CRACK PROPAGATION ON SPHERICAL PRESSURE VESSELS

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SUMMARY

The risk presented by a crack on a pressure vessel built with a ductile steel cannot be well evaluated by simple application of the rules of Linear Elastic Fracture Mechanics, which only apply to brittle materials.

When ductile steels are used, large-scale yielding can occur before fracture, and LEFM is not suitable in that case. A rough analysis suggests that the kind of fracture of a cracked vessel depends on the size of the crack. Large-scale vessels behave like brittle ones if a crack with a critical length can appear. For small-scale vessels, a crack only reduces the stressed area and fracture will be of ductile type. In safety assessments the most unfavorable criterion should therefore be known and applied.

Tests were carried out on spherical vessels of three different scales built with the same steel. Cracks of different length were machined through the vessel wall.

From the results obtained, crack initiation stress (beginning of stable propagation) and instable propagation stress may be plotted against the lengths of these cracks. For small and medium size, subject to ductile fracture, the resulting curves are identical, and may be used for ductile fracture prediction.

Brittle rupture was observed on larger vessels and crack propagation occured at lower stress level. Preceeding curves are not usable for fracture analysis. Ultimate pressure can be computed with a good accuracy by using equivalent energy toughness, K_{1cd}, characteristic of the metal plates. Satisfactory measurements have been obtained on thin samples.

The risks of brittle fracture may then be judged by comparing K_{1cd} with the calculated K_1 value, in which corrections for vessel shape are taken into account.

It is thus possible to establish the bursting pressure of cracked spherical vessels, with the help of two rules, one for brittle fracture, the other for ductile instability. A practical method is proposed on the basis of the work reported here.

1. Introduction

The need for high dependability in the construction and operation of pressure vessels, particularly in the nuclear field, is unanimously acknowledged.

Fault investigation by nondestructive tests, carried out during manufacture or in-service inspection, may reveal the presence of defects such as cracks. The problem which confronts the manufacturer or operator is to evaluate the risk involved, so as to decide whether to leave the vessel in service, or whether to repair or discard it.

Linear Elastic Fracture Mechanics (LEFM) appears likely to provide answers to this question [1] [2] [3] [4] . However, it applies exclusively to materials without any notable ductility. As it so happens, most pressure vessels are built of ductile materials.

Studies devoted to such cases [5] [6] [7] [8] [9] have shown that, in most cases, the classic formulas of LEFM are inapplicable.

The results presented here are derived from an experimental study of spherical pressure vessels of different sizes, weakened by notches penetrating the entire wall.

2. Ductile tear and brittle fracture of cracked structures

A few simple reflections enable us to locate the problem.

LEFM equations are of the following form :

$$K_I = \sigma Q \sqrt{\pi a}$$

where : σ: stress applied far from the crack
a: half-length of crack
Q: shape factor (geometry, application of stress).

It is agreed that a crack becomes unstable when K_I is equal to or greater than a value K_{IC}, which is characteristic of the material.

The shape factors Q may be calculated or taken from published tables [10]. The toughness K_{IC} is determined from tests, some of which are standard [11].

Aside from the fact that K_{IC} is only applicable above a certain thickness, it should be emphasized that the critical crack length, namely, the value of 2a such that $K_I = K_{IC}$, is, in this formula, independent of the vessel dimensions (for an identical stress field). Obviously, if this length is significant with respect to the circumference of the pressure vessel, the risk of fracture will exist although LEFM does not indicate this. In this case, fracture will be caused by an inadequate resistant cross-section, leading to ductile tear.

This ductile tear, which does not concern the toughness of the material, obeys the laws of geometric similitude, according to which similar vessels exhibiting geometrically similar cracks will be destroyed at the same pressure levels. This makes it possible to conduct tests on scale models.

However, as the vessels increase in size, the cracks follow suit, incurring the risk of brittle fracture.

Hence cracked pressure vessels run the risk of ductile tear when they are small, and of brittle fracture when they are large. This implies a conflict between two destruction models, and any evaluation should be based on the most unfavorable criteria.

Although various formulas have been proposed previously [7] [12] to do this, the effect of scale factor with respect to the strength of cracked vessels is still only poorly understood.

Certain results reveal the influence of spherical vessel dimensions on their strength, for geometrically similar defects [13]. The definition of criteria enabling prediction of the type of fracture (brittle or ductile) as a function of geometry has been dealt with in [14].

