movable accelerometer mounted on a magnet was used to search for node points.

3. Testing Procedure

Testing for each water filled annular gap for the cylinder in air consisted of three phases. First, natural frequencies were determined by exciting the shell with a wide band random force and inspecting power spectral density plots produced by a Fourier analyzer (Hewlett Packard Model 5451/A) from the time history signals of several accelerometers. Second, the shell was excited with a sinusoidal current applied to the coil, using a range of frequencies in the vicinity of each of the natural vibration frequencies detected by random excitation. For each natural frequency, f_n, the accelerometer signal in a narrow band about this frequency was processed by the Fourier analyzer to provide a more accurate value of the peak frequency and to establish the RMS acceleration, $\bar{A}$. The corresponding displacement was computed as $\bar{D} = \bar{A}/(2\pi f_n)^2$, which was applicable assuming the system to be lightly damped. The displacement information was plotted to identify mode shapes corresponding to the natural frequencies. In the third phase of testing, the transfer function between the RMS displacement and peak coil current was plotted at discrete points in a narrow frequency band about each natural frequency. The equivalent viscous damping ratio was calculated from these frequency response curves by the half power point bandwidth method as outlined by Thomson [6]. Determination of the damping ratio by log decrement methods and pluck testing was difficult because a pure natural frequency vibration could not be retained without applying an exciting current.

4. Analytical Methods

Analytical solutions to the structural problems were obtained using the NASTRAN [7] finite element analysis program with corroborating solutions obtained by using the SAP IV [8] code and through a Rayleigh-Ritz solution developed by Chung [9]. The structural dimensions used for the analytical work were: height (i.e., length), $\ell = 20.125$ inches; radius to the mid-surface of the shell, $r = 8.505$ inches; shell thickness, $t = 0.058$ inch; elastic modulus, $E = 26.5 \times 10^6$ psi; Poisson's ratio, $\mu = 0.3$; and, shell material mass density, $\rho_s = 701 \times 10^{-6}$ lb-sec^2/in.^{4}. Analytical solutions for the coupled fluid-structure problem were obtained from the NASTRAN code only, using the following additional parameters: fluid mass density, $\rho_f = 93.6 \times 10^{-6}$ lb-sec^2/in.^{4}; bulk modulus, $B = 0.3 \times 10^6$ psi; and, gap sizes as noted above. The basic grid, as used in the reported structural finite element formulations, consisted of 10 divisions vertically and 9 divisions over a quarter of the shell circumferentially (10° segments). Some comparison runs using a finer mesh were made to establish the adequacy of the mesh for the purposes of this study. For example, using a mesh size with twice as many divisions in each direction it was established that the basic grid gives results with the minimum frequencies (which were those with the most error) about 2% too high. Similarly it was found that using a grid with divisions three times larger than the basic grid gives errors of 30 to 50 percent.

The basic grid used in the fluid-structural analyses had 5 divisions vertically, 6 divisions circumferentially (15° segments) and 4 divisions through the fluid in the radial direction. In solving the coupled problem NASTRAN uses a finite element representation of the fluid region. Compressible fluid with small motion is assumed giving a linear (acoustic type) formulation. The fluid is also assumed to be irrotational so that a scalar potential function (pressure) can be used as the solution variable in place of the three components of displacement.

5. Vibration in Air

Before considering the shell vibrating with a fluid, the shell in a vacuum was studied to establish the significance of variations in the boundary conditions and to establish the degree of correspondence to be expected between analytical and experimental results. Figure 2 presents a summary of these results. In all cases the top boundary of the shell was considered free. In comparing analytical results for the fixed base condition (rigidly clamped) to the simply supported base condition (all edge displacements prevented but unrestrained rotation about the tangent), little difference was found. The degree of rotational restraint at the base does not affect the overall results significantly as it is primarily a local flexure condition. Hence it is not considered further. The base boundary condition was then further relaxed by allowing edge motion in the axial direction against an elastic spring restraint, K_Z; using K_Z = infinite, 10^6, 10^5 and 10 lb/in. per inch of circumference. The K_Z infinite case corresponds to the simply supported condition, whereas $K_Z = 10$ which is effectively $K_Z = 0$ corresponds to a shear diaphragm boundary condition. The curves corresponding to each of these conditions are shown in the figure. Comparable results obtained using the SAP IV program gave frequencies about 3% lower; frequencies obtained by the Rayleigh-Ritz method were up to 5% lower.

