THE ANALYSIS OF CRACKED STRUCTURES

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SUMMARY

1. The methods to be described were derived in order that PCRVs could be analysed when they were loaded up to failure, and they have been justified by reference to experiments on model PCRVs. The methods have a very wide application, in particular to reinforced concrete structures, which must crack before they can function. Cases of dynamic loading e.g. explosion, earthquakes, can also be analysed accurately.

2. The only acceptable way of representing a crack is by introducing a pair of new boundaries to the structure, which form the sides of the crack. This consideration makes it impossible to use the finite element method. Finite difference equations are therefore used, solved by dynamic relaxation. New boundaries can be introduced with no trouble at all, and conditions on them can be accurately controlled. A very great advantage is that the method is exceedingly simple, and all the equations depend only on first principles and elementary logic. Programming for the computer is also elementary. Experience has shown that any engineering graduate can easily write his own programme to suit the particular needs of the work in hand.

3. The well-known method of dynamic relaxation has been simplified and systematised, and code numbers have been introduced to describe the boundary conditions of the various blocks. The width of all cracks is calculated directly, and their compatible length is determined by inspection of the crack opening displacement, i.e. the apparent angle at the tip of the crack. It has been found by numerous examples that this method is perfectly reliable, and the computer can establish the length of each crack automatically. The directions of the cracks can be checked in a very simple manner.

4. In the case of a reinforced concrete structure, the ability to calculate the widths of the cracks allows the determination of the forces in the bars. These can then be applied directly, allowing an accurate analysis of such a structure, which is not generally available. Only two or three extra statements are required for this case. Similarly other additional relations can readily be included, e.g. the effect of shear acting across a crack, the extension and displacement of prestressing tendons, etc.

5. Three programmes have been written. The first deals with 2-dimensional structures, the second with 3-dimensions and axial symmetry, and the third is a full 3-dimensional analysis in cylindrical coordinates. Results of the third programme when applied to a multi-cavity PCRV for an HTR showed good agreement with model tests. Six radial sections were used with 252 blocks on each section: three direct stresses, three shear stresses and three deflections were calculated in each block, together with all the crack widths. The computer running time was about 20 minutes, and the effects of gas pressure in cracks, bonded reinforcement and tendon extension were included.

6. A simple, efficient and accurate method has been described for the comprehensive analysis of any cracked, or uncracked, structure in which the various boundary conditions can be reproduced correctly, together with any other contingent relations. Dynamic loading can be used, and temperature distributions and stresses can be calculated.

1. Introduction.

1.1. The particular purpose of this paper is to describe exceedingly simple but powerful methods for producing a comprehensive analysis of the stresses and strains in a three-dimensional structure, described in cylindrical coordinates, which may contain cracks.

1.2. The original purpose for deriving these methods was in the case of Prestressed Concrete Reactor Vessels, where it was necessary to continue the analysis beyond the linear elastic limit, when model vessels were tested to failure. Modern design codes for PCRVs, which demand investigation of loading cases which cannot be modelled, such as gas pressure in cracks, assumed loss of material strength, etc. make such an analysis a vital design tool.

1.3. But it must not be forgotten that cracked structures occur very commonly. For instance, no reinforced concrete structure can function until it has cracked, because it is the stretching of the reinforcing bars at the cracks which produces the forces in them, and it is these forces which enable the structure to carry its loads. The methods to be described permit a comprehensive analysis of reinforced concrete structures in cases where the normal approximate methods are not considered adequate.

1.4. The basic problem is to represent the cracks properly in the analysis, and the only true representation is to introduce a pair of new boundaries into the structure for each crack. This requirement becomes prohibitively expensive if a finite element analysis is used, because the stiffness matrix changes every time any crack extends. Therefore the analysis which has been chosen uses finite difference equations solved by dynamic relaxation. This method is so exceedingly simple, effective and accurate that in the author's opinion there is no longer any justification for using approximate methods for analysing cracked structures.

1.5. The author attaches considerable importance to the quality of simplicity. The analysis uses only the fundamental laws of elasticity, Newton's Law, and simple logic; in order to apply the analysis only a very elementary understanding of computer programming is required. The intention is to enable any design engineer to write his own programme, which he understands thoroughly. It is a very simple matter to rearrange the programme to suit any particular requirements, and it may be used with great advantage as a tool by which the design of a structure may be progressively refined. This policy has been tried out with post-graduate students and found to function well.

2. Basic principles.

2.1. The general method of dynamic relaxation has been fully explained on many occasions (references 1,2,3) and its application to cracked structures was the subject of reference 4. However the method is far less widely known than it merits, and a very brief resume is now given for completeness. The structure to be analysed is covered by a regular rectangular grid to give an array of individual blocks; in the case of the full three dimensional analysis the above arrays occur on as many radial sections as are required. The angles between the radial sections may vary in order to give the best representation of the actual structure, but a regular mesh has been chosen on each radial plane for simplicity. It is easier to provide the required detail by reducing the size and increasing the number of blocks.

