A SIMPLIFIED DYNAMIC ANALYSIS FOR REACTOR PIPING SYSTEMS UNDER BLOWDOWN CONDITIONS

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SUMMARY

In the design of pipelines in a nuclear power plant for blowdown conditions, it is customary to conduct dynamic analysis of the piping system to obtain the responses and the resulting stresses. Calculations are repeated for each design modification in piping geometry or supporting system until the design codes are met. The numerical calculations are, in general, very costly and time consuming. Until now, there have been no simple means for calculating the dynamic responses for the design. The proposed method reduces the dynamic calculation to a quasi-static one, and can be beneficially used for the preliminary design.

The equations of motion for a lumped parameter dynamic model of a piping system subject to a typical blowdown loading function can be written as

$$[M]\{\ddot{U}\}+[C]\{\dot{U}\}+[K]\{U\} = \{F\} \tag{1}$$

where [M] = Mass matrix, [C] = Damping matrix, [K] = Stiffness matrix of the pipe system, {U} = Nodal displacements, and {F} = Blowdown forcing functions. The conventional normal mode method is preferred because damping in pipelines is light. Introducing the transformation, {U} = [Z] {q}, Eq. (1) for [C] = 0, yields

$$\{\ddot{q}\}+[\Omega]\{q\} = [Z]^T\{F\} \tag{2}$$

where $[\Omega] = [Z]^T [K] [Z]$, a diagonal matrix, [Z] = Matrix composed of normalized eigenvectors such that $[Z]^T [M] [Z] = [I]$, in which [I] = Identity matrix and $[Z]^T$ = Transpose matrix of [Z].

By Eq. (2), equation of motion for the modified system is

$$\{\Delta\ddot{q}\}+[\Delta\Omega]\{q\}+([\Omega]+[\Delta\Omega])\{\Delta q\} = \{0\}. \tag{3}$$

Equation (3) can be used to evaluate $\{\Delta q\}$ for given $[\Delta\Omega]$ or to evaluate $[\Delta\Omega]$, given $\{\Delta q\}$. For the first case, it is assumed that when a modification of the piping structure, $[\Delta K]$, is made the change in inertia force is small and can be neglected. Then

$$\{\Delta q\} = -[\Omega+\Delta\Omega]^{-1}[\Delta\Omega]\{q\}. \tag{4}$$

For the latter case, Eq. (3) yields

$$\{\Delta\Omega\} = -[q+\Delta q]^{-1}([\Delta q]\{\Omega\}+\{\Delta\ddot{q}\}) \tag{5}$$

from which $\{\Delta\Omega\}$ can be evaluated. Eqs. (4) and (5) represent the incremental response due to a change in structural stiffness, and vice versa. The method is used in preliminary design calculations, and then followed by a complete dynamical analysis to improve the final results. The new formulations greatly simplify the numerical computation and provide design guides. When used to design a given piping system, the method saved approximately one order of magnitude of computer time. The approach can also be used for other types of structures.

1. Introduction

In the design of pipelines in a nuclear power plant for blowdown conditions, it is customary to conduct the dynamic analysis of the piping system to obtain the responses and the resulting stresses. The calculations are generally repeated for each design modification either in piping geometry or in its supporting system until the design code is met. The numerical calculations are, in general, very costly and time consuming. There has been no simple means for calculating the dynamic responses for the design. The proposed method is aimed for this purpose. The basic procedure, which is based on small perturbation theory, greatly simplifies the calculation. The procedure can be used to determine the incremental response due to a structural modification and to determine the required structural changes as a result of specified changes of dynamic response. The method can be beneficially used in the preliminary design.

The approach has been demonstrated for a piping system of a nuclear reactor. Good agreements and considerable time savings are achieved. The mathematical derivation will be shown in the following section.

2. Analysis

Consider a nuclear pipeline system consisting of elbows, runs as shown in Figure 1. During a blowdown operation, the elbows are subjected to various loading functions as shown in Figure 2.