3. Description of tests

Tests were conducted on spheres of three different sizes :

5 spheres	diameter 363 mm	thickness 3 mm	}	theoretical dimensions
1 sphere	diameter 918 mm	thickness 7 mm	}	
1 sphere	diameter 1800 mm	thickness 15 mm	}	

Spheres diameters 363 mm and 918 mm consisted of stamped hemispheres connected by an equatorial weld, while sphere diameter 1800 mm was built of six welded panels.

Pre-test (ultrasonic) and post-mortem (direct) thickness measurements showed substantial deviations in real thickness from theoretical thickness, together with significant variations between the various measurements taken on a single sphere.

The spheres were built of AMMO steel, from the Marrel steelworks, having the following composition :

C	Mn	Mo	Va
≤ 0.17	0.9 to 1.35	0.4 to 0.6	0.07

The following test procedure was followed for each sphere :

- Measurement of maximum pressure (appearance of plastic expansion) of the untested sphere, by the application of an internal pressure which is not exceeded in subsequent tests conducted on the same sphere.
- A through wall flaw is cut by means of a milling cutter.
- The notch ends are finished with a fine saw ($r < 0.1$ mm).
- The notch is covered on the inside by a patch consisting of the following material, starting from the inner wall :
  - 1 sheet of teflon
  - 1 thin sheet of stainless steel
  - 1 sheet of perbunan cemented to the wall.
- The hydraulic pressure is raised progressively by means of a hand pump.
- After the test, the notch edges are reshaped and weld-filled.

Several tests of this type can be conducted on the 363 mm and 1800 mm diameter spheres. On the other hand, the 1800 mm diameter sphere can only be subjected to a single test, owing to its rigidity.

Initiation and growth of the cracks were observed under a binocular magnifier fitted with appropriate cross hairs.

4. Results

The partial results published previously [15] [16] are given here in full.

It is only appropriate to start off by defining the characteristic values involved in interpretation. These are the following :

(a) Dimensions of the sphere :
diameter ..... D
thickness .... e
notch length . 2a

(b) Characteristics of the material :
yield strength to 0.2 %. σy
ultimate strength ...... σR

(c) Two significant stresses σ_i and σ_p derived from the behavior of a crack when pressure is applied (see figure 1) :
up to a pressure P_i, the initial notch length is unchanged ; at pressure P_i, a crack is initiated with stable propagation ; during this phase, the length of the crack increases by a finite quantity, as a function of the increase in pressure. Finally, at pressure P_p, unstable crack propagation ensues, leading to fracture if this pressure is maintained.
Two characteristic stresses can then be calculated for each test :

Initiation stress $\sigma_i = \dfrac{P_i \times d}{4e}$ éq. (1)

Unstable propagation stress $\sigma_p = \dfrac{P_p \times d}{4e}$ éq. (2)

The results are given in tables 1 and 2. All tests were carried out at ambient temperature (15 < t < 20 °C). The mechanical characteristics of sphere n° 9 were not measured, but were assumed to be identical to those of sphere n° 13 for interpretation of the test results.

The values of the two stresses σ_i and σ_p, related to the yield strength and ultimate strength of the material respectively, are plotted in figures 2 and 3, taking account of the relative length of the crack.

Also plotted in these figures are the test results reported by Burdekin and Taylor [17]. Excellent agreement can be observed in results obtained with small and medium-sized spheres.

5. Discussion Part 1 Ductile tear

It can be seen that the results obtained with large spheres (1470 mm and 1800 mm diameter) stand apart from the other results. An examination of the fracture of the 1800 mm diameter sphere shows that a typical brittle fracture occurs, whereas the small and medium-sized spheres reveal ductile type fracture. An examination of the two foregoing figures agrees perfectly with this observation. The results concerning small and medium-sized spheres, when grouped together, satisfy the aspect of geometrical similitude which goes along with ductile tear.

These results make it possible to determine the risk of ductile tear incurred by a crack, with respect to spherical vessels. The same results are shown in figure 4, which gives the following :

$$\frac{\sigma_P}{\sigma_R} \text{ as a function of } \frac{2a}{\sqrt{R.E}}$$

where : 2a .... crack length

R .... radius of sphere

e .... thickness of sphere

The calculation can be carried out with this figure. A result concerning a large sphere is given for information purposes, in order to show that the fracture stress is far lower than that which may be expected in case of ductile fracture.

6. Discussion Part 2 Brittle fracture 1800 mm diameter sphere

One generally tends to estimate brittleness on the basis of Charpy impact tests. We measured the impact strength KCV of specimens taken from the 1800 mm diameter sphere, and the results are plotted in figure 5. It can be seen that the test on the sphere, carried out at ambient temperature, occurs in a low impact strength zone of the metal. Unfortunately, this does not permit determination of the fracture pressure, as the impact strength represents a simple reference point.

