# A COMPARISON OF EXPERIMENTAL AND THEORETICAL INVESTIGATIONS OF EMBEDMENT EFFECTS ON SEISMIC RESPONSE

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## SUMMARY

A research program has been conducted to both gain insight into the effects of embedment on the seismic response of nuclear power plant containment structures and to verify current analytical methods for the prediction of their seismic response.

Tests were first performed on small wooden cylinders, rectangles, and cubes using a shake table to conduct parametric studies in the effects of geometry, height to diameter ratio, and amount of embedment. The effects of the shake table were isolated by repeating the tests on specimens buried in actual soils. Next, a larger-scale concrete model (heigth—2 meters, weight—1700 kg) was fabricated and tested at varying embedments using an eccentric mass structural vibrator. These tests were conducted at a field site where the dynamic properties of the soil for low strains were well known.

Additionally, an attempt was made to correlate the experimental results with simplified and finite element models that included both embedment and nonlinear effects. The embedment effects in the simplified model were taken as an increase of the impedance coefficients used for structures on the ground. Nonlinear effects were incorporated by calculating strain levels and using published data to adjust the site soil parameters.

The results reported in the paper are applicable to cases where the foundation materials under and around the structure are similar. It was found that when very low strains ($10^{-5}$-$10^{-3}$%) were applied, the embedment effect was progressively more important as the depth-to-diameter ratio increased. However, with larger strain ($10^{-2}$-$10^{-1}$%), where the response of the cylinder was in the range of 0.75 to 1.25 g's, the effect of embedment was negligible up to a depth-to-diameter ratio of 0.5. This phenomenon can be explained by the plastic deformation of the surrounding soil and loss of effective contact with the cylinder.

In conclusion, both the analytical and experimental work indicate that the effect of embedment can be incorporated into the seismic analysis of structures by simple models. In particular, embedment effects can be neglected during seismic excitations if the depth-to-diameter ratio is less than 0.5. For deeper embedments good agreement with experimental results can be achieved with simplified models and by selection of soil parameters evaluated at the appropriate strain levels.

## 1. Introduction

One of the critical areas in the seismic design of nuclear power plant structures is the estimation of soil-foundation interaction effects, particularly where massive structures are embedded. At the present time this is an area where improvements in the state-of-the-art are required [1]. Recent work indicates that errors can result if care is not paid to both the parameters and methods used in analysis [2]. To gain insight into this problem, experimental studies and analyses were undertaken to study the effect of various parameters on the dynamic response of embedded structures [3,4].

The first phase of the project involved dynamic testing of rigid model (4" - 12" high) structures. The models were founded on several types of soil under varying conditions of embedment and horizontal and vertical shaking. Shake table tests were conducted on families of geometric shapes to evaluate possible geometric, soil medium, and embedment effects, as well as the effects of combined horizontal-vertical accelerations upon model resonance. In addition to resonant frequencies and damping values, overturning (or "break loose") from the supporting soil was investigated. Free field harmonic excitation tests of a selected model were conducted, at two different sites, to examine its response and the influence of embedment, soil, and amplitude upon the modal frequencies and damping.

The objectives of the second phase of the project were to perform field tests and additional studies on embedded structures. In this phase, the size of the model was increased by a factor of ten, from typically six inches high in the laboratory tests to seventy two inches high in the field tests.

The field test model was fabricated from a concrete pipe 6 ft (1.8m) long, and 42.5" (1.1m) outside diameter. A concrete plug was poured in the bottom of the pipe to simulate the mass distribution of a concrete containment structure. A steel support structure was attached to the top of the pipe for connecting an eccentric mass shaker. The shaker was rigidly attached to the top of the cylinder. Accelerometers were attached to the structure and placed on the soil at appropriate locations. This assembly was then embedded at different depths at a field site where the soil properties have been documented by extensive testing. The objective of the project was to establish the effect of embedment under varying test conditions. The test results were then compared with theoretical values obtained by various calculational procedures.

## 2. Test Methods

The theory used will first be developed for a one-degree-of-freedom system. With minor modifications, it can be extended to multi-degree-of-freedom systems. The differential equation which defines the dynamic response of this system is:

$$M\ddot{x}[t] + C\dot{x}[t] + Kx[t] = F[t] \tag{1}$$

where

$x[t]$ is the displacement of the system from its equilibrium position,

$M$ is the mass of the system,

C is the coefficient of viscous damping.

K is the spring constant or stiffness, and

F[t] is the input or driving force.

