AN INELASTIC ANALYSIS OF A HEAT EXCHANGER PIPING USING THE FINITE ELEMENT METHOD

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SUMMARY

According to ASME Code Case 1592, an inelastic analysis of nuclear power plant components is required when they are used under creep and plastic range at elevated temperature. In the case of High Temperature Gas Reactor (HTGR), inelastic analysis is inevitable since its normal operating temperature is well within the range where creep and plasticity of the materials used cannot be neglected.

In this paper, an elastic-plastic creep analysis based on the finite element method is performed to determine the structural response of a heat exchanger piping subjected to mechanical and thermal loading. The piping is comprised of a set of straight frame members. For the plastic analysis, the isotropic and Ziegler's kinematic hardening rules are employed. The creep law under uniaxial stress state is extended to give a generalized expression for multiaxial stress state. The combined strain-hardening and time-hardening procedure proposed by Cozzarelli and Shaw is adopted for the creep behavior under varying stress state. An incremental equilibrium equation is obtained using the virtual displacement principle. Element stiffness and nodal forces are calculated by numerical integration. The method to control the amount of increment is proposed since it highly influences the precision and the stability of calculations. Geometrical nonlinearity is considered to discuss the structural instability phenomena.

A computer program, considering the above mentioned factors, is developed to analyse the creep buckling, ratchetting and other phenomena which are very important in the design of nuclear power plant components. The feasibility of this formulation is shown through several numerical examples.

1. Introduction

Recently more heat exchangers have been designed for the operation under higher temperatures and the requirements for their thermal capacities have become severer with the advance in nuclear reactor technologies. Under these conditions, creep and plastic properties of materials have to be considered in refined structural analyses so as to assure the integrity and the safety of heat exchangers.

In this paper, attention is focused on the heat exchanger piping which is exposed to thermally severe conditions. The piping is replaced by a set of straight frame members. The formulations for the inelastic analysis of a frame are derived based on the finite element method. A computer program is developped applying these formulations and calculations are carried out as to the creep buckling and thermal ratchetting problems which are important factors in the designs of heat exchanger piping. The numerical results are compared with analytical solutions and the validity of these formulations are shown.

2. Constitutive relations

In inelastic analyses, small changes of material properties sometimes cause considerable differances on the computed results so that it is the important and difficult problem to construct the constitutive relations which well represent the actual material behaviours. Since the incremental approach is adopted in present study, incremental constitutive relations are established in this section.

According to the beam theory, a strain vector $\{\varepsilon\}$ is given by;

$$\{\varepsilon\} = (\varepsilon_x, \gamma_{xy}, \gamma_{xz})^T$$

where ε_x is the axial strain and γ_{xy}, γ_{xz} are the shear strains due to torsion. The superscript T denotes the transposition of the vector and matrix. Now, total strain increments $\{\Delta\varepsilon\}$ are decomposed to;

$$\{\Delta\varepsilon\} = \{\Delta\varepsilon^\theta\} + \{\Delta\varepsilon^e\} + \{\Delta\varepsilon^p\} + \{\Delta\varepsilon^c\} \quad (1)$$

where superscripts θ, e, p and c denote thermal, elastic, plastic and creep components respectively.

The Thermal Component

Thermal strains are defined by; $\{\varepsilon^\theta\} = \{\alpha\}\theta$

where $\{\alpha\} = (\alpha, 0, 0)^T$ is the linear coefficient vector of thermal expansion. Differentiating the above equation and approximating the differential increments by finite increments, the thermal strain increments are obtained as;

$$\{\Delta\varepsilon^\theta\} = \frac{\partial\{\alpha\}}{\partial\theta}\Delta\theta\cdot\theta + \{\alpha\}\Delta\theta = \{B^{\theta\theta}\}\Delta\theta \quad (2)$$

