EARTHQUAKE ANALYSIS OF NUCLEAR REACTOR BUILDINGS INCLUDING FOUNDATION INTERACTION

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SUMMARY

Presented is a general method of analysis which provides a unified treatment for all types of idealization for the foundation, and overcomes the deficiencies in the commonly used procedures for including foundation interaction effects in earthquake analysis of nuclear reactor buildings.

The method is based on the substructure concept. The dynamic equilibrium equations for the structure, idealized as a finite element system, including foundation interaction effects are expressed in the frequency domain. Foundation interaction introduces a frequency-dependent foundation stiffness matrix in the structural equations. This matrix for the foundation is associated only with the degrees of freedom at the structure-foundation interface; it is obtained by analysis of the foundation separately for unit harmonic ($=e^{i\omega t}$) load or displacement applied in each interface degree of freedom. In carrying out this analysis, it would be appropriate to use a finite element idealization if the foundation consists of a relatively flexible medium underlain by much stiffer media which can be assumed as rigid. At many sites the material near the ground surface may extend to large depths; idealization of the foundation as viscoelastic half space would be more appropriate for such situations. Results for the frequency dependent stiffness coefficients for both types of idealizations of the foundation are summarized.

This substructure approach directly treats the free-field motion at all the degrees of freedom on the structure-foundation interface as the excitation. Fourier decomposition of the free-field ground motion, solution of the structural equations including the foundation interaction terms to determine the steady-state harmonic response over a range of excitation frequencies and Fourier synthesis of the harmonic responses leads to response of the structure including foundation interaction effects to arbitrary earthquake excitation.

Before proceeding with solution of the equations, the structural displacements are represented as the sum of the quasi-static displacements, associated with the displacements of all the degrees of freedom at the structure-foundation interface, and the dynamic displacements, with the latter expressed as a linear combination of the first few mode shapes of the structure on *rigid* foundation. It is demonstrated that excellent accuracy is obtained by including only the first few modes, leading to a drastic reduction in the number of equations to be solved for each excitation frequency.

It is concluded that this method permits better modelling of the structure-foundation system, and is very efficient because the use of modal coordinates leads to drastic reduction in computational effort.

1. Introduction

Analysis of response of nuclear reactor buildings to earthquake ground motion is especially complicated because structure-foundation interaction needs to be included. Much effort has been expended in developing procedures and computer programs to tackle the practical problems that arise in the nuclear industry. However, some of the important features are still not treated in a satisfactory manner and significant deficiencies remain in the available methods of analysis.

The approach most commonly used in the nuclear industry is to directly analyze a finite element idealization of the combined structure-foundation system. (Sometimes a number of linear analyses of the complete response are preformed to iteratively account for the nonlinear soil properties). In addition to the enormous computational requirements and difficulties arising in obtaining reliable results for large finite element systems, such a procedure has two important drawbacks: (1) The boundary hypothesized at some depth in the foundation is usually assumed to be rigid. For sites where essentially the same foundation material extends to large depths and there is no obvious "rigid" boundary such as a soil-rock interface, the location of the rigid boundary introduced in the analysis is often quite arbitrary (2) For nuclear reactor buildings, the design earthquake is typically specified either at the surface of the ground or at the foundation level of the major structures. In a direct finite element analysis, it becomes necessary to first determine the motions at the base of the finite element mesh by deconvolution of the surface or foundation level motions -- a process with considerable conceptual problems.

Another approach used in industry treats the foundation as a half space and the base of the structure as a rigid plate. Because the foundation is treated as a half space, the difficulties associated with introduction of a rigid boundary in the foundation do not arise. However, the assumption of a rigid plate may be inappropriate in many situations. Various solutions relating the interaction forces to displacements are available for a rigid plate on a half space. The cases for which results have been reported in the literature includes circular as well as rectangular plates and elastic as well as viscoelastic foundation materials. The frequency-dependent dynamic stiffness (or impedance) functions obtained from the half space solution enter into the equations of the structure. The problem is thus solved in two stages by a "substructure" approach.

To overcome the limitations of the two approaches, there is the need to develop a general method of analysis which can (1) handle finite element idealizations of the structure coupled with foundations idealized as finite element systems or treated as continua, finite or semi-infinite in size -- without introducing the limitation of a rigid base plate in the continuum approach -- and (2) directly work with the prescribed ground motions, eliminating the deconvolution calculations. The substructure method of analysis [1] is promising in fulfilling this need. Research in this general area is continuing at the University of California, Berkeley. This paper presents a brief progress report on selected phases of the project.