For an N degree of freedom, undamped system, the equations of motion for the above piping system can be expressed as follows:

$$m_1\ddot{u}_1 + k_{11}u_1 + k_{12}u_2 + \dots\dots\dots\dots + k_{1n}u_n = f_1(t)$$

$$\dots\dots\dots\dots$$

$$m_r\ddot{u}_r + k_{r1}u_1 + k_{r2}u_2 + \dots\dots\dots\dots + k_{rn}u_r = f_r(t)$$

$$\dots\dots\dots\dots$$

$$m_n\ddot{u}_n + k_{n1}u_1 + k_{n2}u_2 + \dots\dots\dots\dots + k_{nn}u_n = f_n(t) \qquad (1)$$

where $f_i(t)$'s represent loading functions at the elbows only. Eq. (1) can be written in matrix form as

$$\underset{\sim}{M}\ddot{\underset{\sim}{U}} + \underset{\sim}{K}\underset{\sim}{U} = \underset{\sim}{F}(t) \qquad (2)$$

where $\underset{\sim}{M}$ = Mass matrix of the pipeline
$\underset{\sim}{K}$ = Stiffness matrix of the pipeline
$\underset{\sim}{U}$ = Vector of nodal displacements
$\ddot{\underset{\sim}{U}}$ = Vector of nodal accelerations
$\underset{\sim}{F}^T(t)$ = Row vector of forcing functions at the elbows resulting from a blowdown condition
$= [f_1(t), f_2(t), \dots\ f_n(t)]$

Note that for each elbow node, there are three forcing functions, one in spatial direction, and that for nodes other than the elbows, $f_i(t) = 0$.

The solution of eq. (2) can be obtained by using the normal-mode method or the frequency-response method. In the first method, normal modes of the piping system are determined and the response due to the forcing function in each mode is then found. The total response is obtained by summing the responses in the normal modes. This method is preferred because the damping in the piping system is usually light [1].

The homogeneous equation of eq. (2) is:

$$\underset{\sim}{M}\ddot{\underset{\sim}{U}} + \underset{\sim}{K}\underset{\sim}{U} = 0 \tag{3}$$

The free vibrations are harmonic and we obtain:

$$\ddot{\underset{\sim}{U}} = -\omega^2 \underset{\sim}{U} \tag{4}$$

where ω is the natural frequency of vibrations. The substitution of eq. (4) into eq. (3) yields:

$$\underset{\sim}{M}\underset{\sim}{U} = \frac{1}{\omega^2} \underset{\sim}{K}\underset{\sim}{U} \tag{5}$$

where matrices $\underset{\sim}{M}$ and $\underset{\sim}{K}$ are of the Nth order. For an undamped piping system, the eigenvalues, ω_1^2, ω_2^2, ... ω_n^2, and the eigenvectors, $\underset{\sim}{U}^{(r)}$, can be obtained by standard computer codes.

Let us further introduce that

$$\underset{\sim}{U} = \underset{\sim}{Z}\underset{\sim}{q} \tag{6}$$

where Z = Matrix composed of normalized vectors of mode shapes such that:

$$\underset{\sim}{Z}^T \underset{\sim}{M}\underset{\sim}{Z} = \underset{\sim}{I} \tag{7}$$

in which $\underset{\sim}{I}$ = Identity matrix

$\underset{\sim}{Z}^T$ = Transpose matrix of $\underset{\sim}{Z}$

Substituting eq. (6) into eq. (1), we obtain:

$$\underset{\sim}{M}\underset{\sim}{Z}\ddot{\underset{\sim}{q}} + \underset{\sim}{K}\underset{\sim}{Z}\underset{\sim}{q} = \underset{\sim}{F}(t)$$

Premultiplying the above equations by $\underset{\sim}{Z}^T$, we obtain:

$$\underset{\sim}{Z}^T\underset{\sim}{M}\underset{\sim}{Z}\ddot{\underset{\sim}{q}} + \underset{\sim}{Z}^T\underset{\sim}{K}\underset{\sim}{Z}\underset{\sim}{q} = \underset{\sim}{Z}^T\underset{\sim}{F}(t) \tag{8}$$