The Elastic Component

Elastic strains are related to the stresses by Hooke's law;

$$\{\varepsilon^e\} = [C^e]\ \{\sigma\}$$

where $\{\sigma\} = (\sigma_x, \tau_{xy}, \tau_{xz})^T$ is the stress vector corresponding to the strain vector $\{\varepsilon\}$. Then, elastic strain increments are given by;

$$\{\Delta\varepsilon^e\} = [C^e]\{\Delta\sigma\} + \frac{\partial[C^e]}{\partial\theta}\{\sigma\}\Delta\theta = [C^e]\{\Delta\sigma\} + \{B^{e\theta}\}\Delta\theta \tag{3}$$

The Plastic Component

The plastic strain increments are derived using the von Mises yield criterion and the associated flow rule. Both isotropic and kinematic hardening laws are considered. In the case of isotropic hardening, the yield condition is given by;

$$f(\sigma_{ij}, \theta, \varepsilon^p_{ij}) = \bar{\sigma}^2 - \sigma_0^2(\theta, \overline{\varepsilon^p}) = 0 \tag{4}$$

where σ_{ij} and $\varepsilon_{ij}{}^p$ are the stress and plastic strain tensors. The terms $\bar{\sigma}$ and $\overline{\varepsilon^p}$ are the equivalent stress and the equivalent plastic strain defined by;

$$\bar{\sigma}^2 = \frac{3}{2} S_{ij}{}^2 = \sigma_x{}^2 + 3\tau_{xy}{}^2 + 3\tau_{xz}{}^2 \tag{5}$$

$$\overline{\varepsilon^p} = \int \overline{d\varepsilon^p} \ , \ \overline{d\varepsilon^p} = (\tfrac{2}{3}(d\varepsilon^p_{ij})^2)^{\frac{1}{2}} = (d\varepsilon^{p\,2}_x + \tfrac{1}{3}\, d\gamma_{xy}{}^{p\,2} + \tfrac{1}{3} d\gamma_{xz}^{p\,2})^{\frac{1}{2}} \tag{6}$$

where S_{ij} is the deviatoric stress tensor.

The function $\sigma_0(\theta, \overline{\varepsilon^p})$ is the yield stress function which is obtained from the uni-axial experimental data.

Using the normal flow rule, $\quad d\varepsilon^p_{ij} = d\lambda\frac{\partial f}{\partial\sigma_{ij}} = d\lambda \ . \ 3S_{ij} \qquad (7)$

where $d\lambda$ is a positive scalar and given by substituting eq.(7) into eq.(6);

$$d\lambda = \frac{\overline{d\varepsilon^p}}{2\bar{\sigma}} \tag{8}$$

An incremental loading causes expansion of the yield surface and the stress point remains on the expanded surface during a loading process. Then,

$$df = 3S_{ij}\, dS_{ij} - 2\sigma_0(\frac{\partial\sigma_0}{\partial\theta}d\theta + \frac{\partial\sigma_0}{\partial\overline{\varepsilon^p}}\overline{d\varepsilon^p}) = 0 \tag{9}$$

From eqs.(7), (8) and (9) the plastic strain increments are given by;

$$\Delta\varepsilon^p_{ij} = \frac{3S_{ij}}{2\bar{\sigma}} \ \frac{\frac{3}{2}S_{kl}\Delta S_{kl} - \sigma_0\frac{\partial\sigma_0}{\partial\theta}\Delta\theta}{\sigma_0\frac{\partial\sigma_0}{\partial\overline{\varepsilon^p}}}$$

It should be noted here that $\bar{\sigma}$ and $\frac{\partial\sigma_0}{\partial\overline{\varepsilon^p}}$ are σ_0 itself and the hardening modulus H during a loading process. Then the plastic incremental constitutive relation for the isotropic hardening law is given as follows;

$$\{\Delta\varepsilon^p\} = [C^p]\{\Delta\sigma\} - \{B^{p\theta}\}\Delta\theta \tag{10}$$

where, $\quad [C^p] = \frac{1}{\bar{\sigma}^2 H}\begin{bmatrix} \sigma_x{}^2 & 3\sigma_x\tau_{xy} & 3\sigma_x\tau_{xz} \\ & 9\tau_{xy}{}^2 & 9\tau_{xy}\tau_{xz} \\ \text{SYM} & & 9\tau_{xz}{}^2 \end{bmatrix}, \quad \{B^{p\theta}\} = \frac{\frac{\partial\sigma_0}{\partial\theta}}{\bar{\sigma}\ H}\begin{Bmatrix} \sigma_x \\ 3\tau_{xy} \\ 3\tau_{xz} \end{Bmatrix}$