Equation [1] can be transformed to the frequency domain and written as a transfer function relating the response of the system and input motion as described below:

$$G_1\ [\omega] = \frac{\ddot{x}}{\ddot{x}_1} = \frac{r_n^{\ 4}}{\sqrt{[1 - r_n^{\ 2}]^2 + (2\beta r_n)^2}} \qquad (2)$$

where

$\beta = \frac{C}{C_c}$ is the damping ratio, or fraction of critical damping,

$C_c = 2M\omega_n$ being the critical value of damping,

$\omega_n =$ K/M is the natural frequency of the system,

$r_n = \omega/\omega_n$ is the ratio of frequencies, and

$\ddot{x}_1$ = a reference acceleration which is proportional to the applied force.

The transfer function given by equation (2) describes a resonant system. It is obvious from a graph of this function that the shape of the curve is determined by the eigenparameters (i.e., by the damping and natural frequency) and that the higher values of damping decrease the maximum response and cause a shift of the frequency at which the maximum response occurs. Experimental data obtained from tests could be graphed in the same way and the dynamic parameters could be evaluated from the data.

For a single-degree-of-freedom system with small damping values, the damping ratio is determined from the width of the peak response as follows:

$$\beta = \frac{\Delta\omega}{2\omega_n} \qquad (3)$$

where $\Delta\omega$ is the bandwidth in frequency measured at 0.707 of the peak response and $\omega_n$ is the natural frequency of the system. For small values of damping, i.e., less than 20% of critical, the peak frequency is essentially equal to the eigenfrequency. If the damping is larger, more elaborate techniques should be used. Equation (3) is true for steady-state input excitation of the type

$x_1(f) = A\sin\omega t$

where A is the constant, and $\omega$ is the input motion frequency.

The laboratory tests were performed using a Horizontal Watt's Linkage (HWL) shake table. It was designed and built on the principal of Watt's Linkage which develops linear horizontal motion over several inches. Steady-state sinusoidal motion of the table is induced by eccentric mass structural vibrators mounted directly on the table. This type of vibrator also has been extensively used to test large structures [5]. The vibrator used has an adjustable eccentric moment with a maximum value of 45 lb-in. It is driven by a DC motor controlled by a solid-state feedback system capable of frequency control and measurement to ±0.01 Hz in the range of 0-30 Hz.

In a typical laboratory test, the test specifmen was embedded in soil contained in a box mounted on the shake table. Accelerometers were placed on the table and at selected locations on the test specimen. The frequency was varied in the range 1-30 Hz in finite increments, while the response was recorded on strip chart recorders. The eccentric moment, and hence the level of excitation, was varied to cause different levels of response and to test the structure in greater detail. This was particularly useful for examining nonlinear effects.

For field testing an eccentric mass vibrator was attached to the top of the model structure. A small compact vibrator was used with the small models; for the 1.8m high conrete model, the same vibrator used to drive the shake table was employed. During the forced vibration tests, the response of the structure was measured using accelerometers, strip-chart recorders and a real-time spectrum analyzer.

The entire data acquisition system was field calibrated using a tilt table giving a 0.10 g or 1.0 g signal. The tilt table is leveled by reversing an accelerometer in its level position. For greatest sensitivity, all of the recorder amplifiers can be operated at their maximum gain settings. This results in different sensitivities for each transducer depending upon the particular combination of accelerometer and recorder channel. Because of the range of frequencies covered in the tests, some accelerometers were occasionally operated beyond the range in which their response is independent of frequency. A calibration curve for each accelerometer has been obtained. The computer programs used for data reduction automatically correct the data if the accelerometers have been used in the range in which their response is frequency dependent.

As a guide to detailed testing, to verify that no significant range of frequencies has been overlooked, and to establish the correct attenuator settings for each recording channel, first a "sweep" is made of the entire frequency range attainable with a given set-up. During the sweep, the frequency of vibration was gradually but continuously varied and the response was continuously recorded at some slow recorder speed. The envelope of the resulting record corresponds to the desired response curve. The subsequent detailed testing was then concentrated in those ranges of frequency which were of most interest. During the detailed testing, the recorder was operated at a slow paper speed while incremental changes in frequency were made. In this way, the transition between one frequency setting and the next was monitored and the frequency increment was adjusted to study the response with as much detail as was necessary.