The results obtained experimentally for the shell vibrating in air (i.e., effectively in a vacuum) are shown as data points in Figure 2. Although the experimental setup was intended to simulate a fixed condition (K_Z infinite) at the base, it can be seen that effectively it is behaving as a shell supported by elastic springs in the axial direction with $K_Z = 10^5$ lb/in.^{2}. Note that both for $m = 1$ and $m = 2$ the modes corresponding to the lower n values were not detected. Difficulties were encountered in this regard and explanations posed here and elsewhere are not totally satisfactory. Efforts to get response of these modes included the use of the local electromagnetic coil described previously as well as acoustic excitation at various amplitude levels applied by a loud speaker. Fourier analyzer processing of the acceleration response to a random excitation did show a frequency at 140 Hz which probably corresponds to the $m = 1$ $n = 3$ mode, but no corresponding response was generated when a single harmonic excitation at this frequency was applied. Frequency response analysis performed by the NASTRAN program indicated that the modes which failed to respond are of somewhat lesser intensity than modes at neighboring frequencies with high n values, but this difference was not large enough to justify the difficulties encountered in detecting these modes. One possible explanation is in the relation between flexural strain energy (associated with high n values) and membrane strain energy as discussed by Croll [10]. The masking effect of the higher n value modes at frequencies close to that of the low n value mode may also be the difficulty. This is particularly suspect in the tests using the electromagnetic exciter since with a pure harmonic current applied to the device the resulting force function carries higher harmonic frequencies of an amplitude up to 20% of the fundamental harmonic. Finally, it should be noted that the frequencies of the low n value mode shapes are highly dependent on the stiffness K_Z, as clearly shown in Fig. 2, so that the relatively undeterminable and possibly nonlinear nature of this restraint may greatly reduce the sharpness of the associated response. However, for the modes that gave a clear response, increasing the driving force by a factor of four lowered the peak response frequency by at most 1/2 % ; hence, support nonlinearity appears to be negligible.

6. Vibration in Water

An estimate of the vibration frequencies of the shell with a fluid gap was first approximated by applying the added mass technique developed by Chen [4] for infinitely long shells to the frequencies obtained for the in air vibration case. A comparison between the experimental results and these extrapolations is shown in Fig. 3. This method appears to give frequency predictions that are somewhat low but generally within 10% of the experimental values.

Figure 4 shows the frequencies predicted by the NASTRAN program for the shell vibrating with four selected water gaps, with the in vacuum solutions shown for comparison. These frequencies are compared to the experimentally measured frequencies in Fig. 3. The directly computed values are between 20 to 30 percent higher than the experimental results; however, by comparing results for problems run at several different mesh sizes, the mesh size used here can be expected to give values between 5 and 20 percent high. A refined computer analysis would probably give results quite adequate for design purposes; however, the present cost of running a fluid-solid interaction problem through NASTRAN with a much finer mesh than that used here, is prohibitive.

7. Damping

For purposes of determining the equivalent viscous damping ratios, the frequency response curve was first determined in the (0.1 - 0.5 g) acceleration range and then re-determined at the highest acceleration level compatible with the available equipment. The ratio of acceleration levels was at least two and usually five to ten. The damping ratio variation with amplitude level was in the 10 to 40 percent range. Figure 5 shows selected average values of damping for the shell vibrating with various water gaps and in air. Clearly the smaller fluid gaps are associated with higher damping ratios than those measured in air, typically twice as large. Generally larger damping is associated with smaller water gaps. For those few cases which do not follow this later trend, the frequency response curves probably were distorted (broadened) by superposition of the response corresponding to an adjacent natural frequency which could not be accounted for in the half power bandwidth computation.