In the case of kinematic hardening, the yield condition is given by;

$$f(\sigma_{ij}\ ,\ \alpha_{ij}\ ,\ \theta) = \bar{\sigma}(\alpha_{ij})^2 - \sigma_0{}^2(\theta) = 0 \tag{11}$$

where α_{ij} is the shift tensor which represents the translation of the yield surface in the stress space. The equivalent stress $\bar{\sigma}(\alpha_{ij})$ is defined by;

$$\sigma(\alpha_{ij})^2 = \frac{3}{2}\, S'^{\,2}_{ij} = (\sigma_x - \alpha_x)^2 + 3(\tau_{xy} - \alpha_{xy})^2 + 3(\tau_{xz} - \alpha_{xz})^2 \tag{12}$$

where, $\quad S'_{ij} = S_{ij} - (\alpha_{ij} - \frac{1}{3}\ \alpha mm\ \delta_{ij})$

From the flow rule, $\quad d\varepsilon^p_{ij} = d\lambda\frac{\partial f}{\partial\sigma} = d\lambda\ 3S'_{ij} \qquad (13)$

During a loading, it is assumed that the yield surface does not rotate but only translates without distortion. According to the Ziegler's kinematic hardening rule [1] which is adopted in this study, the yield surface moves toward the direction connecting its center with the stress point, namely;

$$d\alpha_{ij} = d\mu(\sigma_{ij} - \alpha_{ij}) \tag{14}$$

where $d\mu$ is a positive scalar. During a loading,

$$df = 3\, S_{ij}'\, dS_{ij}' - 2\sigma_0 \frac{\partial \sigma_0}{\partial \theta} \sigma\theta = 0$$

Substituting eq.(14) into above equation, $d\mu$ is obtained as;

$$d\mu = \frac{\frac{3}{2} S_{ij}'\, dS_{ij} - \sigma_0 \frac{\partial \sigma_0}{\partial \theta} d\theta}{\bar{\sigma}(\alpha_{ij})^2} \tag{15}$$

The positive scalar $d\lambda$ of eq.(13) is obtained from the assumption used by Inoue et al. [2] that the plastic strain increment $K\, d\varepsilon_{ij}{}^p$ is the projection of $d\alpha_{ij}$ on the exterior normal to the yield surface.

That is ; $d\alpha_{ij}\, d\varepsilon_{ij}{}^p = K\, d\varepsilon_{ij}{}^p\, d\varepsilon_{ij}{}^p$ (16)

The left hand term of the above equation can be transformed as;

$$d\alpha_{ij}\, d\varepsilon_{ij}{}^p = d\mu\, (\sigma_{ij} - \alpha_{ij}) \cdot d\lambda \cdot 3S_{ij}'$$

$$= d\mu d\lambda\; 3S_{ij}'\, S_{ij}' = \frac{d\mu}{3d\lambda} d\varepsilon_{ij}^p d\varepsilon_{ij}^p$$

Therefore; $d\lambda = \frac{d\mu}{3K}$ (17)

Using eqs. (13), (15) and (17), the plastic strain increments are obtained as;

$$d\varepsilon_{ij}{}^p = \frac{S_{ij}'(\frac{3}{2} S_{kl}'\, dS_{kl} - \sigma_0 \frac{\partial \sigma_0}{\partial \theta} d\theta)}{K\, \bar{\sigma}\, (\alpha_{ij})^2} \tag{18}$$

The value of constant K is obtained by applying eq.(18) to the isothermal uni-axial test results and given as $K = \frac{2}{3}H$.

