NUMERICAL ANALYSIS ORIENTED BIAXIAL STRESS-STRAIN RELATION AND FAILURE CRITERION OF PLAIN CONCRETE

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SUMMARY

For a numerical analysis, e.g. by Finite Differences or Finite Element Method, there is still a lack of knowledge on the level of incorporating a material behaviour law for concrete beyond the linear elastic range.

In the presented paper, a biaxial stress-strain relation and failure criterion is proposed, which is applicable to the above mentioned structural analysis methods.

The formulation of material behaviour of plain concrete in biaxial stress-state was developed on the basis of test results, given by Kupfer/Hilsdorf/Rüsch. A nonlinear elastic, anisotropic stress-strain relation was derived with two moduli of elasticity, E_1, E_2, and Poisson's ratios, ν_1, ν_2, which depend on the prevailing biaxial stress state. The stress-strain relation is valid in the whole biaxial stress field, that means with a smooth transition between the domains of tension/tension, tension/compression and compression/compression. The stress-dependent moduli E_1, E_2 and the Poisson's ratios ν_1, ν_2 are approximated by polynomials, trigonometrical and exponential functions. They were established not only as secant-, but also as tangent-functions for arbitrary usein the numerical procedure.

A failure criterion was defined by approximating the test results of the biaxial ultimate concrete strength with a 7th degree polynomial, which is also valid in the whole biaxial stress domain. The definition of the state of failure is given as a function of stresses as well as strains.

Initial parameters of the formulation of the biaxial material behaviour are the uniaxial cylindrical strength of concrete and the initial values of Young's modulus and Poisson's ratio. A simple expansion of this formulation makes it applicable not only to normal but also to lightweight concrete.

Comparison of numerically calculated stress-strain curves up to the ultimate biaxial stresses which indicate the failure criteria with those obtained from tests show a very good agreement. It is shown, that the biaxial stress-strain relation can be extended for use in cases of triaxial tension/tension/compression stress state. Numerical examples of analysis of concrete slabs show the importance of incorporation of a realistic material behaviour for better safety estimations.

1. Introduction

In the numerical analysis of reinforced concrete structures mainly two ways of implementing a material behaviour law are known, both on the phenomenological level of material characterization. One is based on an integrated description of the material behaviour of both concrete and reinforcement, the other considers the respective material characteristics of concrete and reinforced steel bars seperately. This paper refers to the latter way and is restricted to the material behaviour of plain concrete only.

In the last few years numerous papers had been published on this subject, some of which are cited later. Up to now most of the proposed f a i l u r e c r i t e r i a for biaxially loaded concrete consist in a set of functions, each valid only in parts of the biaxial stress-strain domain. The load dependence was always chosen in terms of stress only. Except for the quadrilinear s t r e s s - s t r a i n r e l a t i o n of Romstad et al.[1] , in all other formulations of biaxial deformation behaviour the differential representation of the non-linear stress-strain relations is preferred, being a more adequate way to fit test data and to take account of the incremental numerical procedure.

Berg [2] used a modified representation of Kupfer's [3] test data and calculated the tangent values of the elasticity matrix by interpolation, in dependance of the prevailing stress- and strain-increments. Nelissen [4] and Wegner [5] developed a strain-stress, respectively stress-strain relation, dependent on the current state of stresses, respectively strains, and several further parameters or functions. Stress-strain relations in octahedral form were proposed by Girijavallabhan/Metha [6] , Khan et al. [7] , Grünberg [8] and Kupfer/Gerstle [9] . They all presented tangent values of the shear modulus, while Kupfer/Gerstle [9] derived tangent as well as secant values of both shear and bulk moduli. It was stated by most of the above mentioned authors, that it is sufficient to introduce a stress or strain dependent shear modulus and to regard the bulk modulus as a constant value. The above cited formulations of deformation behaviour are isotropic ones, while the stress-strain behaviour of biaxially loaded concrete indicates stress- or strain-induced anisotropy beyond the linear elastic range. The anisotropy results from the nonlinear stress-strain response in the two directions considered and leads in general to a non-symmetrical elasticity matrix containing the appropriate material properties. Liu et al. [10] and Darwin [11] suggested anisotropic stress-strain relations, and employed special assumptions to obtain a symmetrical elasticity matrix.

The herein proposed formulation of biaxial deformation behaviour provides a stress-induced anisotropical presentation too. Symmetry of the material matrix is achieved by assumption f. , listed below. Main assumptions incorporated in the material law in order to obtain characteristical values on the basis of test results and to describe the biaxial, nonlinear stress-strain behaviour of concrete are as follows:

a. plane stress state ($\sigma_{33} = 0$); (extension to $\sigma_{33} \neq 0$ see chapter 4)
b. restriction to plain concrete, reinforcement regarded separately
c. homogeneous, quasi-isotropic material, with stress-induced anisotropy
d. short-term loading
e. hyperelasticity
f. existence of an elastic potential for each stress- and strain-increment.

The dependence of all material characteristic values on biaxial loading, incorporated in the developed formulations, relies on the test results obtained by Kupfer/Hilsdorf/Rüsch [3], [9] and Stegbauer/Linse [12], with tests performed on concrete and lightweight concrete specimens.