We carried out measurements of K_{IC} on CT specimens in accordance with [11], but with an insufficient maximum thickness to obtain a valid measurement. Hence the results were treated as in the work of Witt and Mager [18] [19]. The question is one of evaluating K_{ICd}*, which characterizes the toughness of the metal plate in question (material and thickness).

This method of using equivalent energy sometimes raises the problem of determining the maximum load point, as shown by figure 6. It can be seen that point P_A, concerning a relatively thick specimen A, is clearly defined, whereas its position becomes harder to determine when the thickness of specimen B is very low (point P_B).

The values measured with specimens of different dimensions are shown in figure 7.

It may be noted that for a thickness of 12 mm, approaching that of the sphere (14 mm), the value of K_{ICd} is 515 $kg/mm^2 \sqrt{mm}$. It can also be seen that the order of magnitude remains the same with very thin specimens.

The value of K_{ICd} should be compared with that of K_I calculated for the fracture conditions of the 1800 mm diameter sphere. K_I is calculated by

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* We retained the notation of Witt and Mager, in which d is the specimen thickness. This should not be confused with "dynamic".

introducing the correction of Folias [20] into the LEFM formula. This gives the following :

$$K_I = \sigma Q \sqrt{\pi a} \qquad \text{éq. (3)}$$

$$\text{with } Q = \left(1 + \frac{1.9a^2}{R.e}\right)^{\frac{1}{2}}$$

This gives $K_I = 660\ kg/mm^2 . \sqrt{mm}$, which falls well within the range of K_{ICd} values measured with the specimens.

7. Conclusions

The foregoing discussion and the experimental studies mentioned above indicate the existence of a practical method for estimating the strength of a spherical vessel exhibiting a through crack. Two verifications must be made:

(a) With respect to ductile tear, by using curves such as those in figures 2 to 4.

(b) With respect to brittle fracture, by measuring the K_{ICd} of the vessel plate, and by applying the classic formulas of LEFM.

We wish to express our thanks to Mr. D. COSTES and Mr. A. DORÉ (Nuclear Safety Department) for the interest they showed in this work, and to Mr. R. BERNARD and Mr. C. SONNET (Mechanical and Thermal Studies Department) for their substantial participation in the experimental phase of this study.

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TABLE 1

D (mm)	Sphère n°	Mechanical Characteristics σ_y 0,2 (hb)	Mechanical Characteristics $\sigma_{R(hb)}$	e (mm)	2a (mm)	P (bars) P_i	P (bars) P_p	σ (hb) σ_i	σ (hb) σ_p
	9			2,45	35	90	98	33	36,2
				2,25	70	51	63	20,5	25,5
				2,20	105	32	42	13,2	17,3
	10	36,2	51,8	2,70	40	95	99	31,8	33,2
				2,70	59	60	68	20	22,8
	13	36,2	52,5	2,15	35	80	84	33,6	36,2
				2,35	50	66	76	25,4	29,2
				2,50	65	60	72	21,7	26
				2,52	72	50	65	18	23,6
				2,50	80	46	56	16,6	20,5
363				2,20	95	32		13,2	
				2,10	102	29		12,5	
				2,20	110	28	37	11,5	15,4
				2,35	125	26	33	10	12,7
	15	39,7	52,5	2,17	15	88	86	36,8	41,5
				2,12	25	99	92	36,8	39,5
	19	36,4	52	2,45	56	55	76	20,3	28
				2,80	61	63	73	20,4	23,5
				2,55	75	52	63	18,5	22,5
				2,80	99	35	50	11,3	15,5

TABLE 2

D (mm)	Sphère n°	Mechanical Characteristics σy 0,2 (hb)	σR (hb)	e (mm)	2 a (mm)	P (bars) P_i	P_p	σ (hb) σ_i	σ_p
918	1	28	48,5	4,70	110	50	56	24	27
				4,75	140	38	50	18,3	23,8
				4,90	170	33	46	15,6	21,6
				4,70	205	25	37	12	18,1
				5	230	24	38	11	17,5
				4,75	260	22	33	10,2	16
				4,90	318	16	30	7,5	13,8
1800	1	50,5	65,7	14	400	32	32	10,4	10,4

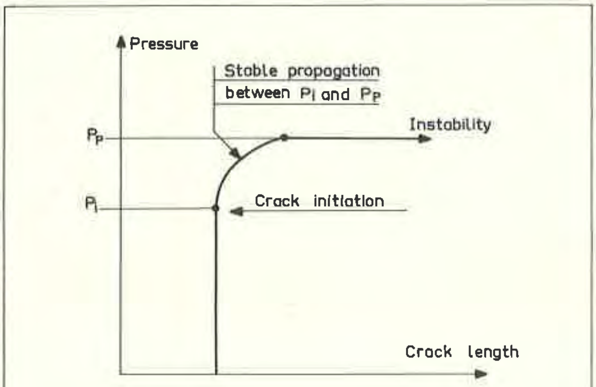

FIG. 1

Diagrammatic relation of pressure versus crack length during a test.