After all, plastic incremental constitutive relation for the Ziegler's kinematic hardening rule is given in the matrix form as;

$$\{\Delta\varepsilon^p\} = [C^p]\{\Delta\sigma\} - \{B^{p\theta}\}\Delta\theta \tag{19}$$

where, $$[C^p] = \frac{1}{\bar{\sigma}(\alpha_{ij})^2 H} \begin{bmatrix} \sigma_x'^2 & 3\sigma_x'\tau_{xy}' & 3\sigma_x'\tau_{xz}' \\ & 9\tau_{xy}'^2 & 9\tau_{xy}'\tau_{xz}' \\ \text{SYM} & & 9\tau_{xz}'^2 \end{bmatrix}, \quad \{B^{p\theta}\} = \frac{\frac{\partial \sigma_0}{\partial \theta}}{\sigma(\alpha_{ij})H} \begin{Bmatrix} \sigma_x' \\ 3\tau_{xy}' \\ 3\tau_{xz}' \end{Bmatrix}$$

It is noticeable that eq.(19) can be obtained by replacing $\bar{\sigma}$, σ_x, τ_{xy} and τ_{xz} with $\sigma(\alpha_{ij})$, σ_x' , τ_{xy}' and τ_{xz}' in eq. (10).

Here, it is necessary to define the loading and unloading criteria. Obviously, loading and unloading occur when $d\lambda$ of eq.(7) (or eq.(13)) is positive and negative respectively and $d\lambda = 0$ corresponds to a neutral state. These conditions are given for the case of isotropic hardening law as;

$$\left.\begin{matrix} \text{loading} \\ \text{unloading} \\ \text{neutral} \end{matrix}\right\} : \frac{3}{2}S_{kl}\, \Delta S_{kl} - \sigma_0 \frac{\partial \sigma_0}{\partial \theta} \Delta\theta \left\{\begin{matrix} > 0 \\ < 0 \\ = 0 \end{matrix}\right. \tag{20}$$

Similar criteria for the kinematic hardening law can be obtained by substituting S_{kl} with S_{kl}' in the above formulae.

The Creep Component

In this section, a uni-axial creep law is discussed first and then generalized to a multi-axial case. For a uni-axial stress state, the creep strain rate is considered to be decomposed into transient and steady terms;

$$\dot{\varepsilon}^c = \dot{\varepsilon}^t + \dot{\varepsilon}^s \tag{21}$$

Norton's law is utilized for the steady creep law;

$$\dot{\varepsilon}^s = (\frac{\sigma}{\lambda})^n \tag{22}$$

The transient creep rate under constant stress state is expressed as;

$$\dot{\varepsilon}^t = (\frac{\sigma}{\mu})^q (1 - e^{-t/\tau}) \tag{23}$$

where μ, q and τ are the transient creep parameter, power and retardation time. In most cases, either time-hardening or strain-hardening law is applied in order to generalize the transient creep law under constant stress state to variable stress conditions. According to Cozzarelli and Shaw [3], the best fits with experimental results can be obtained by the combined time and strain-hardening procedure. This is expressed as;

$$\dot{\varepsilon}^t = \dot{\varepsilon}^{th} + \dot{\varepsilon}^{sh}$$

$$\dot{\varepsilon}^{th} = \frac{1}{\tau}(\frac{\sigma}{\mu^d})^q e^{-t/\tau}, \quad \dot{\varepsilon}^{sh} = \frac{1}{\tau}((\frac{\sigma}{\mu^k})^q - \varepsilon^{sh}) \tag{24}$$

where, $\mu^d = \frac{\mu}{(1-a)^{1/q}}$, $\mu^k = \frac{\mu}{(a)^{1/q}}$, $0 \leqq a \leqq 1$