2. Substructure Method

2.1 Governing Equations

In the substructure method it is most convenient to formulate the problem in the frequency domain. For steady state harmonic motion at frequency ω, any structural response quantity $Z(t) = \hat{Z}(\omega)\, e^{i\omega t}$ where $\hat{Z}(\omega)$ is the complex valued amplitude. The matrix equation governing the response amplitudes for a structure idealized as a finite element system is

$$\left(-\omega^2 \begin{bmatrix} \underline{M}_{ss} & \underline{O} \\ \underline{O} & \underline{M}_{bb} \end{bmatrix} + (1 + i\gamma) \begin{bmatrix} \underline{K}_{ss} & \underline{K}_{sb} \\ \underline{K}_{bs} & \underline{K}_{bb} \end{bmatrix}\right) \begin{Bmatrix} \hat{\underline{r}}_s^t(\omega) \\ \hat{\underline{r}}_b^t(\omega) \end{Bmatrix} = \begin{Bmatrix} \underline{O} \\ \hat{\underline{R}}_b(\omega) \end{Bmatrix} \tag{1}$$

$\hat{\underline{r}}^t$ represents the vector of total displacements in all the degrees of freedom of the nodal points. The subscript b refers to the nodal points on the structure-foundation interface and s to those in the structure but not on this interface. γ is the constant-hysteretic-damping coefficient and $i = \sqrt{-1}$. $\underline{M}$ and $\underline{K}$ are the mass and stiffness matrices. $\hat{\underline{R}}_b$ is the vector of interaction forces on the nodal points on the structure-foundation interface.

Eq. 1 is general in the sense that it applies to any type of finite element system. In a plane stress or plane strain system each nodal point has two degrees of freedom: horizontal and vertical displacements. In an axisymmetric system subjected to horizontal ground motion each nodal circle has three degrees of freedom: radial, vertical and circumferential displacements [2]. The mass and stiffness matrices for any type of finite element system can be determined by well established procedures [3]. Usually the most effective representation of the inertial properties is by a diagonal mass matrix, as in eq. 1; however, a consistent mass matrix can be introduced instead without any conceptual complication.

The foundation forces $\hat{\underline{R}}_f(\omega)$ and displacements $\hat{\underline{r}}_f^t(\omega)$ in the degrees of freedom on the structure-foundation interface are related [1]:

$$\hat{\underline{R}}_f(\omega) = \underline{X}_f(\omega)\, [\hat{\underline{r}}_f^t(\omega) - \hat{\underline{r}}_o(\omega)] \tag{2}$$

where $\hat{\underline{r}}_o(\omega)$ is the Fourier transform of the free-field displacements due to the earthquake ground motion and $\underline{X}_f(\omega)$ is a dynamic stiffness (or impedance) matrix for the foundation; its complete definition and evaluation will be discussed later.

Utilizing the equations of equilibrium: $\hat{\underline{R}}_f(\omega) = -\hat{\underline{R}}_b(\omega)$ and of compatibility: $\hat{\underline{r}}_f^t(\omega) = \hat{\underline{r}}_b^t(\omega)$ for the structure-foundation interface, eqs. 1 and 2 may be combined, leading to

$$\left(-\omega^2 \begin{bmatrix} \underline{M}_{ss} & \underline{O} \\ \underline{O} & \underline{M}_{bb} \end{bmatrix} + (1 + i\gamma) \begin{bmatrix} \underline{K}_{ss} & \underline{K}_{sb} \\ \underline{K}_{bs} & [\underline{K}_{bb} + (1 + i\gamma)^{-1}\, \underline{X}_f(\omega)] \end{bmatrix}\right) \begin{Bmatrix} \hat{\underline{r}}_s^t(\omega) \\ \hat{\underline{r}}_b^t(\omega) \end{Bmatrix} = \begin{Bmatrix} \underline{O} \\ \underline{X}_f(\omega)\, \hat{\underline{r}}_o(\omega) \end{Bmatrix} \tag{3}$$

2.2 Dynamic Foundation Stiffness Matrix

$\underline{X}_f(\omega)$ is a square matrix whose i-j element represents the complex amplitude of the harmonic force required, corresponding to degree of freedom i, if a displacement $e^{i\omega t}$ is imposed at degree of freedom j; all other connection degrees of freedom between the structure and foundation being kept fixed (Fig. 1).

Under the assumption -- commonly made in analysis of nuclear reactor buildings -- that the base of the structure is a rigid footing, the interface has three degrees of freedom in planar motion (Fig. 2). The associated $\underline{X}_f(\omega)$ can be obtained from solutions for steady state response of the rigid plate attached to the foundation medium and subjected to harmonic force applied separately in each degree of freedom. Many investigations have been concerned with this general problem. Circular as well as rectangular footings and various types of foundations: homogeneous elastic or viscoelastic halfspace, homogeneous viscoelastic layer overlying a rigid medium or a homogeneous viscoelastic halfspace, etc. have been considered; see for example, references [4-9].