2. Stress-strain relations for concrete in biaxial loading

Considering the above listed assumptions, it is possible to reduce the yet unknown material characteristic values in the elasticity matrix to a total of four. These four values may be interpreted as two moduli of elasticity, E_1 and E_2, the index indicating principal directions of biaxial load action, and two corresponding Poisson's ratios, ν_1 and ν_2. In order to satisfy assumption f., eq. (1) is obtained, which shows that now only three material

$$\frac{\nu_1}{E_1} = \frac{\nu_2}{E_2} \tag{1}$$

characteristic values, dependent on the current stress-strain state, are necessary at all. Eq. (2) shows the elasticity matrix valid for principal directions 1 and 2 and for tangent values of the material parameters being

$$\Delta\sigma^{(1,2)} = \mathbf{D}^{(1,2)} \cdot \Delta\varepsilon^{(1,2)} \tag{2}$$

$$\mathbf{D}^{(1,2)} = \frac{1}{1-\nu_1\nu_2}\begin{bmatrix} E_1 & \nu_1 E_2 & 0 \\ \nu_2 E_1 & E_2 & 0 \\ 0 & 0 & \frac{E_1(1-\nu_1\nu_2)}{1+2\nu_1+\nu_1/\nu_2} \end{bmatrix} = \frac{E_1}{1-\nu_1\nu_2}\begin{bmatrix} 1 & \nu_2 & 0 \\ \nu_2 & \frac{\nu_2}{\nu_1} & 0 \\ 0 & 0 & \frac{1-\nu_1\nu_2}{1+2\nu_1+\nu_1/\nu_2} \end{bmatrix}$$

constant inside each increment. Incremental stress-strain and strain-stress relations for arbitrary directions x and y, x inclined with an angle α to the 1-axis, are obtained with eqs. (3) and (4) by simple transformation using the abbreviations indicated above.

Eqs. (2) to (4) hold for secant values of the material characteristic parameters too, if the total stresses and strains and not only their increments Δ, are inserted and if the secant values E_i^s and ν_i^s, i = 1,2, are used instead of the tangent values E_i and ν_i.

$s = \sin\alpha \qquad s2 = \sin^2\alpha \qquad s3 = \sin^3\alpha$

$c = \cos\alpha \qquad c2 = \cos^2\alpha \qquad c3 = \cos^3\alpha$

$A1 = 1 + 2\nu_1 + \nu_1/\nu_2 \qquad A3 = -1 + \nu_1 - \nu_2 + \nu_1/\nu_2$

$A2 = 2 - \nu_1/\nu_2 - \nu_2/\nu_1 \qquad A4 = 1 + \nu_1 - \nu_2 - \nu_2/\nu_1$

$$\Delta\boldsymbol{\sigma}^{(x,y)} = \mathbf{D}^{(x,y)} \cdot \Delta\boldsymbol{\varepsilon}^{(x,y)} \tag{3}$$

$$\begin{bmatrix} \Delta\sigma_{xx} \\ \Delta\sigma_{yy} \\ \Delta\sigma_{xy} \end{bmatrix} = \frac{E_1 / A1}{1-\nu_1 \nu_2} \begin{bmatrix} A1\ (\frac{\nu_2}{\nu_1}\ s2 + c2) + A2\ s2\ c2 & \nu_2\ A1 - A2\ s2\ c2 & A3\ s\ c3 + A4\ s3\ c \\ & A1\ (s2 + \frac{\nu_2}{\nu_1}\ c2) + A2\ s2\ c2 & A3\ s3\ c + A4\ s\ c3 \\ \text{symm.} & & (1-\nu_1 \nu_2) - A2\ s2\ c2 \end{bmatrix} \begin{bmatrix} \Delta\varepsilon_{xx} \\ \Delta\varepsilon_{yy} \\ 2\Delta\varepsilon_{xy} \end{bmatrix}$$

$$\Delta\boldsymbol{\varepsilon}^{(x,y)} = [\ \mathbf{D}^{(x,y)}\ ]^{-1} \cdot \Delta\boldsymbol{\sigma}^{(x,y)} \tag{4}$$

$$\begin{bmatrix} \Delta\varepsilon_{xx} \\ \Delta\varepsilon_{yy} \\ 2\Delta\varepsilon_{xy} \end{bmatrix} = \frac{1}{E_1} \begin{bmatrix} \frac{\nu_1}{\nu_2}\ s2 + c2 & -\nu_1 & (1-\frac{\nu_1}{\nu_2})\ s\ c \\ & s2 + \frac{\nu_1}{\nu_2}\ c2 & (1-\frac{\nu_1}{\nu_2})\ s\ c \\ \text{symm.} & & 1+2\nu_1+\frac{\nu_1}{\nu_2} \end{bmatrix} \begin{bmatrix} \Delta\sigma_{xx} \\ \Delta\sigma_{yy} \\ \Delta\sigma_{xy} \end{bmatrix}$$

$$\Delta\varepsilon_{zz} = \frac{1}{E_1} [\ -\nu_1 \quad -\nu_1 \quad 0\ ] \cdot \Delta\boldsymbol{\sigma}^{(x,y)}$$

By close-fitting approximation of Kupfer's [3], [9] parameter representation of the secant values E_1^S and ν_1^S, dependent on the two principal stresses, σ_{11} and σ_{22}, related to the uniaxial cylindrical strength of concrete f_c, the functions E_2^S (≙FEM2), ν_1^S(≙FQU1) and ν_2^S(≙FQU2) were developed (see table I, a - c). The numerical procedure may be more efficient, when secant as well as tangent values of the material characteristic functions are available in the computer program. The use of tangent values is more convenient during incremental loading, but secant values are advantageous when partial unloading effects, e.g. cracking of concrete under tension stresses, have to be considered. A parameter representation of the tangent values was derived similar to Kupfer's secant one. The functions of the tangent values E_1 (≙ FET1), E_2 (≙ FET2) and ν_1 (≙ FQT1), resulting of approximation by least squares, are presented in table I, d - f.

The respective fourth material characteristic value, missing in table I, can be obtained by application of eq.(1).

The whole formulation of the nonlinear stress-strain behaviour is presented in dependence of the biaxial related **to** f_c principal stresses in form of FORTRAN - function subroutines.

Figs. 1 and 2 point out, that the functions are valid in the entire biaxial stress domain and show a smooth transition between the compression/compression, compression/tension and tension/tension zones. They are limited by the failure criteria described in chapter 3.