## 3. Laboratory Tests

The purpose of this phase of the project was to obtain test data on the response of embedded model structures under simulated conditions of ground shaking. The laboratory tests involved a series of parametric studies using small wooden models. The areas examined included:

- An investigation of the effects of geometry on overturning of structures
- Determination of the effect of horizontal excitation versus combined horizontal-vertical excitation.
- Evaluation of the influence of the aspect ratio (height-to-base ratio) on overturning.
- Evaluation of the effect of gross soil properties on test results.
- Determination of the effect associated with different amounts of embedment.

Nuclear power plant containment buildings have gross specific weights in the range of 40 to 55 $lbf/ft^3$ (0.64 to 0.88 $gm/cm^3$). The models consisted of spheres, cubes, cylinders, and right parallelepipeds made from ash or mahogany with effective specific weights (including instrumentation) in the range of 36 to 50 $lbf/ft^3$. There were 17 models of varying sizes ranging from 4 inches (10 cm) diameter or width up to 10 inches (25 cm) maximum diameter or width and 12 inches (30 cm) maximum height. Two types of "soils" were used; the first was a clean (noncohesive) No. 30 white silica sand. The second was natural soil from a Los Angeles site. Both soils were subjected to direct shear tests to determine shear strengths at low normal loads. The input (shake table) motions occurred over the frequency range of 1 to 30 Hz and had amplitudes of 0.7 g horizontal and 0.5 g vertical or greater.

During testing the input (base) motion was gradually increased. It was observed that the response of the models increased gradually to a certain point, and then suddenly increased. The response was similar to a nonlinear "jump" phenomenon and was followed by a still greater response and eventual overturning of the models.

Subsequent investigations indicated that overturning of the structural models during the shake table testing was preceded by local failure of the soil adjacent to the model. This phenomenon was called "break loose" and an attempt was made to determine the conditions under which it occurred. The tests indicated that lateral acceleration and percent embedment were the critical variables. Combined horizontal and vertical acceleration seemed to be only slightly more effective than strong horizontal shaking alone. Geometry had an effect on response, but it was a second order effect (except for the spheres, which were significantly less stable). The aspect ratio (height-to-base ratio) was an important parameter; low ratios performed better.

Based on these tests, the optimum model structure for avoiding "break loose" conditions would be a low profile, cylindrical geometry, with $1.0 < H/D < 2.0$ and embedment equal to 50% H.

The type of soil had an important effect on both the dominant frequencies of response and the amplitude (through damping and other effects), and increased soil motions resulted in increased damping and a decrease in natural frequency.

In anticipation of future field studies of embedment effects upon partially buried rigid bodies, a series of field tests were conducted with one model (the 12" high, 6" diameter cylinder) at two different sites. One site

(Venice Beach, Los Angeles) was selected to provide an effective sand half-space similar to the No. 30 sand used in the laboratory vibration studies and the other site was the location from which the second lot of soil (referred to herein as "Los Angeles soil") was obtained for the shake table study. The model was mounted with both an accelerometer and an ANC Mark 10 shaker.

The field testing investigated two areas: (1) the effect of embedment upon the first mode response of the model in different soil media and (2) comparison of measured damping ratios for the two soil media studied.

The first mode rocking-translation frequencies increased dramatically (in a nonlinear fashion) with percent embedment; for both sites, the frequency approximately tripled as the model went from zero embedment (surface test) to 50% embedment. The frequencies for the Los Angeles soil were always lower than for the sand site. At the Los Angeles site for 50% embedment without backfill, a 23.5 Hz first mode and 40.8 Hz second mode were observed as opposed to surface results of 16.6 Hz and 21.1 Hz. This phenomenon was attributed to the hardened surface of the undisturbed soil.

The frequency response curves for two different force inputs illustrate the nonlinear behavior of the soil. Increasing the input force by a factor of 3.3 produced a 20% reduction in natural frequency, a factor of 3.95 increase in amplitude and at least a 60% increase (to $\beta \simeq 16\%$) in apparent damping. The trend of all these effects are as would be expected for an elasto-plastic medium under dynamic loading. Of particular interest to seismic design is the large increase in damping at the higher levels of response.

## 4. Field Tests

Three types of tests were performed at each depth of embedment. The first test was a low level test without backfill. The level of response was maintained at 0.025 inch or less. The second test was to install a compacted backfill and repeat the low level test, but now with the added restraining effect of the backfill. The maximum displacements at the top of the cylinder during the low level tests were in the range of 5-25 mils. The second series of tests, that is low level force with backfill, resulted in a roughly constant maximum displacement of 15 mils. The third series of tests involved increasing the force by a factor of ten and repeating the test with backfill. Under these conditions, the top of the cylinder responded in the range of 0.75 to 1.25 g. The displacements ranged as high as 0.23 inch but were typically around 1/10 inch.