2.2. A typical block in the array is represented in figure 2.1.

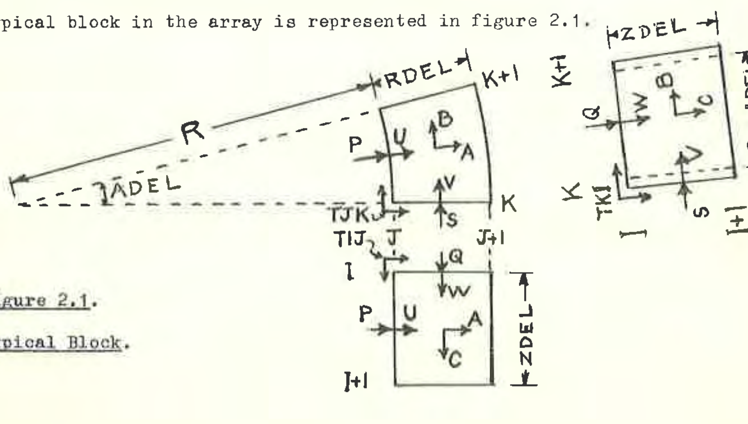


Figure 2.1.

Typical Block.

It will be seen that each block is identified by the co-ordinates (I,J,K) of the top left hand corner: because this is a FORTRAN array the co-ordinates increase in the directions shown in the figure. A list of the variable names is given in appendix 1, from which which it will be seen that A, B, and C are the three direct stresses, assumed constant across each block, and U, V and W are the velocities at the left hand and top sides of the block. DU, DV and DW (not shown) are the deflections at the same points as the velocities. The three shears are TIJ, TJK and TKI, measured at the top left hand corner of the block, and these are assumed constant between the mid-points of the sides of adjacent blocks. The positive direction of all variables is shown in figure 2.1.

3. Dynamic relaxation.

3.1. The finite difference equations relating the direct stresses in each block to the deflections in it, and in the adjacent blocks, are a simple matter of the basic laws of elasticity, and similarly for the shear stress - deflection equations. Thus an array of such equations can very easily be set up to cover the structure to be analysed. In order solve this system of simultaneous equations a method of successive approximation is used, which is called Dynamic Relaxation. This method has a very simple physical analogy which can be described as follows. Assume the unloaded structure to be without stresses or deflections. Now apply the loads suddenly and examine every block in turn. It is a simple matter of using Newton's Law to establish the acceleration of every block to which a load is applied. Some short time interval is now chosen so that the deflection of each accelerating block at the end of the time interval can readily be found. The whole array of deflected blocks is now re-considered and the stresses produced by the deflections are calculated in every block, using the finite difference equations already mentioned. This cycle of calculations will be called an iteration.

3.2. Now a new value of acceration is calculated for every block , which is now subjected to the external loads plus the stresses calculated at the last iteration. From these new accelerations, new deflections are calculated, and hence a new set of stresses. This process of alternately calculating deflections and stresses is continued until the vibrating structure comes to rest and convergence is achieved. At every iteration all the external forces are in equilibrium with the internal stresses plus the inertia loads, and at convergence the inertia loads have become zero. Also at every iteration the deflections and stresses are compatible, so that at the end of the dynamic analogy the correct solution to the static problem has been achieved. In order to hasten the process of convergence a viscous damping is applied at each iteration.

3.3. It is convenient to do the calculations in terms of velocities, deflections and stresses, so that only first order finite difference equations are required, and the basic equations for the typical block which is illustrated in figure 2.1. will be found to be as follows (the very simple derivation of these equations is given in reference 4).

$$A_a=A_b+G6\left[\frac{U-U(J+1)}{RDEL}\right]-\frac{G4}{R+RDEL/2}\left[\frac{U+U(J+1)}{2}-\frac{V-V(K+1)}{ADEL}\right]+G4\left[\frac{W-W(I+1)}{ZDEL}\right] \quad (3.1)$$

$$B_a=B_b+G4\left[\text{ ditto }\right]-\frac{G6}{R+RDEL/2}\left[\text{ditto}\right]+G4\left[\text{ditto}\right] \quad (3.2)$$

$$C_a=C_b+G4\left[\text{ ditto }\right]-\frac{G4}{R+RDEL/2}\left[\text{ ditto }\right]+G6\left[\text{ ditto }\right] \quad (3.3)$$

$$TIJ_a=TIJ_b+G5\left[\frac{U(I-1)-U}{ZDEL}+\frac{W(J-1)-W}{RDEL}\right] \quad (3.4)$$

$$TJK_a=TJK_b+G5\left[\frac{V(J-1)-V}{RDEL}-\frac{V(J-1)+V}{2*R}+\frac{U(K-1)-U}{R*(ADEL+ADEL(K-1))/2}\right] \quad (3.5)$$

$$TKI_a=TKI_b+G5\left[\frac{V(I-1)-V}{ZDEL}+\frac{W(K-1)-W}{(R+RDEL/2)*(ADEL+ADEL(K-1))/2}\right] \quad (3.6)$$

$$U_a=U_b*G1+G3\left[\frac{P+A(J-1)-A}{RDEL}-\frac{A+A(J-1)-B-B(J-1)}{2*R}+\frac{TIJ-TIJ(I+1)}{ZDEL}+\frac{TJK-TJK(K+1)}{R*ADEL}\right] \quad (3.7)$$

$$V_a=V_b*G1+G3\left[\frac{S+B(K-1)-B}{(R+RDEL/2)*(ADEL+ADEL(K-1))/2}+\frac{TJK-TJK(J+1)}{RDEL}-\frac{TJK+TJK(J+1)}{2*(R+RDEL/2)}+\frac{TKI-TKI(I+1)}{ZDEL}\right] \quad (3.8)$$

$$W_a=W_b*G1 + G3\left[\frac{Q+C(I-1)-C}{ZDEL} + \frac{TIJ-TIJ(J+1)}{RDEL} - \frac{TIJ+TIJ(J+1)}{2*(R+RDEL/2)} + \frac{TKI-TKI(K+1)}{(R+RDEL/2)*ADEL}\right] \quad (3.9)$$