The proposed stress-strain relation presented in table I is applicable with good agreement to test results of normal concrete of various strengths when the proper initial parameters of Young's modulus, E_o (≙EMO), and Poisson's ratio , ν_o (≙ QUO) are inserted. Fig. 3 shows the good agreement of calculated stress-strain curves with those obtained in the experiment. Eq. (5) indicates the extension of the stress-strain relation obtained of normal concrete

$$E_i^{s,LC} = E_o^{LC} \left[0.7 + 0.3 \left(\frac{E_i^{s,NC}}{E_o} \right) \right]$$

$$\nu_i^{s,LC} = \nu_o^{LC} \left[0.7 + 0.3 \left(\frac{\nu_i^{s,NC}}{\nu_o} \right) \right]$$

$$E_i^{LC} = E_o^{LC} \left[0.6 + 0.4 \left(\frac{E_i^{NC}}{E_o} \right) \right] \qquad (5)$$

$$\nu_i^{LC} = \nu_o^{LC} \left(\frac{\nu_i^{NC}}{\nu_o} \right)$$

with i=1,2 and material values in round brackets according to table I

(NC) to ligthweight concrete (LC). The extension was performed by simple curve-fitting of the above outlined dependence of the material characteristic values on biaxial loading related to those reported in the test results of Stegbauer/Linse [12]. Calculated stress-strain curves for lightweight concrete (Fig. 4) show good correspondence with the test results.

3. Failure criteria for concrete in biaxial loading

Since the failure limit is symmetrical to the equisectrix ($\sigma_{11} = \sigma_{22}$, or $\epsilon_{11} = \epsilon_{22}$), the formulation of a failure criterion may be restricted to one half of the biaxial stress (or strain) domain. This is realized by approximating the test results of ultimate concrete strength in a X-Y, respectively U-V coordinate system inclined to the σ_{11}/f_c-, respectively $\epsilon_{11}/\epsilon_{c,U}$-axis with an angle of 45^o. Using polynomials, as stated in eqs. (6) and (7), the coefficients compiled in table II are obtained. They refer to the three

$$Y = \sum_{n=0}^{n=7} A_n X^n + A_8 \left(0.594 \sin\frac{\pi X}{1.7} - 0.00192 \tan\frac{\pi X}{3.3}\right) \qquad (6)$$

$$V = \sum_{n=0}^{n=6} B_n U^n \qquad (7)$$

different concrete strengths used in Kupfer's tests [3], [9], to the lightweight concrete investigated by Stegbauer/Linse [12] and to an average failure curve, valid for all strengths of normal concrete. Failure criteria for normal and lightweight concretes, according to eq. (6) and dependent on biaxial principal stresses related to the uniaxial strength, are compared with several test curves in Fig. 5. The related to f_c stress dependent failure curves of different concrete strength, e.g. Kupfer's test results included in Fig. 5, only differ scarcely. On the other hand strong dependence on concrete strength is obvious through Fig. 6 by the failure criteria as functions of strains, related to the uniaxial ultimate strain, received from eq. (7). This evident dependence on concrete strength gave rise to the stress-dependent formulation of the material characteristic values outlined in chapter 2. It is the formulation of biaxial material behaviour in terms of stresses that allows for this advantageous, uniformly defined material law to be applied for all concrete strengths in the numerical procedure.

4. Extension of the stress-strain relation proposed in chapter 2 to special triaxial stress states

The stress-strain behaviour of concrete, subjected to pure tension load, is almost linear. The material characteristic functions, given in table I, confirm this linear elastic behaviour in the biaxial tension/tension domain yielding material characterisitic values nearby the initial parameters E_o and ν_o. An extension of the proposed biaxial stress-strain relation to a triaxial one, may therefore be easily obtained in the compr./compr./tens., compr./tens./tens. and tens./tens./tension domain. Since stresses and strains are algebraically arranged the material characteristic functions according to table I are related to the principal directions 1 and 2. The pertinent principal direction 3 remains in tension loading throughout all the above mentioned special triaxial stress states. Therefore it becomes obvious to apply the initial parameters E_o and ν_o to principal direction 3 upon derivation of a special triaxial stress-strain relation. The obtained strain-

$$\Delta \varepsilon^{(1,3)} = [\mathbf{D}^{(1,3)}]^{-1} \Delta \sigma^{(1,3)} \qquad (8)$$

$$[\mathbf{D}^{(1,3)}]^{-1} = \frac{1}{E_1} \begin{bmatrix} 1 & -\nu_1 & -\nu_1 & 0 & 0 & 0 \\ & \nu_1/\nu_2 & -\nu_1 & 0 & 0 & 0 \\ & & E_1/E_o & 0 & 0 & 0 \\ & & & 1+2\nu_1+\frac{\nu_1}{\nu_2} & 0 & 0 \\ & \text{symm.} & & & 2(1+\nu_o)\frac{E_1}{E_o} & 0 \\ & & & & & 2(1+\nu_o)\frac{E_1}{E_o} \end{bmatrix}$$

stress relation, valid for principal directions and applicable to the above defined special triaxial domain, is given in eq. (8). Matrix-inversion of eq. (8) yields the elasticity matrix stated in eq. (9). Comparison of

$$\Delta \sigma^{(1,3)} = \mathbf{D}^{(1,3)} \cdot \Delta \varepsilon^{(1,3)} \qquad (9)$$

$$\mathbf{D}^{(1,3)} = \frac{E_1}{A5} \begin{bmatrix} \nu_1\frac{E_1}{\nu_2 E_o}-\nu_1^2 & \nu_1\frac{E_1}{E_o}+\nu_1^2 & \nu_1\frac{\nu_1}{\nu_2}+\nu_1^2 & 0 & 0 & 0 \\ & \frac{E_1}{E_o}-\nu_1^2 & \nu_1(1+\nu_1) & 0 & 0 & 0 \\ & & \frac{\nu_1}{\nu_2}-\nu_1^2 & 0 & 0 & 0 \\ & \text{symm.} & & \frac{A5}{1+2\nu_1+\nu_1/\nu_2} & 0 & 0 \\ & & & & \frac{A5\ E_o/E_1}{2(1+\nu_o)} & 0 \\ & & & & & \frac{A5\ E_o/E_1}{2(1+\nu_o)} \end{bmatrix}$$