The 6 foot cylinder, tested under actual field conditions, was found to respond in a manner similar to the small models which were tested in the laboratory on the shake table [3]. As the frequency of shaking was slowly increased, the response of the cylinder would increase slowly until a critical level was reached. At this point, the response would suddenly increase dramatically and the cylinder would undergo large amplitude response. Additional tests were performed to explore the significance of the "breakloose" frequency. These tests used the ANC MK-10 vibrator, a small low force, high frequency shaker, to insure elastic response. The initial eccentricity was ∿0.1 in-lb. This was subsequently reduced to 0.03 in-lb. Tests were conducted on the 6' cylinder embedded 56 inches. At these low force levels (strain = $10^{-5}$ - $10^{-4}$%), the soil-structure system remained elastic and the response resembled a typical resonance curve. Modes were observed at 38, 50, and 80 Hz. The first mode (rocking at 38 Hz) exhibited relative damping of 6% of critical. Based on these results, it is concluded that the "breakloose" frequency characterizes response at the rocking mode when the applied force is sufficient ot cause generalized yielding of the soil around the embedded structure. Table I summarizes the experimental results.

Figure 1 illustrates how the rocking frequency varies as a function of embedment for both the high level and low level tests with and without backfill. So long as the embedment is less than 70% of the diameter of the cylinder (50% of the height of the cylinder), the backfill has little effect on the high level test results. In other words, for shallow embedments (70% of the diameter) when the soil surrounding the cylinder is undergoing large deformations, the effect of the soil is quite slight. At the higher level, the soil undergoes greater responses and a large extent of yielding. It appears that the response of the system can be modeled by neglecting embedment effects. In the low level, no backfill case, the stiffness is primarily that of the cylinder rocking on its flat base, and the stiffness of the soil is one which corresponds to the small deflections being experienced.

When backfill is provided and the level of excitation is still low, the rocking frequency increases in proportion to the depth of embedment. This is also true for high level tests when the depth of embedment is greater than 30 inches (70% of the diameter), although the rate of increase is not so great as with the low level tests.

Using the applied forces and measured displacements, effective stiffnesses can be found and compared with calculated values (Table II). In the low level no backfill tests, the effective stiffness is attributed primarily to rocking. It remains relatively constant with depth. If the soil was ideally uniform and isotropic, it would be expected to remain constant. During the tests, it was observed that there were slight differences in the soil as a function of depth and this may explain, in part, the observed changes in the low level, no backfill stiffnesses. The low level backfill curve gives the highest values of effective stiffness, as would be expected, since the responses under this condition are still low. The high level backfill results show effective stiffnesses less than the low level no backfill case for embedments up to 30 inches. For embedments greater than 30 inches, the effect of the backfill is significant in increasing the effective stiffness of the system.

### 4.1 Comparison with Theory

Two methods were used to calculate both the eignefrequencies and damping of the structure at different levels of embedment. The first method [6], for simplicity of analysis, increases the impedance coefficients of the surface structure by a factor as given in Figure 1. The curves in this figure are derived from the solution of the embedded structure as reported by Beredugo and Novak [7,8] with certain modifications. The second approach used the methodology of References 7 and 8 without any modifications. The strain dependent soil properties were estimated based on generalized curves published for this purpose by Seed and Idriss [9].

a. Eigenfrequencies

Figure 2 compares experimental eigenfrequencies with calculated values for various embedments. The theoretical values using geophysical soil data overestimated the measured values in all cases. This was attributed to the fact that even the low level tests resulted in high strain levels and inelastic response.

To check this hypothesis, a supplemental test was performed (see Figure). In this test, a smaller shaker was used and the strain level was about 2 x $10^{-5}$%, or well in the elastic range. This point falls in the midband of the results obtained with the two theoretical models used.

In the 56 inch embedded cases, where results were obtained at three force levels, ranging from 5 lbf to 1250 lbf, the nonlinear softening effect of the soils is clearly evident.