The parameter a is considered to be a material constant expressing the degree of combination of hardening states. Next, the equivalent creep rate and the equivalent strain-hardening creep strain are defined to establish a multi-axial creep law as follows;

$$\overline{\dot{\varepsilon}^c} = (\frac{2}{3}\dot{\varepsilon}_{ij}^{c2})^{\frac{1}{2}} = (\dot{\varepsilon}_x^{c2} + \frac{1}{3}\dot{\gamma}_{xy}^{c2} + \frac{1}{3}\dot{\gamma}_{xz}^{c2})^{\frac{1}{2}} \tag{25}$$

$$\overline{\varepsilon^{sh}} = (\frac{2}{3}\varepsilon_{ij}^{sh2})^{\frac{1}{2}} = (\varepsilon_x^{sh2} + \frac{1}{3}\gamma_{xy}^{sh2} + \frac{1}{3}\gamma_{xz}^{sh2})^{\frac{1}{2}} \tag{26}$$

Now, uni-axial creep law obtained from eqs.(21), (22) and (24) is written again as $\dot{\varepsilon}^c = F^c(\sigma, \varepsilon^{sh}, \theta, t)$.

Here, it is assumed that the above equation is generalized to a multi-axial state as;

$$\overline{\dot{\varepsilon}^c} = F(\bar{\sigma}, \overline{\varepsilon^{sh}}, \theta, t) \tag{27}$$

The flow rule which assumes the creep potential and incompressibility of creep strain is; $\dot{\varepsilon}_{ij}^c = \dot{\lambda}^c S_{ij}$ (28)

By substituting eq.(28) into (25); $\dot{\lambda}^c = \frac{3}{2}\frac{\overline{\dot{\varepsilon}^c}}{\bar{\sigma}}$ (29)

Then, the creep law for the multi-axial stress state is;

$$\dot{\varepsilon}_{ij}^c = \frac{3}{2\bar{\sigma}} F^c(\bar{\sigma}, \overline{\varepsilon^{sh}}, \theta, t,) S_{ij} \tag{30}$$

The incremental form of eq.(30) is written as follows;

$$\{\Delta\varepsilon^c\} = \frac{F(\bar{\sigma}, \overline{\varepsilon^{sh}}, \theta, t)}{\bar{\sigma}} \begin{Bmatrix} \sigma_x \\ 3\tau_{xy} \\ 3\tau_{xz} \end{Bmatrix} \Delta t = \{B^c\}\Delta t \tag{31}$$

Stress increments

The stress increments are computed from the stress-strain relations which is obtained by substituting eqs.(2), (3), (10) (or(19)) and (31) into eq.(1) and expressed for the thermo-elastic-plastic creep condition as;

$$\{\Delta\sigma\} = [D^p]\{\Delta\varepsilon\} - \{H^{p\theta}\}\Delta\theta - \{H^{pc}\}\Delta t \tag{32}$$

where, $[D^p] = ([C^e] + [C^p])^{-1}$

$$\{H^{p\theta}\} = [D^p]\ (\{B^{\theta\theta}\} + \{B^{e\theta}\} - \{B^{p\theta}\}),\ \{H^{pc}\} = [D^p]\{B^c\}$$

When the elastic part of the structure crosses the yield surface during the relevant load increment, it is called the transition from elastic state to plastic one and the stress-strain relationship is modified by weighting those of the elastic and elastic-plastic states as discussed by Marcal et al. [4].

3. Incremental equilibrium equation

Since we deal with the piping as a set of straight frame member, the displacement modes for the beam, which are assumed to be linear for axial displacement u and twist ϕ and cubic for deflection v and w due to bending, are employed in the incremental form as;

$$\left.\begin{aligned} \Delta u^c &= (1,\ x)\ (a_1,\ a_2)^T \\ \Delta v^c &= (1,\ x,\ x^2,\ x^3)\ (a_3,\ a_4,\ a_5,\ a_6)^T \\ \Delta w^c &= (1,\ x,\ x^2,\ x^3)\ (a_7,\ a_8,\ a_9,\ a_{10})^T \\ \Delta \phi^c &= (1,\ x)\ (a_{11},\ a_{12})^T \end{aligned}\right\} \tag{33}$$

where Δu^c, Δv^c, Δw^c and $\Delta\phi^c$ denote the incremental displacements at the centroid of each member as shown in Fig. 1. The generalized displacement vector $\{a\}$ is connected with the nodal incremental displacement vector$\{\Delta U\}$ as; $\{a\} = [\Gamma au]\{\Delta U\}$ (34)