$$\text{with:} \quad A5 = \left(\frac{\nu_1}{\nu_2} - \nu_1^2\right)\frac{E_1}{E_o} - \nu_1^2\left(1+2\nu_1+\frac{\nu_1}{\nu_2}\right)$$

$$[\Delta \sigma^{(1,3)}]^T = [\Delta\sigma_{11} \quad \Delta\sigma_{22} \quad \Delta\sigma_{33} \quad \Delta\sigma_{12} \quad \Delta\sigma_{13} \quad \Delta\sigma_{23}]$$

$$[\Delta \varepsilon^{(1,3)}]^T = [\Delta\varepsilon_{11} \quad \Delta\varepsilon_{22} \quad \Delta\varepsilon_{33} \quad 2\Delta\varepsilon_{12} \quad 2\Delta\varepsilon_{13} \quad 2\Delta\varepsilon_{23}]$$

calculated stress-strain curves with those obtained in tests will be presented at the conference.

5. Finite element analysis of a reinforced concrete slab

The influence of different assumptions concerning the stress-strain and failure behaviour of concrete was investigated with the numerical analysis of a reinforced concrete slab. Fig. 7 shows the calculated load deflection curves compared with test results obtained by Franz [13]. The comparison of various assumptions, described in Fig. 7, indicates the importance especially for the ultimate load range of incorporating a realistic formulation of biaxial concrete behaviour. Not only the ultimate load, but also the deflections near ultimate load are influenced by different assumptions of concrete behaviour. More detailed results of the numerical analysis are comprised elsewhere [14].

6. Notations, References

$\sigma_{11}, \sigma_{22}, \sigma_{33}$: principal stresses, arranged algebraically (but: in the biaxial domain: $\sigma_{33}=0$)

$\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}$: corresponding principal strains

α : angle between arbitrary x-y, and principal 1-2 coordinate system; counter-clockwise positive

f_c : uniaxial, compressive cylindrical concrete strength

$\varepsilon_{c,U}$: ultimate strain corresponding to uniaxial cyl. strength f_c

E_o , ν_o : initial values (at beginning of uniaxial loading) of Young's modulus and Poisson's ratio

E_1, E_2, ν_1, ν_2: stress-dependent elasticity moduli and Poisson's ratios of the anisotropic stress-strain formulation, the principal directions 1 and 2 beeing the axes of stress-induced anisotropy(tangent value, superscript 's' stands for secant one)

[1] ROMSTAD, K.M./TAYLOR, M.A./HERRMANN, L.R., "Numerical biaxial characterization for concrete", Proc. ASCE, EM5, 935 - 948, (1974)

[2] BERG, S., "Nonlinear Finite Element Analysis of reinforced concrete plates", Div.of Struct.Mech., Univ. of Trondheim, Rep. No. 73-1, (1973)

[3] KUPFER, H.B./HILSDORF, K.A./RÜSCH, H., "Behaviour of concrete under biaxial stresses", Proc. ACI, 66, 656-666, (1969) and: Lehrstuhl für Massivbau der Techn. Univ. München, Bericht Nr. 78, (1969)

[4] NELISSEN, L.J.M., "Twee-assig onderzoek van grindbeton", Diss.Techn.Hochschule Delft, (1970)

[5] WEGNER, R.,"Tragverhalten von Stahlbetonplatten mit nichtlinearen Materialgesetzen im gerissenen Zustand-Finite Element Methode", Diss. Techn. Univ. Braunschweig, (1974), Bericht No. 74-11 aus dem Institut für Statik der Techn. Univ. Braunschweig

[6] GIRIJAVALLABHAN, Ch.V./METHA, K.G., "Stress-strain relationship from compression tests on nonlinear materials". Proc. of the Symp. on Applic. of FEM in Civ.Eng., Vanderbilt Univ., Nov. 13-14, 457-480, (1969)

[7] KHAN, M.A. et al., "Analyse tridimensionale du comportement non-linéaire d'un caisson de réacteur nucléaire en beton précontrainte", Proc.1st.Intl.Conf. on SMIRT, 4, H3/5, (1971) and: Preprints 2nd. Intl. Conf. on SMIRT, III, H2/5, (1973)

[8] GRÜNBERG, J.,"Berechnung von ebenen Stahlbetonflächentragwerken im gerissenen Zustand mit der Methode der finiten Elemente", Diss.Techn.Univ.Hannover, (1973) and: Werner-Verlag, Düsseldorf, (1974)

[9] KUPFER, H.B./GERSTLE, K.H. "The non-linear behaviour of concrete under biaxial stress states", Nat.Struct.Eng.Meeting, ASCE, Cleveland, Ohio, April 24-28, (1972) and: Schriftenreihe des Deutschen Ausschusses für Stahlbeton (DAfStb), Heft 229, (1973)

[10] LIU,T.C.Y./NILSON, A.H./SLATE, F.O., "Biaxial stress-strain relations for concrete", Proc. ASCE, ST 5, 1025-1034, (1972)

[11] DARWIN,D./Pecknold, D.A.W., "Inelastic model for cyclic biaxial loading of reinforced concrete", UILU-ENG-74-2018, (July 1974), University of Illinois, Urbana

[12] STEGBAUER, A./LINSE, D., "Das Verhalten von Leichtbeton, Gasbeton, Zementstein und Gips unter zweiachsiger Beanspruchung", Lehrstuhl für Massivbau der Techn.Univ.München, Bericht AIF, Nr. 1842, (1972)

[13] FRANZ, G., "Über die Beanspruchung der Bewehrung bei kreuzweise bewehrten, vierseitig frei drehbar gelagerten Rechteckplatten aus Stahlbeton", Inst.für Beton und Stahlbeton, Univ. Karlsruhe, Untersuchungsbericht zum Forschungsauftrag F 505, (1970)

[14] LINK, J., "Eine Formulierung des zweiaxialen Verformungs- und Bruchverhaltens von Beton und deren Anwendung auf die wirklichkeitsnahe Berechnung von Stahlbetonplatten", Diss. Techn.Hochschule Darmstadt, D17, (1975) and: Schriftenreihe des DAfStb, (to be published)