Equation 17 of Ref. [6] gave the best prediction of the deeply embedded test results, while Beredugo and Novak's [7,8] model was more accurate for embedments less than 30 inches. For embedments greater than 50% of the OD, and for high strain levels, either method could overestimate the natural frequency by 100% or more unless appropriate strain dependent parameters are used [6]. Neglecting the effect of the backfill, on the other hand, would overestimate (12.5 Hz vs. 7-8 Hz) the high level results for embedments up to about 30 inches (70% of OD), and at the greatest embedment, would underestimate the frequency by about 25% (12.5 Hz vs. 16.5 Hz).

b. Damping Values

Calculated damping values for rocking range from 3.8% to 21% depending on embedment (see Table III). The experimental values ranged from 3% to 14%. In general, the low level backfilled results were higher than the low level no backfill results (typically 7% vs. 3%) although this was not true in every case. The high level tests showed the highest damping values (7-14%) but in one case showed only 3%. The damping data is limited because some of the peaks are not sufficiently well defined. In other cases, once the system "broke loose", the resulting clearance may have caused less energy dissipation.

## 5. Conclusions

These tests investigated the effects of embedment on structural models of nuclear power plant containment structures. Experimental results were compared with theoretical models in which equivalent impedance coefficients which depended on depth were computed.

The experimental results showed substantial increases in resonant frequency as the amount of embedment increased. When soil parameters (shear modulus principally) were evaluated at strain levels consistent with those experienced during the test, satisfactory agreement with experimental values was obtained. These results emphasize the importance of using strain-dependent soil parameters in the seismic analysis of nuclear power plants. Additionally the effect of embedment was negligible for embedments less than about 70% of the diameter of the cylinder during the high strain level tests. More importantly, the analytical methods used were adequate when strain dependent soil parameters were used.

## References

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[7] Beredugo, Y.O., M. Novak, "Coupled Horizontal and Rocking Vibration of Embedded Footing", Canadian Geotechnical Journal, Vol. 9, p. 474, 1972.

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### TABLE I
### SUMMARY OF VIBRATION TEST RESULTS(1)

| Embedment (inches) | Low Level, No Backfill | | | | Low Level With Backfill | | | | High Level With Backfill(2) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | f (Hz) | β (%) | F (lbf) | γ (%) | f (Hz) | β (%) | F (lbf) | γ (%) | f (Hz) | β (%) | F (lbf) | γ (%) |
| 0 | 10 | --- | 46 | 0.003 | -- | --- | --- | --- | 6.5 | 3-7 | 194 | 0.026 |
| 14 | 9.3 | --- | 39 | 0.018 | 13.5 | --- | 84 | 0.017 | 6.5 | 3-11 | 194 | 0.32 |
| 30 | 10 | --- | 46 | 0.028 | 19 | --- | 166 | 0.021 | 8.3 | 3 | 312 | 0.22 |
| 42 | 8.9 | 7 | 37 | 0.033 | 23 | --- | 243 | 0.019 | 13.3 | 14 | 689 | 0.13 |
| 56 | 8.0 | 4 | 29 | 0.035 | 24.5 | 3 | 276 | 0.013 | 16.5 | 7 | 1250 | 0.096 |

Notes:

(1) Symbols: f = rocking frequency; β = damping factor; F = applied force; γ = soil strain

(2) No "backfill" on surface.

### TABLE II
### CALCULATED IMPEDANCE COEFFICIENTS

| Embedment (inches) | Rocking Stiffnesses, $K_\psi$, lb-ft/radian x $10^{-7}$ | | | Translational Stiffnesses $K_X$, lb/ft x $10^{-6}$ | | |
|---|---|---|---|---|---|---|
| | Based on Geophysical Data | | Based on Strain Dependent Data | Based on Geophysical Data | | Based on Strain Dependent Data |
| | Ref. 7 & 8 | Ref. 6 | Ref. 6 | Ref. 7 & 8 | Ref. 6 | Ref. 6 |
| 0 | 6.37 | 1.47 | 0.9 | 5.3 | 5.8 | 3.5 |
| 14 | 8.95 | 2.16 | 0.35 | 8.2 | 7.5 | 1.2 |
| 30 | 10.2 | 3.52 | 0.7 | 11.4 | 9.4 | 1.9 |
| 42 | 10.7 | 5.34 | 1.2 | 13.9 | 10.8 | 2.5 |
| 56 | 13.7 | 8.69 | 2.6 | 16.7 | 12.4 | 3.7 |

| Embedment (inches) | Measured Rocking Stiffness with Backfill $K_\psi$ lb-ft/radian x $10^{-6}$ | |
|---|---|---|
| | Low Level Tests | High Level Tests |
| 0 | --- | 1 |
| 14 | 3.5 | 1 |
| 30 | 5.0 | 1 |
| 42 | 9.0 | 3.5 |
| 56 | 15 | 8.5 |