Kirchhoff's assumption is applied to obtain the incremental displacements at any point within the member as follows;

$$\left.\begin{aligned} \Delta u &= \Delta u^c - y\frac{d\Delta v^c}{dx} - z\,\frac{d\Delta w^c}{dx} \\ \Delta v &= \Delta v^c - z\Delta\phi^c \\ \Delta w &= \Delta w^c + y\Delta\phi^c \end{aligned}\right\} \tag{35}$$

The incremental strain-displacement relation which takes large deflection effect into consideration is given by;

$$\left.\begin{aligned} \Delta\varepsilon_x &= \frac{d\Delta u}{dx} + \left(\frac{dv_0}{dx}\,\frac{d\Delta v}{dx} + \frac{dw_0}{dx}\,\frac{d\Delta w}{dx}\right) \\ &\quad + \frac{1}{2}\left(\frac{d\Delta v^2}{dx} + \frac{d\Delta w^2}{dx}\right) - y\frac{d^2\Delta v}{dx^2} - z\frac{d^2\Delta w}{dx^2} \\ \Delta\gamma_{xy} &= -z\,\frac{d\Delta\phi}{dx} \\ \Delta\gamma_{xz} &= y\,\frac{d\Delta\phi}{dx} \end{aligned}\right\} \tag{36}$$

where v_0 and w_0 are the initial deflections which are calculated by subtracting the displacements due to rigid translations and rotations from total displacements obtained for the previous state. From eqs.(34), (35) and (36), the strain increments are related to nodal incremental displacements as;

$$\{\Delta\varepsilon\} = [M^L][\Gamma au]\{\Delta U\} + \frac{1}{2}\left\{\begin{matrix} \{\Delta U\}^T\ [\Gamma au]^T\ [M^N][\Gamma au]\{\Delta U\} \\ 0 \\ 0 \end{matrix}\right\} \tag{37}$$

The principle of incremental virtual work used by Hofmister et al.[5] is;

$$\int_v \delta\{\Delta\varepsilon\}^T\ \{\sigma + \Delta\sigma\}dv - \int_x (\delta\Delta u,\ \delta\Delta v,\ \delta\Delta w)\{p + \Delta p\}dx$$
$$-\delta\{\Delta U\}^T\ \{F + \Delta F\} = 0$$

where $\{p\} = (p_x,\ p_y,\ p_z)^T$ is the distributed load vector and F is the concentrated force vector. Substituting eq.(37) and the constitutive equation into the above equation and taking the variation, incremental equilibrium equation is obtained as;

$$([K] + [K_G])\{\Delta U\} = (\{P\} + \{F\} - \{R\}) + (\{\Delta P\} + \{\Delta F\} + \{\Delta Q^\theta\} + \{\Delta Q^c\}) \quad (38)$$

where,

$$[K] = [\Gamma au]^T (\int_V [M^L]^T [D][M^L]\, dv)[\Gamma au]$$

$$[K_G] = [\Gamma_{au}]^T (\int_V [M^N]\sigma_x\, dv)[\Gamma_{au}]$$

$$\{R\} = [\Gamma_{au}]^T (\int_V [M^L]^T \{\sigma\} dv)$$

$$\{\Delta Q^\theta\} = [\Gamma_{au}]^T (\int_V [M^L]^T \{H^\theta\}\Delta\theta\, dv)$$

$$\{\Delta Q^c\} = [\Gamma_{au}]^T (\int_V [M^L]^T \{H^c\}\, dv)\Delta t$$

$$\{P\} = [\Gamma_{au}]^T (\int_x (p_x, xp_x, p_y, xp_y, x^2p_y, x^3p_y, p_z, xp_z, x^2p_z, x^3p_z, 0, 0)^T dx)$$

$$\{F\} = (F_{x1}, F_{x2}, F_{y1}, M_{z1}, F_{y2}, M_{z2}, F_{z1}, M_{y1}, F_{z2}, M_{y2}, M_{x1}, M_{x2})^T$$