From eqs. (5) and (6), we have:

$$\underset{\sim}{K}\underset{\sim}{Z} = \underset{\sim}{M}\underset{\sim}{Z}\underset{\sim}{\Omega} \tag{9}$$

where

$$\underset{\sim}{\Omega} = \begin{bmatrix} \omega_1^2 & & & \\ & \omega_2^2 & & \\ & & \ddots & \\ & & & \omega_n^2 \end{bmatrix} \tag{10}$$

Premultiplying eq. (9) by Z^T, we obtain:

$$\underset{\sim}{Z}^T \underset{\approx}{K} \underset{\approx}{Z} = \underset{\sim}{\Omega} \tag{11}$$

By substitutions of eqs. (7) and (11) into eq. (8), we obtain:

$$\ddot{\underset{\sim}{q}} + \underset{\approx}{\Omega}\underset{\sim}{q} = \underset{\sim}{Z}^T \underset{\sim}{F}(t) = \bar{\underset{\sim}{F}}(t) \tag{12}$$

where q is a vector of normal coordinates defined by eq. (6). Eq. (12) can be written for the rth mode as:

$$\ddot{q}_r + \omega_r^2 q_r = z_1^{(r)} f_1(t) + z_2^{(r)} f_2(t) + \ldots z_n^{(r)} f_n(t)$$

$$= \bar{f}_r(t) \tag{13}$$

For rest conditions, the general solution of eq. (13) can be obtained:

$$q_r = \frac{1}{\omega_r} \int_o^t \bar{f}_r(\tau) \sin \omega_r (t-\tau)\, d\tau \tag{14}$$

For forcing functions representing blowdown conditions, eq. (14) can generally be evaluated numerically for each normal mode. The responses for the piping system are obtained through eq. (6).

When a slight modification of the piping system, $\Delta \underset{\sim}{K}$, is made:

$$\Delta \underset{\sim}{\Omega} = \underset{\sim}{Z}^T \Delta \underset{\approx}{K} \underset{\sim}{Z} \tag{15}$$

the change of response, from eq. (12), is governed by the following differential equation [2].

$$\Delta \ddot{\underset{\sim}{q}} + \Delta \underset{\approx}{\Omega} \underset{\sim}{q} + (\underset{\sim}{\Omega} + \Delta \underset{\sim}{\Omega}) \Delta \underset{\sim}{q} = 0 \tag{16}$$

Equation (16) can be used to evaluate $\Delta \underset{\sim}{q}$ for given $\Delta \underset{\sim}{\Omega}$ or to evaluate $\Delta \Omega$, given $\Delta \underset{\sim}{q}$. For the first case, it is assumed that when a modification of the piping structure, $\Delta \underset{\sim}{k}$, is made, the change in inertia force is small and can be neglected. Then,

$$\Delta \underset{\sim}{q} = -(\underset{\sim}{\Omega}+\Delta\underset{\sim}{\Omega})^{-1}\Delta\underset{\sim}{\Omega}\underset{\sim}{q} \tag{17}$$

For the latter case, eq. (16) yields

$$\Delta\underset{\sim}{\Omega} = -(\underset{\sim}{q}+\Delta\underset{\sim}{q})^{-1}(\Delta\underset{\sim}{q}\underset{\sim}{\Omega}+\Delta\underset{\sim}{\ddot{q}}) \tag{18}$$

where $\underset{\sim}{q}$ and $\Delta\underset{\sim}{q}$ can be represented as diagonal matrices, and $\underset{\sim}{\Omega}$ and $\Delta\underset{\sim}{\Omega}$ as column matrices. Eq. (18) can be used to evaluate the structural changes due to changes in responses. Thus, the new response is

$$\underset{\sim}{q}' = \underset{\sim}{q} + \Delta\underset{\sim}{q}, \quad \text{or} \quad \underset{\sim}{U}' = \underset{\sim}{U} + \Delta\underset{\sim}{U} \tag{19}$$