8. Conclusion

Correspondence of experimental and finite element analysis results is within acceptable limits for design purposes. Several vibration modes corresponding to solutions with low n values were not detected; however, the significance of these modes in design, as well as the reasons for the difficulty in detecting them experimentally, are not clear. The feasibility of using coupled fluid-elastic finite element analysis to solve vibration problems involving shells containing a fluid has been demonstrated; however, the computer time costs incurred with the present methods prohibit extensive application of this technique.

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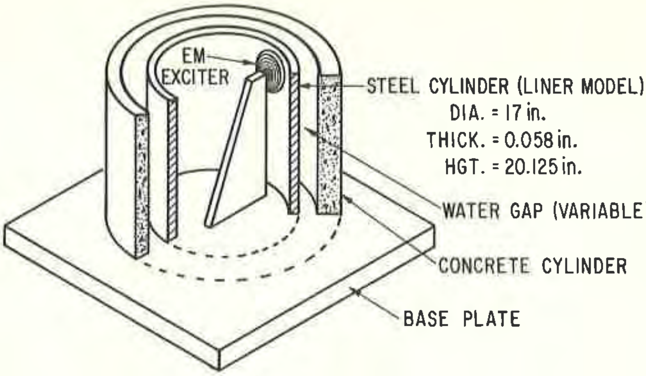

Figure 1. Experimental setup schematic.

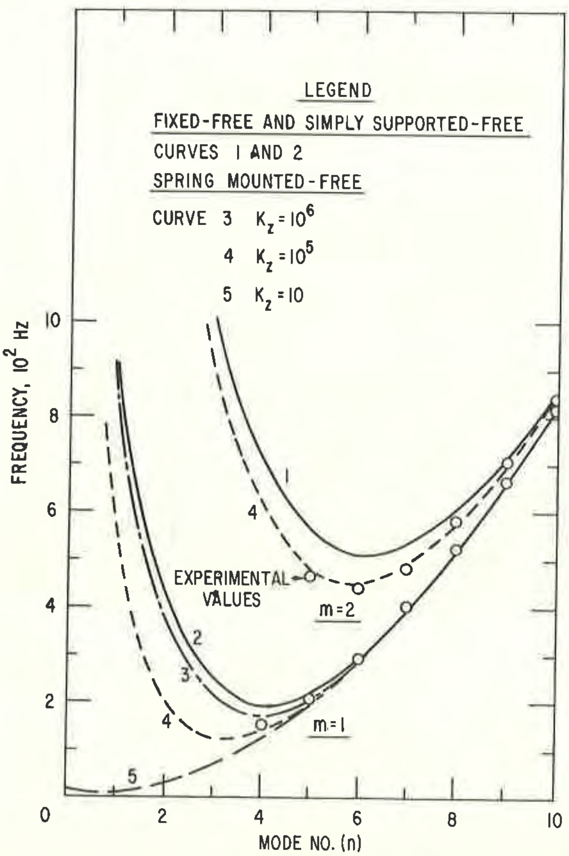

Figure 2. Comparison of experimental and predicted vibration frequencies for the shell without fluid.