3.4. In the above equations the suffixes b and a refer to the values of the variable before and after the iteration. The values of the constants G1, G3, etc will be found in appendix 1, together with the meaning of all the variable names. It will be noticed that variables P, Q and S have been introduced and these represent external pressures which may be applied to any block, as shown in figure 2.1. It is only necessary to solve the velocity equations for each block, and then the direct and shear stress equations, continuing alternately until the velocities become small. The deflections are calculated at each iteration from the equations $DU_a=DU_b+U*TDEL$, etc. A simple visualisation of the process of convergence will be seen in figure 9.1, which shows the velocities at two selected points plotted horizontally, against time, plotted vertically.

4. <u>Boundaries</u>.

4.1. The equations given above are correct for the normal block, but they have to be changed if one or more sides of the block form an external boundary, or form the sides of a crack. On an orthogonal boundary one or more shears will generally be zero, and it is very simple to make them so. Also the velocity across an external boundary must be calculated specially, if there is no shear on the boundary, and also because the velocity block is only a half-block. In these cases the very reasonable approximation is made that the shear varies uniformly from the established value at the mid-point of the block side, to zero at the boundary.

4.2. The only other special case occurs at a re-entrant corner, where there must in fact be some value of shear, which becomes zero along the edges forming the corner. The calculation of the shear rotation is made in the normal manner, but the modulus of rigidity of a block,, from which a quadrant has been removed, is reduced to a value of 0.3 times the normal value. This figure represents an average of a series of values which depend upon the loading distribution. The fact that this is an approximation only affects the stresses in this block, and has a negligible effect on the behaviour of the complete structure.

4.3. The appropriate equations for a special block which has boundaries are given in appendix 2, and the application of the principles stated in the two previous paragraphs will be obvious. It will also be noted that the sides of cracks nearest to the block are treated as those belonging to the block, and the velocities and deflections are referred to as U,W, DU and DW. But the opposite sides of the cracks are specially calculated, and are referred to as UL, WT, DUL and DWT. Obviously the width of each crack is DU - DUL and DW - DWT.

4.4. In a three dimensional structure as complex as a multi-cavity PCRV there are a very large number of possible boundary conditions, and the one which is given in appendix 2 has been selected from over one hundred special blocks which are now in use. The need for special blocks is recnised from time to time, as various different structures come to be analysed, and the appropriate equations can be written down by simple logic, as already shown, and added to the programme. So the programme gradually grows in use, and all that is required is to keep track of the special blocks, and to describe them to the computer. This is very easily done by allocating a code number to each type of block, the normal block shown in figure 2.1 being given the code number 1. Thus the array KODE, which is fed into the computer at the beginning of each run, completely describes the structure to be analysed, and the apprpriate equations will be selected by COMPUTED GO TOs, controlled by KODE. Such an array is illustrated in figure 13.1, which shows a radial section of a multi-cavity PCRV which will be discussed later. It will be noted that code 10 is a zero block.

4.5. The equations for the nine parameters of the special block given in appendix 3 have been written out in full, but it will be appreciated that many of these in fact recur for many other special blocks. In fact only 115 equations suffice, in various combinations, to calculate the nine parameters in over 100 special blocks.

5. <u>Crack compatibility</u>.

5.1. The methods by which a crack may be correctly represented and its width calculated have already been described, but it is necessary to determine the correct length and direction of each crack. Of course the presence of each major crack has a profound effect upon the functioning of the structure. The loads must be applied in increments, as they would in real life, and at each stage the compatible crack system must be established. This forms an essential part of the input for the next succeeding load increment. As regards the compatible length of the crack, it is of no use to examine the calculated stresses ahead of the crack tip. In the presence of the singularity the stress is theoretically infinite, so that the values calculated become inversely proportional to the size of the blocks. Moreover the material properties at the crack tip are no longer linearly elastic. It has been found quite safe to rely on crack opening displacement as a criterion. This is a concept of fracture mechanics, and is measured as an angular rotation at the tip. In our case, because the crack tip is always assumed to be located at the corner of a block, and because these have constant dimensions, it is only necessary to examine the apparent width of the crack half a block from the tip. This is called apparent width because it is calculated on the basis of linear elastic properties, which do not in fact obtain at the crack tip.

5.2. Resulting from various tests on models built with high-strength concrete and 3/8" aggregate, it has been found that if the apparent angle subtended by the crack tip is more than 1 milli-radian then the crack is a real one, and if the apparent angle is more than 3 milli-radians, the crack should be allowed to grow one block (reference 5). After a sufficient number of iterations to allow the crack widths to stabilise to a reasonable extent, the computer examines the crack width in the block where the tip is located. If the apparent angle exceeds 3 milli-radians the code numbers are changed in such a way as to move the crack tip forward one block. After a further number of iterations the new crack width is examined, and the tip extended another block if appropriate. In this way cracks will be extended automatically until the compatible length is achieved, without errors and without increasing the computer running time.