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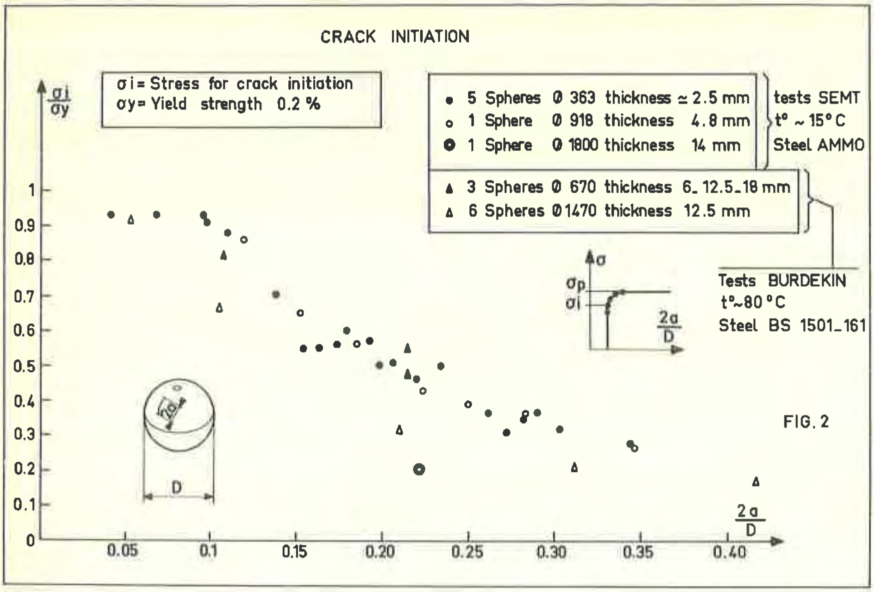
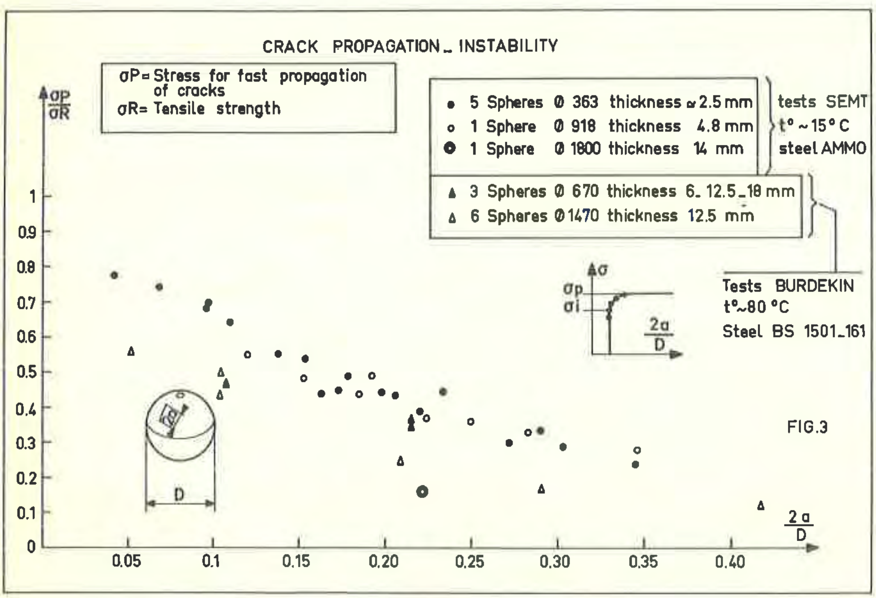
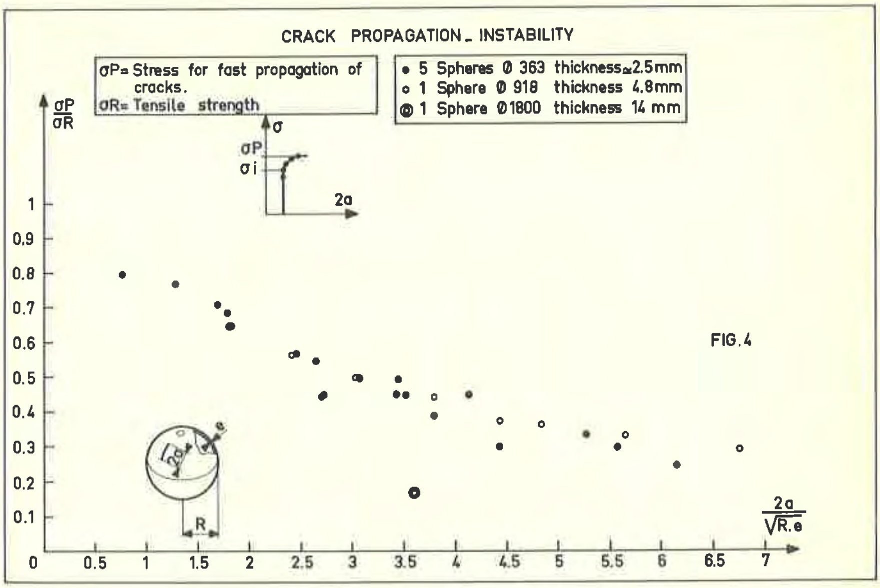