## TABLE III
## DAMPING FACTORS, β (%)

| Embedment (inches) | Rocking Mode $\beta_\psi$ | Translation Mode $\beta_x$ |
|---|---|---|
| 0 | 3.8 | 27 |
| 14 | 5.9 | 31 |
| 30 | 9.5 | 35 |
| 42 | 14 | 37 |
| 56 | 21 | 40 |

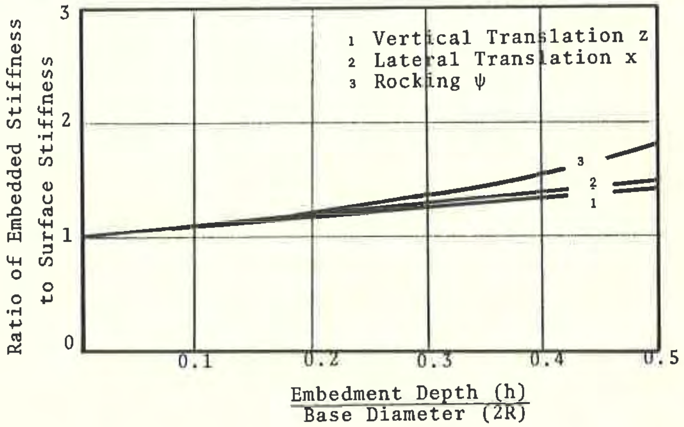

FIGURE 1: MODIFICATION FACTORS DUE TO EMBEDMENT FOR EQUIVALENT FOUNDATION INTERACTION SPRINGS AT UNIFORM SOIL SITES

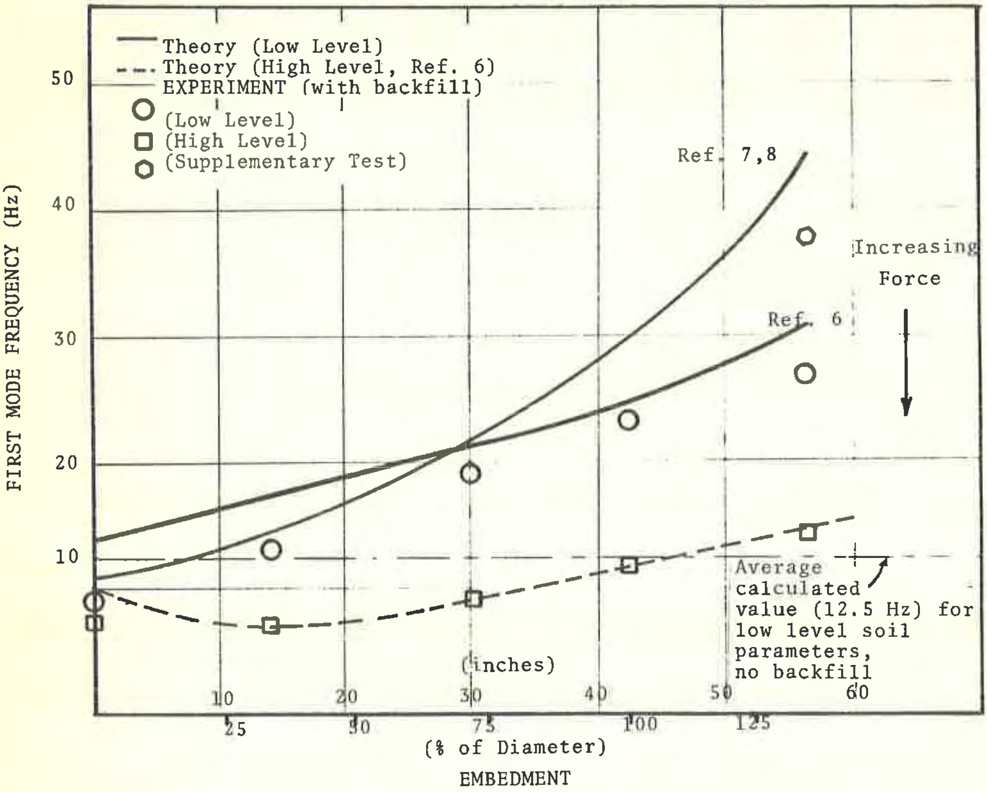

FIGURE 2: COMPARISON OF CALCULATED AND MEASURED EIGENFREQUENCIES