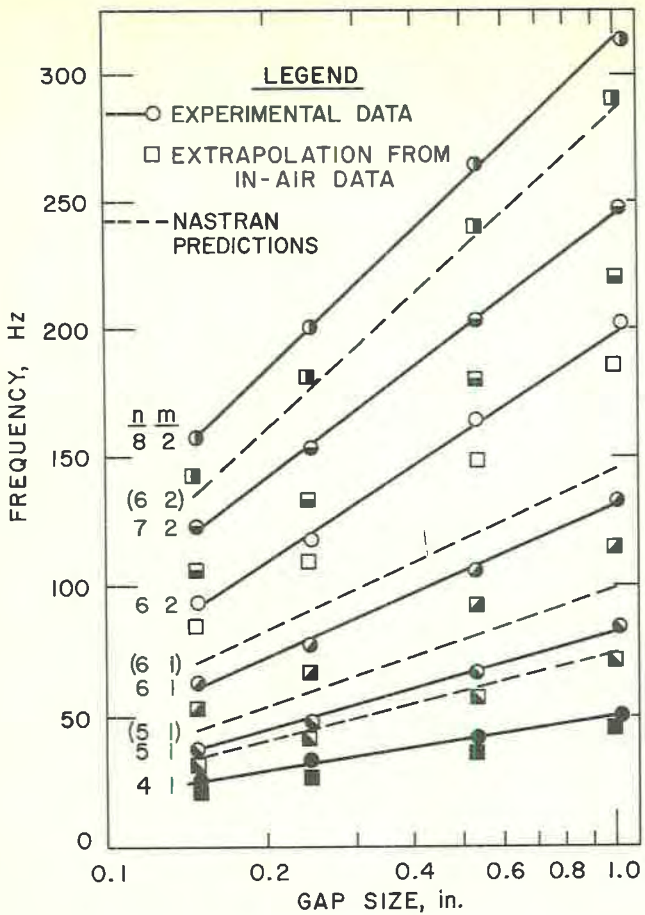

Figure 3.
Comparison of experimental and predicted vibration frequencies for the shell with a fluid filled gap.

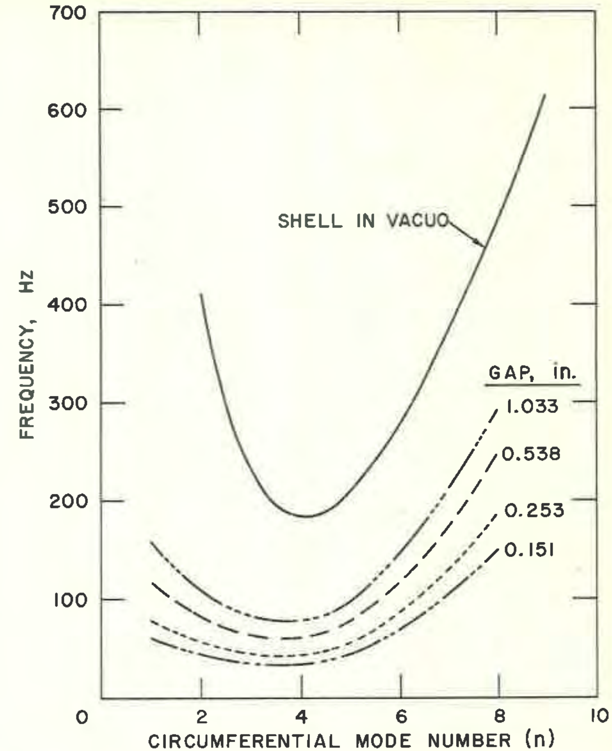

Figure 4.
Computed m = 1 type vibration frequencies for the shell with and without fluid.

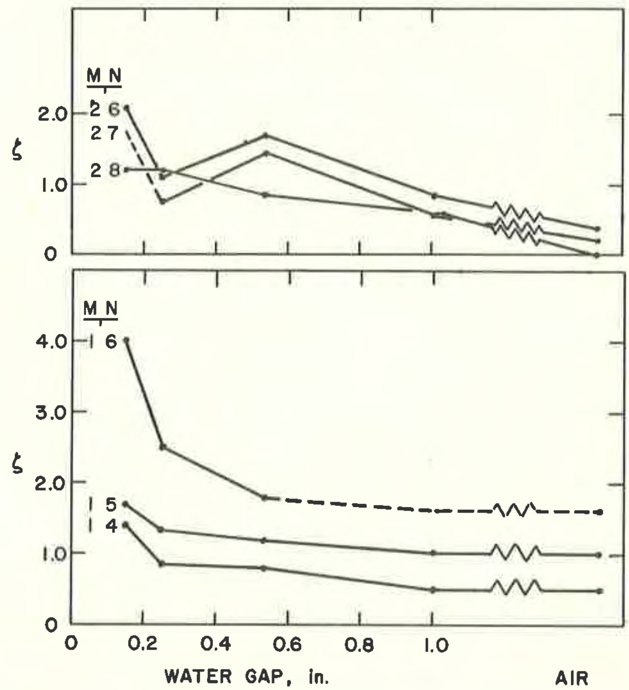

Figure 5. Experimentally determined damping coefficients.