Relation between the initiation stress and the crack length.

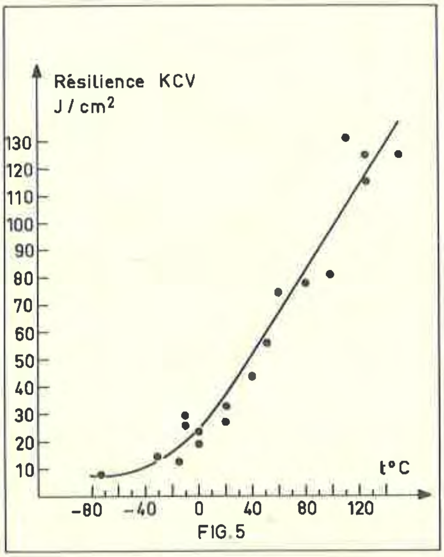


Relation between the instability stress and the crack length.



Relation between the instability stress and the nondimensional.

parameter $\frac{2a}{\sqrt{R.e}}$



Charpy V notch test values versus temperature.

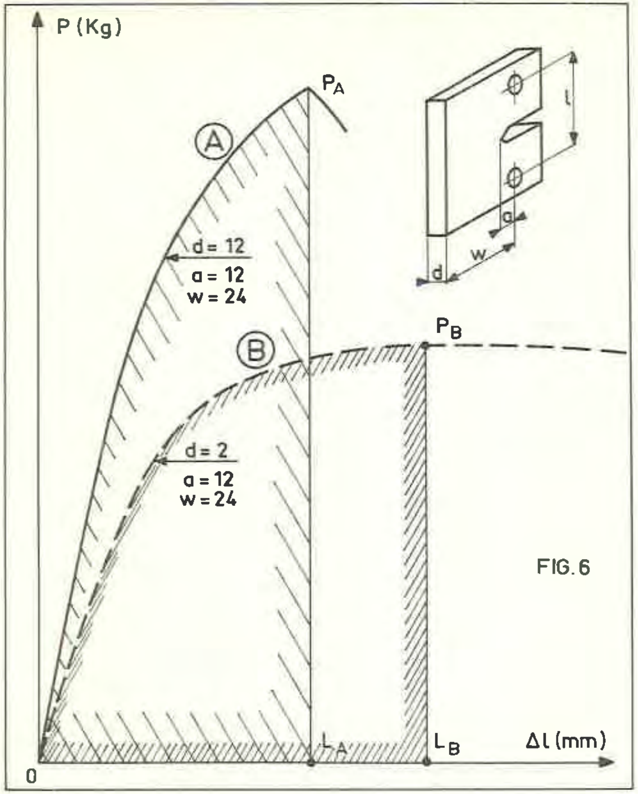


FIG. 6

Typic curves from measurement of K_{ICd}.

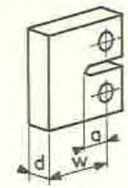


FIG. 7

Sphère	W × a mm	d mm	$K_{I}cd$ ($Kg/mm^2\sqrt{mm}$)	Mean
Ø363 (n°13)	24 × 12	2	570 580 780	<u>640</u>
Ø900 (n°1)	24 × 12	2	775 680 680	<u>705</u>
Ø1800 (n°1)	24 × 12	2	645 570 660	<u>625</u>
		6	645 635 530	<u>600</u>
		12	495 510 550	<u>515</u>
	48 × 24	6	870 705 800	<u>800</u>

Values of K_{ICd} for various specimens.