The Term [K] is the conventional incremental stiffness matrix and the term $[K_G]$ is the incremental geometric stiffness matrix. The terms {P} and {F} are the applied force vectors accumulated up to the previous load step. The term {R} is the reaction force vector and the terms $\{\Delta Q^\theta\}$ and $\{\Delta Q^c\}$ are the equivalent force vectors due to thermal and creep strain increments. The volume integrals are carried out by numerical integration so that the constitutive relation represented by [D], $\{H^\theta\}$ and $\{H^c\}$ can be evaluated for each integral point.

Solution procedures

The terms in eq.(38) is computed for individual element and transformed to global coordinate system. The overall incremental equilibrium equation is solved through step by step scheme. Since the load history of the nuclear power plant components is very complicated and includes cyclic processes, it seems suitable to incorporate a means to specify the amount of the load increments automatically. In this work, a load history is separated into several pathes in which loads are approximated to be varied proportionally with the time passing. The incremental equilibrium equation is solved for the entire loads given in a prescribed load path. The resulting displacements are reduced to some extent so as to satisfy the restrictions of the maximum incremental displacements, the maximum strain increments and the maximum number of transitting integral points. These restrictions assure the numerical precisions and stabilities especially for large deflection, creep and plastic analyses respectively. The reduction ratio is called the load incremental ratio and the calculation for a prescribed load path is finished when the sum of incremental load ratios becomes equal to unity.

4. Numerical examples

Creep buckling of a slightly inaccurate elastic column

As a first example to illustrate the present formulation, a creep buckling of a slightly inaccurate elastic column, which is analysed by Hoff [6], is discussed. The geometries and the constants used in this example are given in Fig.2. The steady creep law $\bar{\dot\varepsilon}^c = 0.01\bar{\sigma}^3$ is adopted in this example. Calculations are done for the columns which have three different initial deflection modes as shown in Fig.2. Hoff analized this problem for the column of Case-I by idealizing the actual section to the two flange model. The numerical results are shown in Fig.3. Comparing these results, it can be said that the initial deflection mode considerably affects the creep deflection. According to the result by Hoff's analysis, it requires only approximately a half time to reach a certain amount of deflection as compared to Case-I. This is supposedly due to the fact that the stiffness of the two flange model is less than that of the actual section as pointed out by Samuelson [7].

Thermal ratchetting of a pipe subjected to axial tension

Next, we consider the thermal ratchetting of a pipe subjected to axial tension. The geometry and the constants are given in Fig.4. The condition shown there corresponds to the R_1 stress regime of the Bree's Diagram [8]. To facilitate the comparison between the results obtained by Bree and present numerical calculations, the material constants are taken to be independent of temperature and bi-linear stress-strain relation is assumed.

Two calculations are carried out by assigning the value of tangent modulus E_T to 10 kg/cm^2 and 1000 kg/cm^2. When E_T is equal to 10 kg/cm^2, the material is almost perfectly plastic so that it can be compared directly with Bree's analysis. Fig.5 shows the axial stress distributions through wall thickness. The positions of a and b in Fig.5 can be calculated from the equilibrium condition and the gradient of thermal stress and were almost equal to those obtained from Bree's analysis. Fig.6 show the ratchet strain growth per each cycle. In the case of E_T equal to 10 kg/cm^2, the ratchet strain growth is a little smaller than Bree's because the pipe of this example can not be the one in the category of thin tube. In the case of E_T equal to 1000 kg/cm^2, ratchet strain growth is decreased cycle by cycle and this suggests that the pipe shakes down eventually.