5.3. In the case of radial cracks it will sometimes be found that the tensile hoop stress patterns are sensibly uniform, in which case it may be more appropriate to control crack onset simply on the basis of tensile stress in the concrete. The user can easily change the programme if necessary. In the matter of crack direction, there is no problem with the radial cracks, and in the case of zig-zag cracks in horizontal and circumferential planes, a very simple check can be applied. Suppose that such a crack is as illustrated in figure 5.1, then if the crack direction is correct:-

$0.9 * \frac{\sum \delta h}{\sum \delta v} = \frac{V}{H}$ where the summation is over the whole length of the crack. If this relationship is not approximately correct, the direction of crack growth should be adjusted as appropriate, before the next load increment is applied.

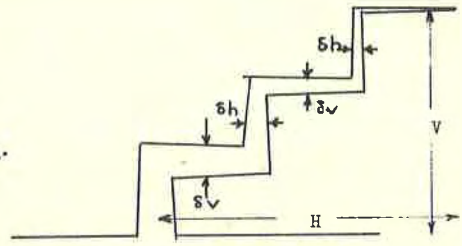


Figure 5.1.

Zig-zag Crack.

6. Reinforced concrete.

6.1. Consider a reinforced concrete beam with a variable moment, then loads will be induced in the reinforcing bars as they are stretched across the cracks which willform in the tensile zone of the beam. There will be a general pattern of bar loads appropriate to the varying bending moment along the beam, and superimposed on that there will be a finer pattern of load variation in response to unknown spacing and width of the actual cracks. The finer pattern of bar loads can be neglected as it is merely a process of averaging produced by a multiplicity of cracks. In the calculations it is much easier to average arithmetically. All that is necessary is to use the already described methods of crack analysis to calculate the width of cracks which are inserted at regular intervals in the tensile zone. These inserted cracks will have a calculated width equal to the sum of the widths of the real cracks, and this must be so because all the conditions of equilibrium and compatibility are being strictly maintained. In practice these inserted cracks can occur at intervals of two blocks, because different code numbers are required on the left and right of each crack.

6.2. The process of calculating the average load in the bar at each inserted crack merely involves dividing the calculated crack width by two block lengths, and multiplying by the elastic modulus of the bar, and by the area of the bar. These bar loads will be calculated at each iteration and will then be applied as forces tending to close each crack, exactly simulating real life.

6.3. The final converged result will show correct equilibrium of bar loads plus stresses with external loads, correct compatibility of crack widths with bar loads, and correct compatibility of deflections with stresses. It is the difficulty of this three-fold relationship which leads to the normal approximate methods of calculating reinforced concrete structures, and its success provides a demonstration of the power of this simple method of analysis, in which any number of contingent relationships, local or general, can be added to the basic structural relationships and a common solution found in which all will be satisfied.

7. <u>Tendon loads</u>.

7.1. Tendon loads provide a further illustration of supplementary calculations. As a tendon is extended by the deflections of the structure the load in it will be increased, and if the line of a tendon is deflected a lateral force will be produced. These factors can be included in a very simple manner; for instance the changing force in each tendon as it is extended can be calculated at regular intervals during the convergence, in relation to the stress-strain curve for the wire as modified by friction. The computer will also simulate the failure of any tendon if it reaches its ultimate extension, by setting the load in it to zero.

8. <u>Relaxation</u>.

8.1. The value of the time interval can be calculated from the formula :-

$$TDEL = \sqrt{\frac{RHC}{(L+2G)} \Big/ \left[\frac{1}{RDEL^2} + \frac{1}{ZDEL^2} + \frac{1}{((R+RDEL/2)*ADEL)^2}\right]} \qquad (8.1)$$

where L and G are the Lamé constants. But a difficulty will be observed as the variable R (radius of the block under consideration) appears in the expression, so that TDEL must be chosen to suit the smallest value of R, which is zero at the polar axis. TDEL will then be unnecessarily small for blocks at a larger radius. A simple solution is to use a fictitious density in the circumferential direction, where the fictitious density varies inversely as $((R+RDEL/2)*ADEL)^2$.

8.2. This means that the constant G3 to be used when calculating V is to be multiplied by $((R+RDEL/2)*ADEL)^2$. If this is done the expression for TDEL becomes

$$TDEL = \sqrt{\frac{RHC}{(L+2G)} \Big/ \left[\frac{1}{RDEL^2} + \frac{1}{ZDEL^2} + 1\right]} \qquad (8.2)$$

The use of fictitious densities does not in any way affect the final values calculated; it merely changes the size of the steps of the convergence. It is sometimes recommended that fictitious densities should be used throughout the arrays to produce a time interval of unity in every block. The author does not find that anything is to be gained by such a procedure, while if the programme is to be used for dynamic calculations in the manner to be described later, no fictitious densities are permissible.

8.3. The formula quoted for TDEL may lead to instability in structures which are considerably cracked or in which there are several cavities, because parasitic oscillations may build up in portions of the structure which are loosely coupled. In such cases it is advisable to reduce the time interval by about 30%, and to increase it gradually in later runs if all is well, as discussed later.