and the new stiffness is

$$\underset{\sim}{K}' = \underset{\sim}{K} + \Delta\underset{\sim}{K} \tag{20}$$

3. Illustrations and Remarks

An example is given to illustrate the proposed procedure. Figure 1 shows a simplified version of a nuclear piping system which consists of 18" schedule 40, SA-106-B carbon steel pipes with I.D. = 16.876", thickness = 0.562", area = 224 sq. in., $E = 27.9 \times 10^6$ psi and $\alpha = 6.81 \times 10^{-6}$ in/in at a temperature of 395°F. The steam properties are: p_o = 202 psig, T_o = 379.6°F (180 psig/T sat.), R = 85.58 ft-lbf/lbm-°R, and k=1.13. For the purpose of demonstrating the procedure, the piping system is reduced to a ten degrees of freedom system as shown in Figure 1 in order to simplify the numerical tabulations. The x-direction responses are considered at nodes 10, 16, 22 and 44. The y-direction responses are considered at nodes 10, 22 and 30, while the z-direction responses are considered at nodes 16, 30 and 44.

The blowdown forcing functions are also simplified which appear only at node 10 in the y-direction, at node 16 in the z-direction, and node 44 in the x-direction. For simplification, these loading functions are pressure differentials. Note that node 44 is the main valve attached to the main steam line (see Figure 1).

The mass and stiffness matrices, eigenvalues and eigenvectors are reported in the computer print-out form (see Tables I and II).

In Table III, Q1 (or U1) denotes the responses of the original system, Q2 (or U2) denotes the responses of the modified system, and DQ1 = Q2 - Q1. Note that the stiffness of the modified system is increased by five percent. In the same tabulation DQ2 denotes $\Delta\underset{\sim}{q}$ obtained from eq. (17). It is to be noted that the error, (DQ1-DQ2), of the present method is practically unnoticeable.

In conclusion, it has been demonstrated that good agreements in responses have been obtained between the complete dynamic analysis and the simplified dynamic analysis. The saving in computer time is of one order of magnitude for the example given. The savings in time will greatly increase when the dynamical system becomes larger. The method is very useful for the preliminary design of nuclear piping systems and other types of structures.

References

[1] ITOH, T., "Damped Vibration Mode Superposition Method of Dynamic Response Analysis," Earthquake Engineering and Structural Dynamics, Vol. 2, pp. 47-57, 1973.

[2] Chen, M. M., "A Simplified Dynamical Analysis for Structural Systems Subject to Arbitrary Forcing Functions," Boston University, Aero. Eng. Rept. (in preparation).

Table I Mass, Stiffness Matrices, and Eigenvalues of the Original Piping System

GLOBAL (REDUCED) STIFFNESS MATRIX k

0.589E 06	0.125E 04	-0.909E 05	-0.265E 05	-0.557E 05	0.801E 04	-0.122E 05	0.247E 05	0.299E 05	-0.867E 04
0.125E 04	0.959E 06	-0.181E 04	-0.104E 06	0.559E 04	-0.146E 05	0.185E 05	-0.288E 03	0.209E 03	-0.175E 04
-0.909E 05	-0.181E 04	0.508E 05	-0.191E 04	-0.226E 05	0.178E 05	-0.121E 05	0.380E 04	0.182E 05	0.791E 04
-0.265E 05	-0.104E 06	-0.191E 04	0.516E 06	0.207E 06	-0.390E 06	0.394E 06	-0.508E 06	-0.123E 06	0.543E 05
-0.557E 05	0.559E 04	-0.226E 05	0.207E 06	0.221E 06	-0.201E 06	0.190E 06	-0.210E 06	-0.181E 06	-0.408E 03
0.801E 04	-0.146E 05	0.178E 05	-0.390E 06	-0.201E 06	0.456E 06	-0.436E 06	0.391E 06	0.957E 05	0.101E 05
-0.122E 05	0.185E 05	-0.121E 05	0.394E 06	0.190E 06	-0.436E 06	0.432E 06	-0.406E 06	-0.476E 05	-0.759E 04
0.247E 05	-0.288E 03	0.380E 04	-0.508E 06	-0.210E 06	0.391E 06	-0.406E 06	0.539E 06	0.122E 06	-0.614E 05
0.299E 05	0.209E 03	0.182E 05	-0.123E 06	-0.181E 06	0.957E 05	-0.476E 05	0.122E 06	0.418E 07	-0.316E 04
-0.867E 04	-0.175E 04	0.791E 04	0.543E 05	-0.408E 03	0.101E 05	-0.759E 04	-0.614E 05	-0.316E 04	0.368E 07