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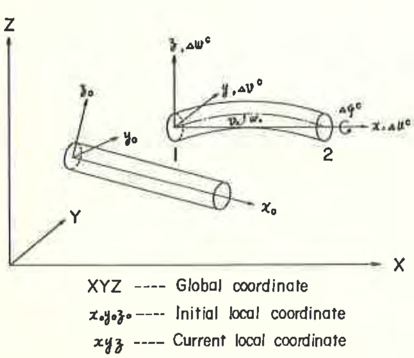


Fig. 1 Coordinate systems

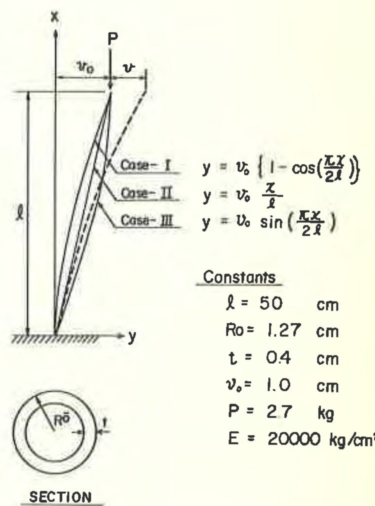


Fig. 2 The geometries and the constants of the column

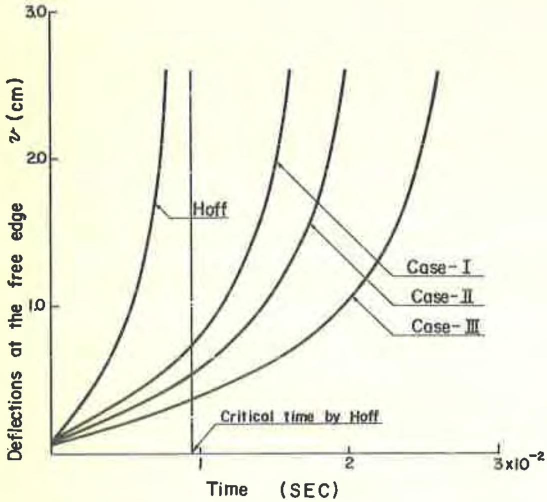
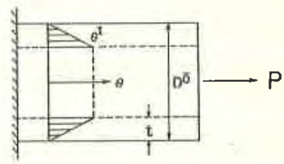


Fig. 3 Variation in time of the deflection at the free edge



Constants

D^{O} = 2.54 cm

t = 0.4 cm

P = 18.9 kg

θ^{I}_{max} = 400 ℃

E = 10000 kg/cm²

α = 10^{-5} /℃

Fig. 4 The geometry and the constants of the pipe

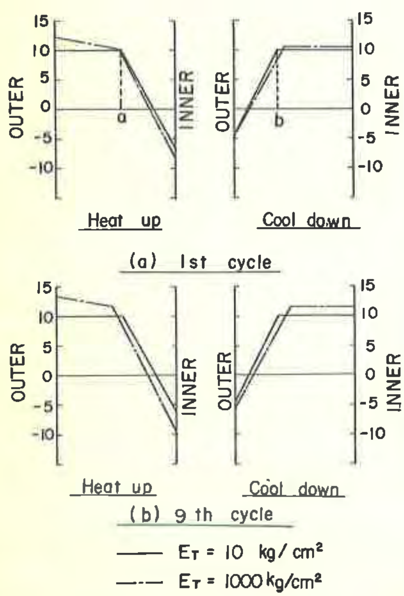


Fig. 5 Stress distributions through wall thickness for the 1st and 9th thermal cycles

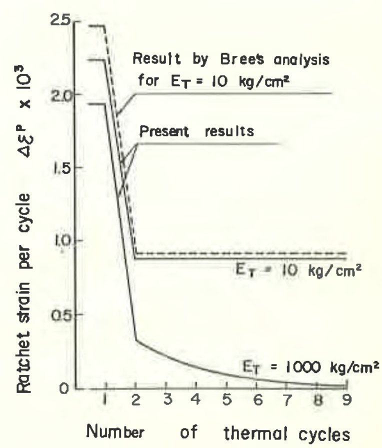


Fig. 6 Ratchet strain per cycle vs. number of thermal cycles