8.4. The second relaxation constant is the value of viscous damping, which should be about 80% of critical. In a simple structure the critical damping can be derived directly from the fundamental circular frequency, but in extensively cracked and multi-cavity structures there is no one fundamental frequency. The best advice that can be given is to start with DAMP = 0.03, and to adjust this in subsequent runs, as described later.

8.5. The final relaxation constant is the number of iterations required for convergence and this will be found to be 12/DAMP in the simple case. The following methods are available in order to confirm that the relaxation constants have been well chosen. If TDEL is too great, instability will occur during the convergence and the computer will stop with a record of the number of iterations achieved, and the location of the instability. The code numbers at that location should be checked, and if all is in order the programme should be re-run with TDEL reduced by 20%. The choice of DAMP and number of iterations are verified by a plot of selected velocities, during the convergence.

9. Velocity plot.

9.1. Two blocks remote from fixed points are chosen, where the velocities are likely to be maxima. The horizontal velocity of the first block, and the vertical velocity of the second, are plotted by the printer at regular intervals during the convergence. If the plot shows dead-beat convergence, and the final velocities are small, as in figure 9.1, then satisfactory convergence has been reached with economy of computer time. Too large a value of DAMP will give a slow asymptotic approach, and too small a value is indicated by a continuing series of oscillations.

9.2. As a final check on the convergence, all the remaining velocities are printed out. If a concrete structure is being analysed in inch, pound, second units, and if the largest remaining velocities do not much exceed 0.5 ins/sec, then a satisfactory convergence has been achieved with stress errors generally less than 5%.

9.3. The print out of final velocities is useful in detecting instability, which may be localised, as already mentioned. The appearance of alternate signs, or alternate large and small quantities, in consecutive rows or columns is a sensitive indicator of incipient instability. A particularly virulent form of instability consists of shear waves, in horizontal or tangential planes, running round the polar axis, and adjacent to it. However these are very small in this area, and an effective cure for the instability is to keep them at zero. This can be done very easily by setting the limits of the DO loop for these shears to J = 3 , JM.

10. Tape.

10.1. As already mentioned, the calculations are normally started from arrays of zero stresses and deflections, but any compatible set would do. When incremental loads are being applied it will obviously save considerable computer time to start from the converged values of the previous run. The arrays DU,DUL,DV,DW,DWT,A,B,C,TIJ,TJK, TKI are therefore recorded on tape at the conclusion of each run. Also the arrays P,Q,S,KODE are recorded, so that any changes in these, produced by crack extension, etc. are correctly fed into the next run.

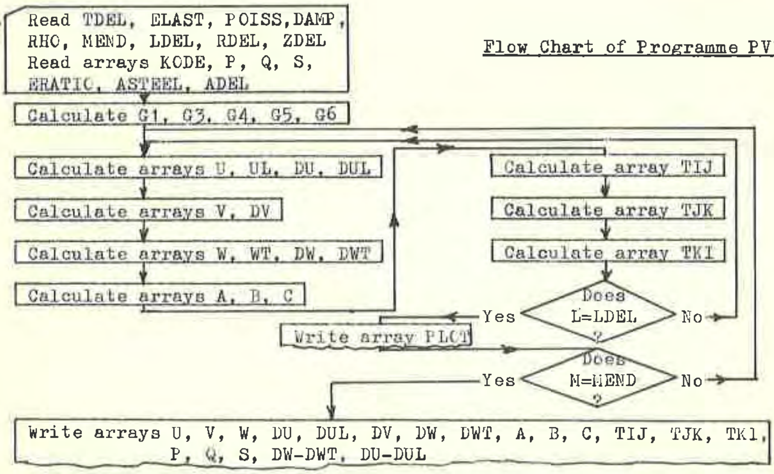
10.2. The use of the tape also means that the advice given in paragraph 8.4 can readily be implemented without loss of computer time. The velocity plot from the first run will show quite plainly what changes are desirable if convergence has not been achieved, and a few iterations with improved constants, starting from the previous result, will produce successful convergence.

11. Flow chart.

11.1. The programme is an exceedingly simple one, and only involves about 700 statements, although over 100 special blocks are included together with the calculation of crack widths, tendon loads, velocity plot, etc. as already mentioned. The computer processing time for a structure containing 1,600 blocks with numerous cracks and reinforcing bars is about 20 minutes.

11.2.

Flow Chart of Programme PV3.



12. Dynamic loads.

12.1. It will be obvious from the physical analogy of dynamic relaxation that the programme can be used, with trivial modifications, to follow the behaviour of a structure, in great detail, under any dynamic loading system. Two points must be watched: the damping must be chosen as appropriate to the structure under the conditions of the analysis, and no fictitious densities can be employed. The necessary modifications to apply the changing loads, after appropriate numbers of iterations, are very simple. Reference 6 describes a prestressed partly- reinforced concrete beam, loaded to an extent sufficient to promote major cracks, and subjected to simple harmonic vertical accelerations at the supports. The applied loads were treated as dead loads, i.e. the forces they imposed were dependent on the accelerations to which they were subjected. A further detailed analysis has been made of the results of an explosion occuring in a prestressed concrete pressure vessel.