GLOBAL (REDUCED) MASS MATRIX M

0.179E 0[illegible]	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.495E 01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.316E 01	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.529E 0[illegible]	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.213E 01	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.213E 01	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.451E 01	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.451E 01	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.246E 02	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.200E 02

EIGEN VALUES OF KU=WMU

0.530363E 06	0.334180E 06	0.196456E 06	0.183753E 06	0.170717E 06	0.514713E 05	0.442892E 05
0.911958E 04	0.321743E 04	0.849083E 03				

Table II Stiffness Matrix, Eigenvalues and Eigenvectors of the Modified Piping System

EIGEN VECTORS OF KU=WMU

-0.134E 00	-0.720E-02	-0.963E-02	0.295E 00	0.390E 00	-0.713E 00	0.331E 00	-0.352E 00	-0.269E-01	0.416E-02
0.990E 00	0.115E-02	-0.907E-01	0.228E-01	-0.379E-01	-0.867E-01	0.373E-01	-0.288E-01	0.459E-02	-0.247E-02
-0.748E-02	0.990E 00	-0.674E-02	-0.103E 00	0.359E-01	-0.759E-01	0.371E-01	0.351E-02	-0.388E-02	-0.329E-01
0.318E-01	0.141E 00	0.139E-01	0.315E-01	-0.418E-01	0.859E-01	-0.322E-01	-0.645E-01	0.113E-01	0.982E 00
-0.113E 00	0.140E-01	0.552E-01	0.613E-01	-0.325E 00	-0.238E 00	0.168E 00	-0.783E-01	0.885E 00	-0.725E-01
0.530E-01	-0.170E-01	-0.185E 00	-0.101E 00	0.948E 00	0.148E 00	-0.109E 00	0.131E 00	0.432E-01	0.515E-02
0.488E-01	0.702E-01	0.714E-01	0.365E 00	0.945E-01	0.730E 00	-0.221E 00	-0.513E 00	0.133E-01	-0.216E-01
0.170E 00	-0.124E-02	0.950E 00	-0.868E-02	0.240E 00	-0.162E-01	0.105E-01	0.976E-01	0.258E-02	0.222E-03
-0.100E-01	0.73[illegible]E-01	-0.492E-01	0.627E 00	-0.493E-01	-0.362E 00	-0.476E 00	0.489E 00	0.524E-02	-0.981E-03
-0.217E-02	0.294E-01	-0.571E-01	0.306E 00	0.273E-01	0.586E 00	0.650E 00	0.369E 00	-0.637E-02	0.152E-02