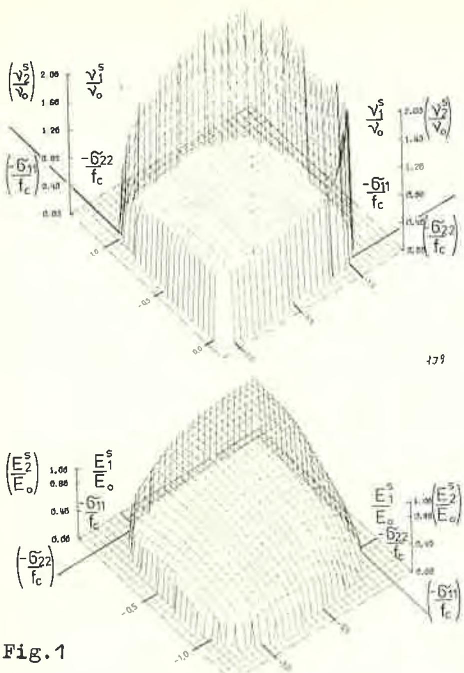

Fig.1

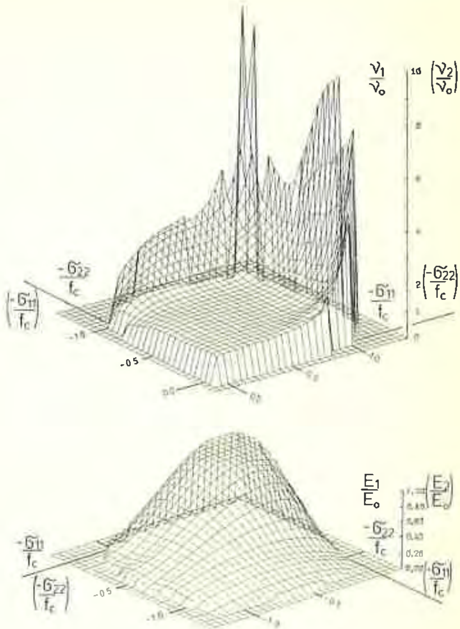

Fig.2

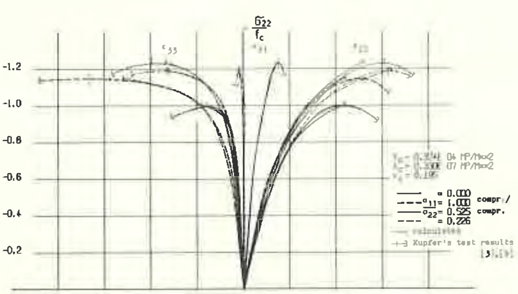

Fig.3

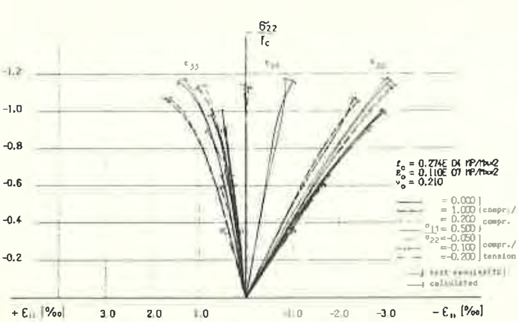

Fig.4

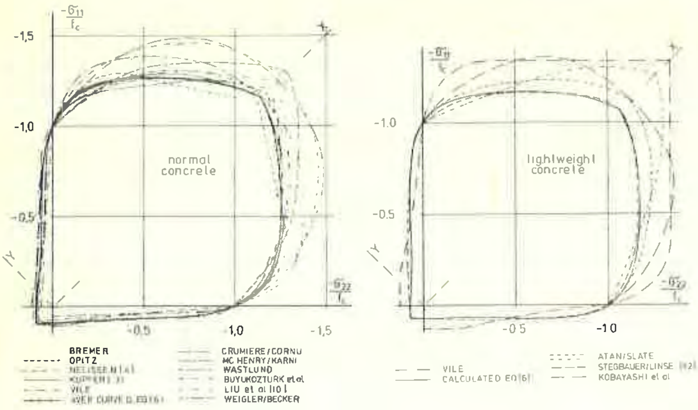

Fig.5

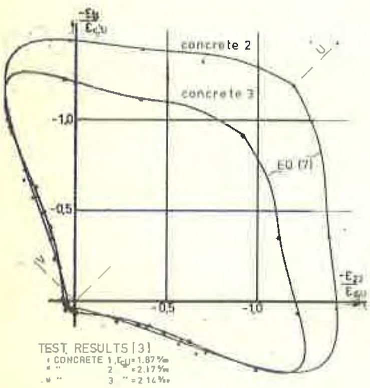

Fig.6

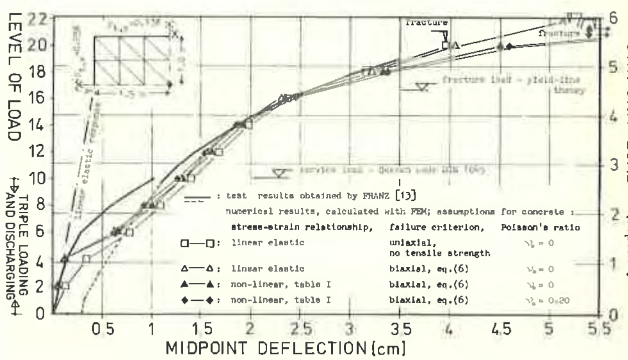

Fig.7

TABLE II

POLYNOMIAL COEFFICIENTS A_i ACCORDING TO EQ.(6)

type	f_c [kp/cm²]	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
concrete 1	192	0.1535	1.0556	-0.3081	-1.7391	6.5294	-9.1747	5.2937	-1.0879	0
concrete 2	324	0.1253	0.9309	-0.2515	0.4045	2.1693	-6.6915	5.3193	-1.3357	0
concrete 3	619	0.1196	0.8414	-1.0010	4.7454	-6.2908	1.4994	1.4218	-0.6206	0
lightweight	274	0.09915	0.9406	-0.3003	3.2897	-6.7550	3.5832	0.1611	-0.3952	0
average curve I		0.1231	0.8335	0.2191	2.0055	-7.1955	7.9627	-4.0104	0.7742	0
average curve II		0.1203	0.3682	-0.1910	2.8430	-5.4875	3.7567	-1.0154	0.06664	0.4335