13. Test results.

13.1. In order to prove the programme PV3, it was used to calculate the stresses, deformations and crack widths in a 1/20 scale model of a multi-cavity PCRV for a High Temperature Reactor, subjected to increasing hydraulic pressure almost to the point of collapse. The model was designed, constucted and tested by the General Atomic Company of San Diego, California in 1969 (reference 7) to demonstrate the operation of a slip plane between barrel and head, and the capacity of the barrel and head. The slip plane concept is not representative of actual vessel configurations. The structure had six equal boiler cavities, with cross-ducts connecting to the central cavity. As the vessel was also symmetrical about its mid-height, the analysis was performed on six radial sections, each comprising a quadrant, as shown in figure 13.1.

13.2. The code numbers which were required to represent the vessel on the radial plane K=1 are shown in figure 13.1, on which the calculated width of the radial cracks at an internal pressure of 1400 psig has also been written. The theoretical width of the crack on the slip plane has been entered on the sector above, and the width of the horizontal crack at the equator is shown at the bottom of the quadrant. The blocks in which the calculations showed the hoop rebar to be stressed to the yield point are also marked, and it will be noted that this has not occured in the blocks nearest to the outer surface. The reason is that according to the analysis the circumferential prestress wire wrapping, acting through friction was sufficient to limit the width of the radial crack at the outer boundary.

13.3. The data shown on figure 13.1 cannot be confirmed directly from the experimental results, although there is general agreement with the crack pattern which was observed. But good indirect confirmation can be obtained from the deflections of the vessel plotted in figure 13.2. Because elastic strains contribute only a little to the total deflections after cracking has started, the agreement which is shown between theoretical and experimental deflections also implies agreement of crack extent and crack widths.

13.4. The radial deflection at the equator which is plotted in figure 13.2 shows considerable differences between the first three tests, and this is thought to be due to unintended adhesion across the slip plane, which became freed at the higher test pressures. In the calculations it was assumed that the slip plane functioned perfectly.

13.5. The results shown in figures 13.1 and 13.2 represent of course a very small part of some 17,000 stresses and deflections which were printed out for each coolant pressure.

14. Conclusion.

14.1. Limitations of space have prevented as full an exposition as the author would have wished, but it is hoped that enough has been said to show that the methods described can provide an accurate, simple and cheap analysis of complex three-dimensional structures, including cracks and other dependent phenomena.

14.2. The author acknowledges with thanks the approval of the Superintendent, Instituto de Energia Atomica, Sao Paulo, Brazil, for the publication of this paper.

References.

1. Otter, JRM. Dynamic Relaxation. Proc. Inst. civ. Engrs. 1966, 35 (Dec) 633-656.

2. Holland, JA. Dynamic Relaxation applied to local effects. Conf. PCPVs. Inst. civ. Engrs. 1968,587-595.

3. Cassel, AC. Sells of revolution. Proc. Inst. civ. Engrs, 1970, 45 (Jan) 65-78.

4. Davidson, I. Analysis of cracked structures. Publ. 337, IEA, CX Postal 11049, Sao Paulo. Brazil.

5. Davidson, I. Modes of behaviour of model PCPVs when pressurised to failure. Paper H 5/1, Nucl Engg. and Design. 1972, 20.

6. Davidson, I. Response of structures to seismic excitation. Journ. Br. Nucl. Engg. Soc. 1973, 12 (Apr) 257-260.

7. Davis,LL and Koerner,RJ. Testing of a podded PCRV model. Proc. Petrol, Mech. Eng, and Pressure Vessels Conf. Denver, Colorado. 1970.

List of variable names and constants. Appendix 1.

A	radial stress
ADEL	angle between radial planes
ASTEEL	area of bonded steel
B	circumferential stress
C	vertical stress
DAMP	viscous damping
DU	radial deflection
DUL	radial deflection left of crack
DV	circumferential deflection
DW	vertical deflection
DWT	vertical deflection top of crack
ELAST	modulus of elasticity
ERATIO	ratio of modulus of elasticity
I,J,K	cylindrical coordinates
IM,JM,KM	maximum values of I,J,K
KODE	describes boundary conditions of block
P	external load in radial direction
POISS	Poisson's ratio
Q	external load in vertical direction
R	radius of block =(J-1)*RDEL
RDEL	radial dimension of block
RHO	mass density
S	external load in circumferential direction
TDEL	time interval
TIJ	shear stress in radial plane
TJK	shear stress in horizontal plane
TKI	shear stress in tangential plane
U,V,W	velocities in radial, tangential and vertical directions
UL,WT	velocities left of crack and top of crack
ZDEL	vertical dimension of blocks

G1 = (1-0.5*DAMP)/(1+0.5*DAMP)

G3 = TDEL/(RHO*(1+0.5*DAMP))

G4 = (ELAST*TDEL*POISS)/((1+PCISS)*(1-2*PCISS))

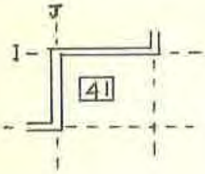
G5 = (ELAST*TDEL)/(2*(1+PCISS))

G6 = (ELAST*TDEL)*(1-PCISS)/((1+PCISS)*(1-2*PCISS))

N.B. Compressive stresses are positive.

Any consistent system of weights and measures can be used.

Appendix 2.



Code Number 41.

Under dagonal crack in horizontal and tangential planes.