GLOBAL (REDUCED) STIFFNESS MATRIX $k^* = k + \Delta k$

0.618E 06	0.131E 04	-0.955E 05	-0.278E 05	-0.584E 05	0.841E 04	-0.128E 05	0.259E 05	0.314E 05	-0.910E 04
0.131E 04	0.101E 07	-0.190E 04	-0.109E 06	0.587E 04	-0.153E 05	0.194E 05	-0.302E 03	0.220E 03	-0.184E 04
-0.955E 05	-0.190E 04	0.533E 05	-0.201E 04	-0.237E 05	0.186E 05	-0.128E 05	0.399E 04	0.192E 05	0.830E 04
-0.278E 05	-0.109E 06	-0.201E 04	0.542E 06	0.217E 06	-0.410E 06	0.414E 06	-0.534E 06	-0.129E 06	0.570E 05
-0.584E 05	0.587E 04	-0.237E 05	0.217E 06	0.232E 06	-0.211E 06	0.199E 06	-0.22[illegible]E 06	-0.190E 06	-0.429E 03
0.841E 04	-0.153E 05	0.186E 05	-0.410E 06	-0.211E 06	0.479E 06	-0.458E 06	0.411E 06	0.100E 06	0.106E 05
-0.128E 05	0.194E 05	-0.128E 05	0.414E 06	0.199E 06	-0.458E 06	0.453E 06	-0.427E 06	-0.500E 05	-0.797E 04
0.259E 05	-0.302E 03	0.399E 04	-0.534E 06	-0.22[illegible]E 06	0.411E 06	-0.427E 06	0.566E 06	0.128E 06	-0.645E 05
0.314E 05	0.220E 03	0.192E 05	-0.129E 06	-0.190E 06	0.100E 06	-0.500E 05	0.128E 06	0.439E 07	-0.332E 04
-0.910E 04	-0.184E 04	0.830E 04	0.570E 05	-0.429E 03	0.106E 05	-0.797E 04	-0.645E 05	-0.332E 04	0.386E 07

EIGEN VALUES OF KU=WMU

0.556881E 06 0.359889E 06 0.206279E 06 0.192941E 06 0.179253E 06 0.540447E 05 0.465037E 05
0.957555E 04 0.337829E 04 0.89[illegible]661E 03

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Table III   Responses (q,q' and U,U') and Differences in Responses (Δq and ΔU)

AT TIME STEP 37   (T = 0.3600E-01 SEC)

LOADINGS AT CORRESPONDING NODES

X10	Y10	X16	Z16	X22	Y22	Y30	Z30	X44	Z44
0.0	0.190E 04	0.0	-0.711E 04	0.0	0.0	0.0	0.0	-0.137E 05	0.0

NODE	Q1	Q2	DQ1	DQ2
1	-0.1908E-02	-0.1317E-02	0.9085E-04	0.9085E-04
2	-0.4957E-03	-0.4721E-03	0.2361E-04	0.2361E-04
3	0.6[illegible]02E-02	0.58[illegible]2E-02	-0.2906E-03	-0.2906E-03
4	-0.1376E-03	-0.1311E-03	0.6543E-05	0.6553E-05
5	-0.1655E-01	-0.1576E-01	0.7879E-03	0.7879E-03
6	0.1196E-02	0.[illegible]39E-02	-0.5694E-04	-0.5693E-04
7	-0.3278E-01	-0.3122E-01	0.1561E-02	0.1561E-02
8	0.1497E-02	0.1426E-02	-0.7132E-04	-0.7130E-04
9	-0.6437E 00	-0.6130E 00	0.3065E-01	0.3065E-01
10	-0.1234E 01	-0.1175E 01	0.5894E-01	0.5894E-01

NODE	U1	U2	DU1	DU2	DU2-DU1
1	0.3918E-02	0.3731E-02	-0.1867E-03	-0.1867E-03	0.1790E-08
2	-0.3952E-01	-0.3765E-01	0.1885E-02	0.1885E-02	0.1537E-07
3	0.5049E-01	0.4808E-01	-0.2409E-02	-0.2409E-02	-0.5407E-07
4	-0.3927E 00	-0.3740E 00	0.1872E-01	0.1872E-01	0.2036E-06
5	-0.2299E-02	-0.2187E-02	0.1117E-03	0.1118E-03	0.6519E-07
6	-0.2746E 00	-0.2614E 00	0.1312E-01	0.1312E-01	0.920[illegible]E-06
7	-0.2662E 00	-0.2535E 00	0.1273E-01	0.1273E-01	0.1036E-05
8	-0.3734E 00	-0.3556E 00	0.1791E-01	0.1791E-01	0.3316E-06
9	-0.1015E-02	-0.9674E-03	0.4780E-04	0.4780E-04	-0.9633E-08
10	-0.3543E-03	-0.3373E-03	0.1702E-04	0.1702E-04	0.4453E-08