POLYNOMIAL COEFFICIENTS B_i ACCORDING TO EQ.(7)

type	$\epsilon_{c,U}$ [°/oo]	B_0	B_1	B_2	B_3	B_4	B_5	B_6
concrete 2	2.17	0.07118	1.6077	2.2264	-1.7707	-5.0898	5.4652	-1.4778
concrete 3	2.14	0.09937	1.2405	-1.6506	27.2072	-66.968	57.244	16.5738

TABLE I

FORTRAN-function subroutines of material characteristic values (secant and tangent functions of principal stresses)

Notations:

SQ11 = σ_{11}/f_c SQ22 = σ_{22}/f_c EMO = E_o QUO = ν_o

E_2^s = FUNCTION FEM2(EMO,SQ11,SQ22)

ν_i^s = FUNCTION FQU1(QUO,SQ11,SQ22)

E_i = FUNCTION FET1(EMO,SQ11,SQ22)

ν_1 = FUNCTION FQT1(QUO,SQ11,SQ22)

I b

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0002       FUNCTION FQU1(QUO,SQ11,SQ22)
      C   * LETZTE AENDERUNG VOM.. 17.JAN. 1975
0003       DIMENSION Z1(8),Z2(8),Z3(8),Z4(8),Z5(8),Z6(8)
0004       DATA Z1 /13,25,50,57,71,73,46,57,13,105,0,954,-0,0867,0,1999/,
          1     Z2 /-131,7,-575,9,-999,5,-870,1,-387,9,-72,45,4,708,7,4/,
          2     Z3 /12,98,49,27,69,61,42,76,7,789,-2,998,-1,675,1,07/,
          3     Z4 /-4,554,-25,28,-54,95,-58,49,-31,01,-7,768,-1,526,0,3/,
          4     Z5 /-549,45,-2471,1,-4462,9,-4102,7,-1967,2,-401,8,25,55,
          5        26,/,
          6     Z6 /7,987,28,65,37,32,20,25,3,394,-0,3857,-0,1919,0,14/
0005       CALL ERRSET(207,999,1,1,1,209)
0006       POL1=0.
0007       POL2=0.
0008       POL3=0.
0009       POL4=0.
0010       POL5=0.
0011       POL6=0.
0012       DO 1 I=1,7
0013       POL1=(POL1+Z1(I))*SQ22
0014       POL2=(POL2+Z2(I))*SQ22
0015       POL3=(POL3+Z3(I))*SQ22
0016       POL4=(POL4+Z4(I))*SQ22
0017       POL5=(POL5+Z5(I))*SQ22
0018       POL6=(POL6+Z6(I))*SQ22
0019     1 CONTINUE
0020       POL1=POL1+Z1(8)
0021       POL2=POL2+Z2(8)
0022       POL3=POL3+Z3(8)
0023       POL4=POL4+Z4(8)
0024       POL5=POL5+Z5(8)
0025       POL6=POL6+Z6(8)
0026       ARG =6,283*(0,525-SQ22)
0027       ARGD=POL2*(POL3+SQ11)
0028       ARGE=POL5*(SQ11-POL6)
0029       ARGF=14,5+10,*SQ11
0030       ARGG=7,4*SQ22-1,036
0031       ARGH=100,*(SQ11-(0,32+0,25*COSH(ARGG)/SINH(ARGG)))
0032       QU1Q=1,029 *POL1+1,043 *(0,8*COSH(ARGD)/SINH(ARGD))+1,099 *POL4*
          1     (1,+COSH(ARGE)/SINH(ARGE))-49,40*(1,-SINH(ARGF)/COSH(ARGF))*
          2     (0,024+0,0608*(0,694*COS(ARG)+0,453*COS(2,*ARG)+0,201*COS
          3     (3,*ARG)+0,0826*COS(4,*ARG)))-0,0404*(1,+TANH(ARGH))
0033       FQU1=QU1Q*QUO
0034       RETURN
0035       END
```

I c

```
0002       FUNCTION FQU2(QUO,SQ11,SQ22)
```

... as FUNCTION FQU1, but SQ11 and SQ22 interchanged

I a

```
0002       FUNCTION FEM2(EMO,SQ11,SQ22)
      C   * LETZTE AENDERUNG VOM.. 17.JAN. 1975
0003       DIMENSION Z1(8)
0004       DATA Z1 /-2,4873,-20,0688,-51,3789,-59,667,-34,5181,
          1          -9,709,-1,4205,0,1/
0005       CALL ERRSET(207,999,1,1,1,209)
0006       POLS=0.
0007       DO 1 I=1,7
0008       POLS=(POLS+Z1(I))*SQ11
0009     1 CONTINUE
0010       POLS=POLS+Z1(8)+SQ22
0011       POLI=1.+POLS
0012       POLZ=0,7+SQ22
0013       ARG1=5,24*(0,7-SQ22)
0014       ARG2=13,*(1,6+0,55*SQ22+SQ11)
0015       ARG3=10,*(1,35+SQ11)
0016       ARG4=6,+5,*SQ11
0017       A1B5=-0,325 *SQ11**2+0,992 *POLI**2*(1,-2,*POLS)+1,999 *POLS**2*
          1     POLI-0,257 *POLS**3*(10,+15,*POLS+6,*POLS**2)-0,990 *SQ11*(1,
          2     +SQ11)**2*EXP(11,*SQ22-0,77)*SIN(0,77-11,*SQ22)
0018       A6B7=-0,325 *SIN(3,141*SQ11/(0,82-2,2*(0,75*SQ22+SQ22**2)))*(EXP(
          1     4,6*SQ22-0,46)*SIN(0,46-4,6*SQ22)+0,065+0,060*(1,5*COS(ARG1)+
          2     0,375*COS(2,*ARG1)+0,0937*COS(4,*ARG1)))-0,294 *(SINH(ARG2)/
          3     COSH(ARG2))*POLZ**3*(1,-2,*POLZ+POLZ**2)
0019       EM2Q=A1B5+A6B7-9,692 *(0,131+0,071*SQ22)*(1,-(SINH(ARG3)/COSH(ARG3
          1     )))+3,134 *EXP(10,*SQ22-1,)*SIN(1,-10,*SQ22)*(1,-(SINH(ARG4)/
          2     COSH(ARG4)))
0020       IF (EM2Q-1,) 11,11,10
0021    10 EM2Q=1,0
0022    11 FEM2=EM2Q*EMO
0023       RETURN
0024       END
```

I e