A = normal

B = normal

C = normal

$$TIJ_a = TIJ_b + 0.3*G5 \left[\frac{U(I-1) - UL}{ZDEL} + \frac{W(J-1) - WT}{RDEL}\right]$$

TJK = 0

TKI = 0

$$U_a = U_b*G1 + 2*G3 \left[\frac{P-A}{RDEL} - \frac{A-B}{2*R} - \frac{TIJ(I+1)}{2*ZDEL} + \frac{TJK(J+1)-TJK(J+1,K+1)}{4*R*ADEL}\right]$$

$$UL_a = UL_b*G1 + 2*G3 \left[\frac{A(J-1) - P}{RDEL} - \frac{A(J-1) - B(J-1)}{2*R} + \frac{TIJ}{2*ZDEL} + \frac{TJK(J-1) - TJK(J-1,K+1)}{4*R*ADEL}\right]$$

V_a = normal

$$W_a = G1*W_b + 2*G3 \left[\frac{Q-C}{ZDEL} - \frac{TIJ(J+1)*(R+RDEL)}{2*RDEL*(R+RDEL/2)} + \frac{TKI(I+1) - TKI(I+1,K+1)}{4*(R+RDEL/2)*ADEL}\right]$$

$$WT_a = G1*WT_b + 2*G3 \left[\frac{C(I-1) - Q}{ZDEL} + \frac{R * TIJ}{2*RDEL*(R+RDEL/2)} + \frac{TKI(I-1) - TKI(I-1,K+1)}{4*(R+RDEL/2)*ADEL}\right]$$

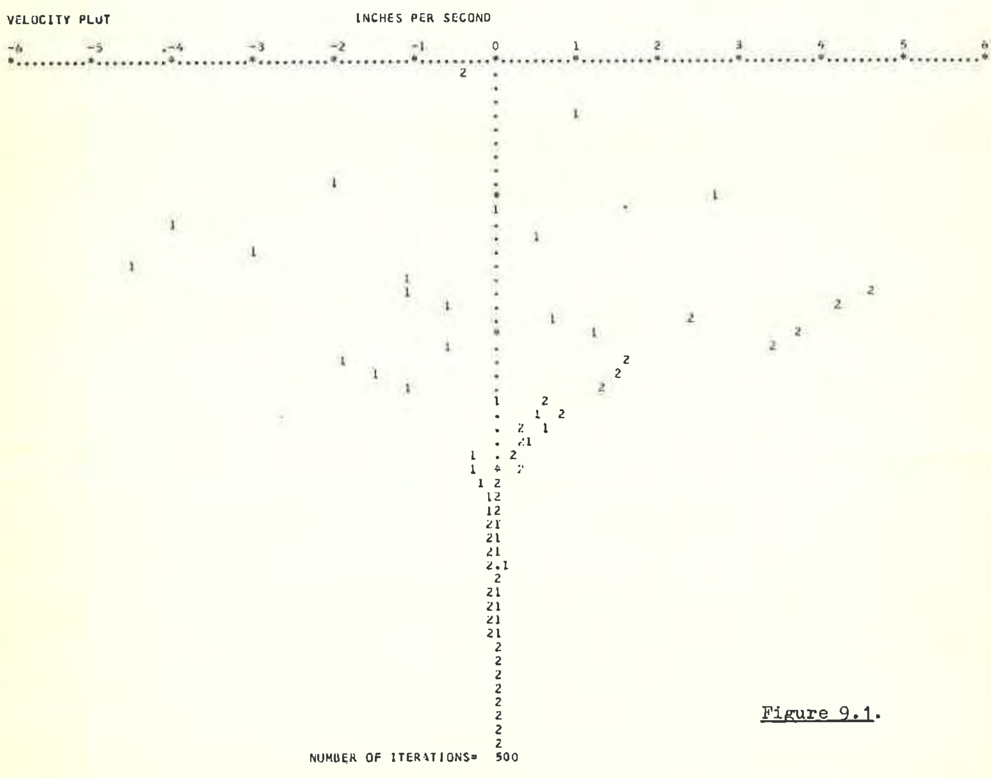
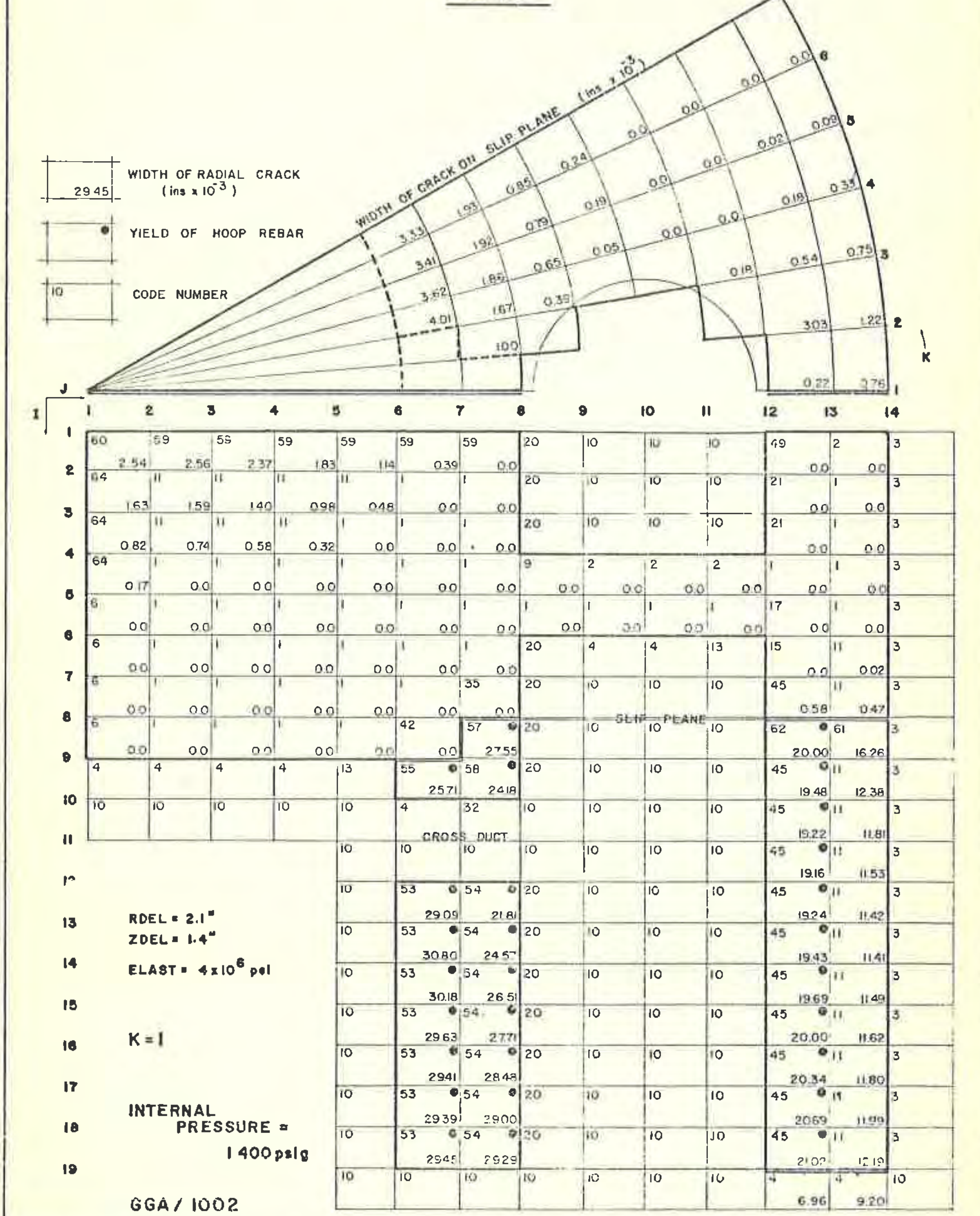