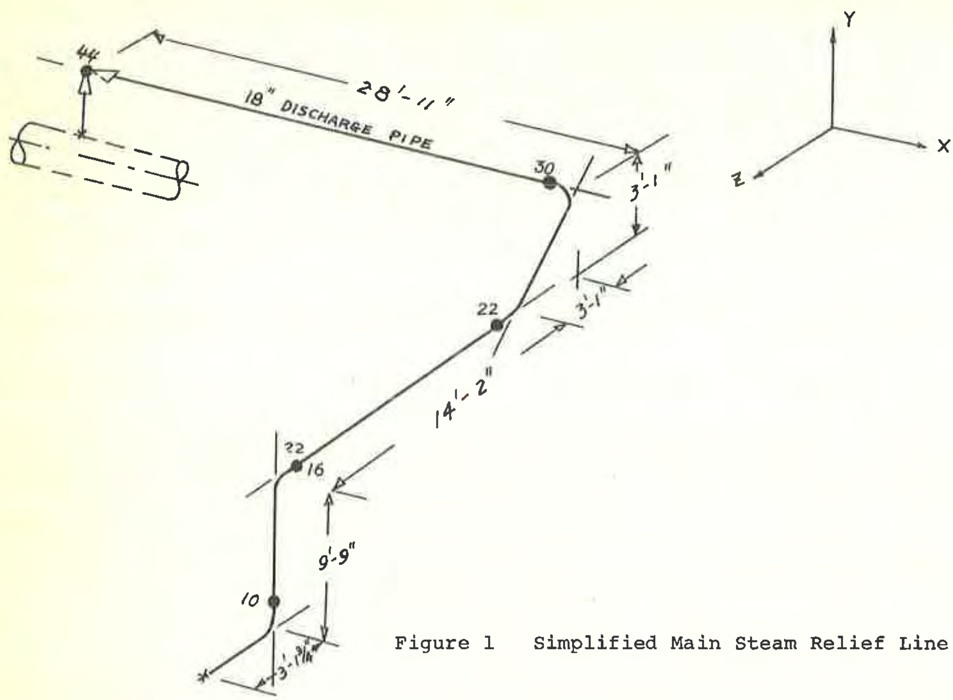

Figure 1 Simplified Main Steam Relief Line

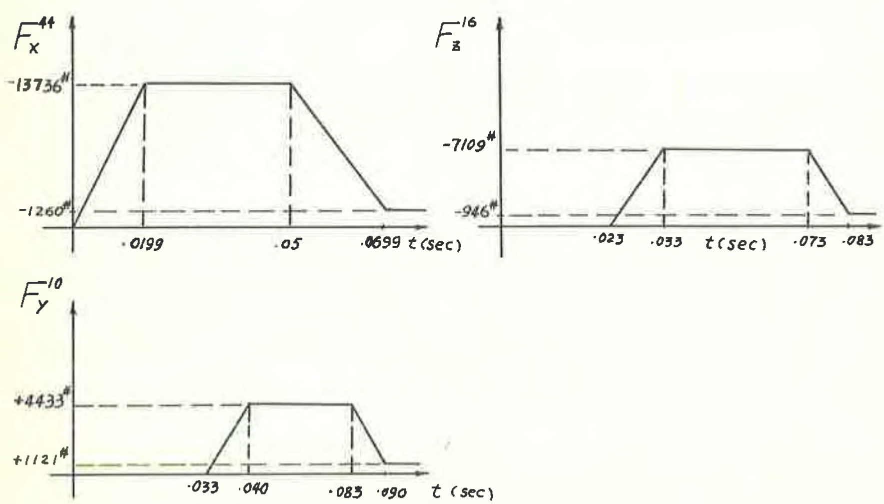

Figure 2 Loading Functions at nodes 10, 16 and 44