```
0002       FUNCTION FET2(EMO,SQ11,SQ22)
      C   * LETZTE AENDERUNG VOM.. 17.JAN. 1975
0003       DIMENSION Z1(10),Z2(10),Z3(10),Z4(10),Z5(10)
0004       DATA Z1 /-94,156,-319,97,-318,73,18,278,185,202,72,186,
          1          -8,2591,-7,7757,-0,76369,0,094584/,
          2     Z2 /14,832,45,248,37,608,-7,6154,-25,032,-16,62,
          3          -8,0318,-0,82671,0,10856,-1,0013/,
          4     Z3 /-8,0483,-32,804,-40,526,-4,7916,18,421,4,6353,
          5          -2,3142,1,2604,0,94234,-0,12004/,
          6     Z4 /62,684,210,52,207,013,-12,785,-121,46,-52,278,
          7          -0,94016,1,5977,0,047917,0,99665/,
          8     Z5 /29,305,70,401,-11,291,-114,45,9,79,161,89,108,43,
          9          16,265,-2,0046,0,11/
      C *** AEXP .. MAX. ARGUMENTE DER BETREFFENDEN FUNKTION, RECHNERABHAENGIG
0005       AEXP=174,6
0006       CALL ERRSET(207,999,1,1,1,209)
0007       X=SQ22
0008       Y=SQ11
```

... as FUNCTION FET1, but SQ11 and SQ22 interchanged as shown

If

```
      FUNCTION FDT1(QU0,SQ11,SQ22)
C     * LETZTE AENDERUNG VOM.. 17.JAN. 1975
      DIMENSION Z1(10),Z2(10),Z3(10)
      DATA Z1 /1408,6998,6602,374,12484,76,12816,256,8594,377,
     1        4269,978,1385,152,178,405,-12,9748,8,/,
     2     Z2 /188,345,-533,592,-4595,21,-8462,36,-5804,47,
     3        -632,77,807,07,222,86,-12,122,8,916/,
     4     Z3 /1236,59,6641,984,14560,541,16656,83,10488,048,
     5        3453,481,465,673,-5,7084,-1,3717,5,14/
C *** AEXP .. MAX, ARGUMENTE DER BETREFFENDEN FUNKTION, RECHNERABHAENGIG
      AEXP=174,6
      CALL ERRSET(207,999,1,1,1,209)
      X=SQ11
      Y=SQ22
      B12=0.
      B13=0.
      B21=0.
      DO 1 I=1,9
      B12=(B12+Z1(I))*Y
      B13=(B13+Z2(I))*Y
      B21=(B21+Z3(I))*Y
    1 CONTINUE
      B12=B12+Z1(10)
      B13=B13+Z2(10)
      B21=B21+Z3(10)
      B22=6,0+9,503*EXP(-0,25*(-9,80-9,533333*Y     )**2)
     1    -1,34*EXP(-0,1*(-18,0-24,0*Y     )**2)
      B72=-0,241*EXP(-0,05*(-65,0-50,0*X     )**2) - Y
      VERV11=-B12*X-B13
      VERV12=-B21+B22 * X
      VERV15=-6,283*(-0,0425*(10,*Y+10,)**2-X)
      VERV67=17,45*B72
      IF(VERV11-AEXP) 11,11,10
   10 VERV11=AEXP
   11 IF(VERV12-AEXP) 13,13,12
   12 VERV12=AEXP
   13 VERV1=1,0 + 1,23*EXP(-0,6*(-6,1-5,0*Y     )**2)
     1       +10,0*EXP(VERV11)
     2       -10,0 * EXP(VERV12)
     3       -0,8*EXP(-0,05*(-90,0*Y     )**2)*EXP(-0,1*(-18,0-20,0*X)**2)
     4     +0,78*EXP(-0,05*(-24,0-27,75*Y     )**2)*EXP(-0,05*(-30,0-
     4       30,0*X     )**2)
     5       -2,19*EXP(-0,05*(-44,0-40,0*Y     )**2)*EXP(VERV15) *
     5       SIN(-VERV15)
      VERV6=1,2*EXP(-0,05*(-180,0-150,0*X     )**2)*EXP(-0,05*(-52,0-39,0
     6       *Y     )**2) + 0,4*EXP(-0,05*(-82,5-75,0*X     )**2) * (EXP(-
     6       0,05*(-36,0-47,4*Y     )**2)+EXP(-0,05*(-44,0-47,4*Y   )**2))
     7     +35,3*EXP(-0,05*(-16,68-53,8*Y     )**2)*EXP(-0,05*(-20,0-
     7       10,0*X     )**2)-11,1*EXP(-VERV67)*SIN(VERV67)*
     7       EXP(-0,05*(-29,555-25,7*X     )**2)
      VERV8=0,38*EXP(-0,05*(-19,5-15,0*X)**2)*EXP(-64,8*((-0,675-Y)/
     8       (1,5*X))**2) + EXP(-0,05*(-112,5 - 90,0*X)**2)*(-1,1*EXP(-
     8       0,05*(-99,0-120,0*Y)**2)+EXP(-0,05*(-117,0-120,0*Y)**2)-0,15
     8       *EXP(-0,05*(-60,0-200,0*Y)**2)+0,20*EXP(-0,05*(-57,5-145,5*Y
     8       )**2) )
     8     +0,39*EXP(-0,05*(-41,6-40,0*X)**2)*EXP(-0,05*(-208,0-200,0*Y)
     8       **2) - 0,4*EXP(-0,05*(-63,0-120,0*Y)**2)*EXP(-0,1*(-25,0-20,
     8       0*X)**2)
      VERV9=EXP(-0,05*(-129,375-112,5*X)**2)* (0,35*EXP(-0,05*(-84,375-
     9     112,5*Y)**2)+0,40*EXP(-0,05*(-93,375-112,5*Y)**2)+0,20*EXP(-
     9       0,05*(- 103,5-112,5*Y)**2) ) + 0,30*EXP(-0,05*(-181,5-150,0*
     9       X)**2) * EXP(-0,05*(-84,375-112,5*Y)**2)
     1       +0,40*EXP(-0,1*(-37,0-40,0*X)**2) * EXP(-0,05*( 36,999+
     1       49,999*X-300,0*Y)**2)
      QT1Q=VERV1+VERV6+VERV8+VERV9
      FQT1=QT1Q * QU0
      RETURN
      END
```

Id