Figure 9.1.

- FIG. 13.1. -


WIDTH OF CRACK ON SLIP PLANE (ins x 10^-3)
WIDTH OF RADIAL CRACK (ins x 10^-3)
YIELD OF HOOP REBAR
CODE NUMBER
SLIP PLANE
CROSS DUCT
RDEL = 2.1"
ZDEL = 1.4"
ELAST = 4 x 10^6 psi
K = 1
INTERNAL PRESSURE = 1400 psig
GGA / 1002

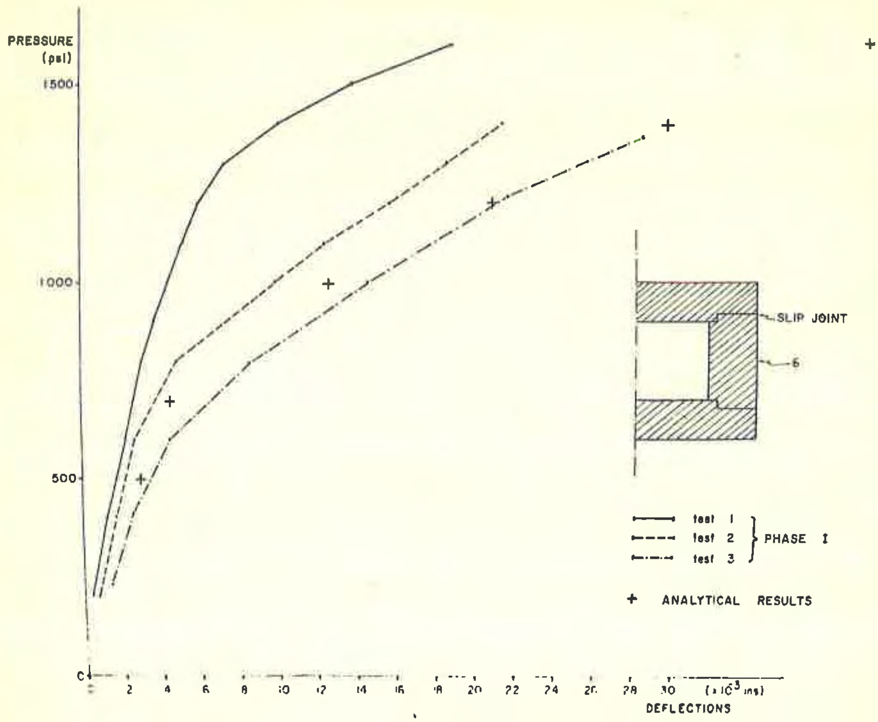


FIG 13 2 – BARREL DILATATION PLOTTED AGAINST INTERNAL PRESSURE