```
      FUNCTION FET1(EMO,SQ11,SQ22)
C     * LETZTE AENDERUNG VOM.. 17.JAN. 1975
      DIMENSION Z1(10),Z2(10),Z3(10),Z4(10),Z5(10)
      DATA Z1 /-94,156,-319,97,-318,73,18,278,185,202,72,186,
     1        -8,2591,-7,7757,-0,76369,0,094584/,
     2     Z2 /14,832,45,248,37,606,-7,6154,-25,032,-16,82,
     3        -8,0318,-0,82671,0,10856,-1,0013/,
     4     Z3 /-8,0483,-32,804,-40,526,-4,7916,18,421,4,6353,
     5        -2,3142,1,2604,0,94234,-0,12004/,
     6     Z4 /62,684,210,52,207,013,-12,785,-121,46,-52,278,
     7        -0,94016,1,5977,0,047917,0,99665/,
     8     Z5 /29,305,70,401,-11,291,-114,45,9,79,161,89,108,43,
     9        16,265,-2,0046,0,11/
C *** AEXP .. MAX, ARGUMENTE DER BETREFFENDEN FUNKTION, RECHNERABHAENGIG
      AEXP=174,6
      CALL ERRSET(207,999,1,1,1,209)
      X=SQ11
      Y=SQ22
      A11=0.
      A12=0.
      A13=0.
      A14=0.
      A22=0.
      DO 1 I=1,9
      A11=(A11+Z1(I))*Y
      A12=(A12+Z2(I))*Y
      A13=(A13+Z3(I))*Y
      A14=(A14+Z4(I))*Y
      A22=(A22+Z5(I))*Y
    1 CONTINUE
      A11=A11+Z1(10)
      A12=A12+Z2(10)
      A13=A13+Z3(10)
      A14=A14+Z4(10)
      A22=A22+Z5(10)
     3       +A31*(-1,500+ATAN(3,7*A32-37,0*X     )+0,009 *(2,0+A32+10,0*X))
      A21=0,055+0,04*Y     -0,5*(Y     **3)*(1,0-2,0*Y     +Y     **2)
      POL=5,*3,1415926*(A21-X)
      A31=1,16*EXP(-0,5*(-4,17-7,29*Y     )**2)
      A32= 5,406-ATAN(-8,0-32,0*Y     )-ARCOS((-28,0-32,0*Y     )/SQRT(1,0
     1     +(-28,0-32,0*Y     )**2))
      VERM13=3,7*A32-37,*X
      VERM71=-390,-400,*X
      IF(ABS(VERM13)-AEXP) 31,31,30
   30 VERM13=AEXP
   31 IF(ABS(VERM71)-AEXP) 33,33,32
   32 VERM71=AEXP
   33 CONTINUE
      VERM1=A11+A14*(1,-3,*(A12*X       +A13)**2+2,*(A12*X       +A13)**3)
     2     +A22*EXP(-POL)*SIN(POL)
     3      +A31*(-1,500+ATAN(VERM13          )+0,009 *(2,0+A32+10,0*X))
     4     -0,14*EXP(-0,1*(1,383-27,66*X     )**2)*EXP(-0,5*(-9,0-12,0*
     4     Y     )**2)
     5     -1,57*EXP(-0,5*(-40,0-30,0*Y     )**2)
     6     -2,4*EXP(-0,25*(1,8-20,0*X     )**2)*EXP(-(7,5-50,0*Y     ) )
     6     +1,76*EXP(-0,25*(5,0-70,0*X     )**2)* ( 0,13*EXP(-0,1*(-6,0
     6     -57,0*Y     )**2)-0,23*EXP(-0,1*(-13,05-38,4*Y     )**2)+0,16*
     6     EXP(-0,1*(-23,1-36,26*Y     )**2) )
      VERM7=EXP(-0,1*(-4,5-60,0*Y     )**2)*(-0,0325-0,00206*ATAN(
     7     VERM71     )) + 0,172*EXP(-0,1*(-15,0-10,0*X     )**2) *
     7     EXP(-0,05*(-13,33-26,66*Y     )**2)
     8     +1,76*EXP(-0,25*(5,0-70,0*X     )**2)* (0,13*EXP(-0,1*(-17,0-
     8     100,0*Y     )**2)+0,11*EXP(-0,1*(-32,4-60,0*Y     )**2) )
      ET1Q=VERM1+VERM7
      IF(ET1Q - 1,0) 20,20,10
   10 ET1Q=1,0
   20 FET1=ET1Q * EMO
      RETURN
      END
```

H1/2