In the more general case when the base is not assumed as rigid, the degrees of freedom associated with all connection nodal points need to be considered. For plane stress or plane strain problems, each nodal point has two degrees of freedom: horizontal and vertical displacements. Analysis of the foundation is required for unit harmonic displacement in one degree of freedom at a time with all other connection degrees of freedom kept fixed (Fig. 1). Results of such analysis can then be assembled to obtain $\underline{X}_f(\omega)$. One effective procedure is to first solve the problem for displacements prescribed as zero even outside the structural base (Fig. 3), assemble an expanded version of $\underline{X}_f(\omega)$ including additional degrees of freedom outside the structural base, and finally apply the static condensation process to eliminate the additional degrees of freedom and introduce the condition of zero forces outside the structural base [10]. Results for these displacement boundary value problems (Fig. 3) have been presented for a homogeneous elastic halfspace [10]. Similar results for a homogeneous viscoelastic halfspace will soon be reported. In order to analyze axisymmetric structure-foundation systems, corresponding displacement boundary value problems associated with the three degrees of freedom -- radial, vertical, and circumferential displacements -- of each nodal circle need to be tackled.

It would be appropriate to idealize the foundation as a finite element system provided the physical characteristics at the site are such as to justify introducing in the analysis a rigid boundary at a depth which is not too large. Such would be the case, for example, when the soil profile at the site consists of layers of relatively soft soils overlying stiff rock. A finite element idealization can handle arbitrary geometry and nonhomogeneities of the foundation medium, whereas, at the present time, the assumption of homogeneity is necessary when the foundation is treated as a halfspace. A method for obtaining $\underline{X}_f(\omega)$ for foundations idealized as planar finite element systems has been developed [1]. Research is continuing to improve this method and to extend it to axisymmetric systems.

2.3 Reduction of Degrees of Freedom

Eq. 3 in the frequency domain governs the response of the structure including foundation interaction. Once $\underline{X}_f(\omega)$ is determined, a straight-forward way to obtain the structural response is to directly solve these equations for a large number of discrete values of ω and then evaluate the inverse Fourier transforms of all the displacements. This, however, is not a desirable method of solution for it involves the solution of a large number of simultaneous algebraic equations to be repeated for many values of ω and requires large amounts of storage to save the entire set of displacements for future Fourier synthesis.

For large systems that possess classical normal modes, it is well known that the mode superposition method allows profound numerical simplification: the equations become uncoupled in the modal coordinates and even more important is the feature that earthquake response of most structures can be expressed adequately by retaining only the first few modes of vibration. When foundation interaction is included, the structure, considered as a subsystem, does not possess classical normal modes because of the frequency dependent terms associated with the foundation. However, the number of coordinates to be considered in the analysis can still be reduced by introducing a set of generalized coordinates: normal modes of the structure on rigid foundation along with the degrees of freedom on the structure-foundation interface. These generalized coordinates were demonstrated to be effective for structures on a single rigid footing [11]. However, as reported earlier [1], they were not useful for structures with non-rigid base; the conceptual limitations of the earlier approach have been overcome, resulting in the following formulation.

The total displacements in eq. (3) are separated into three parts:

$$\begin{Bmatrix} \hat{\underline{r}}^t_s \\ \hat{\underline{r}}^t_b \end{Bmatrix} = \begin{Bmatrix} \underline{1}_s \\ \underline{1}_b \end{Bmatrix} \hat{u}_g + \begin{Bmatrix} \hat{\underline{r}}^q_s \\ \hat{\underline{r}}_b \end{Bmatrix} + \begin{Bmatrix} \hat{\underline{r}} \\ \underline{0} \end{Bmatrix} \tag{4}$$

In eq. (4), $\hat{u}_g(\omega)$ is the Fourier transform of the horizontal component of the specified free field acceleration divided by $-\omega^2$, identical for all the interface nodal points; $\underline{1}_s$ and $\underline{1}_b$ are vectors defining the quasi-static displacements in all degrees of freedom due to rigid base motion $u_g = 1$, $\hat{\underline{r}}_b$ represents the interaction displacements on the structure-foundation interface; $\hat{\underline{r}}^q_s$ are the quasi-static displacements due to the interface displacements $\hat{\underline{r}}_b$, i.e.

$$\underline{K}_{ss} \hat{\underline{r}}^q_s + \underline{K}_{sb} \hat{\underline{r}}_b = 0 \tag{5}$$

or

$$\hat{\underline{r}}^q_s = \underline{L} \hat{\underline{r}}_b \tag{6}$$

where

$$\underline{L} = - \underline{K}^{-1}_{ss} \underline{K}_{sb} \tag{7}$$

Eq. 4 can thus be written as

$$\begin{Bmatrix} \hat{\underline{r}}^t_s \\ \hat{\underline{r}}^t_b \end{Bmatrix} = \begin{Bmatrix} \underline{1}_s \\ \underline{1}_b \end{Bmatrix} \hat{u}_g + \begin{bmatrix} \underline{I} & \underline{L} \\ \underline{0} & \underline{I} \end{bmatrix} \begin{Bmatrix} \hat{\underline{r}} \\ \hat{\underline{r}}_b \end{Bmatrix} \tag{8}$$

The natural frequencies and mode shapes of vibration of the structure on rigid foundation are solutions of the eigen-value problem:

$$\underline{K}_{ss} \underline{\phi}_n = \omega^2_n \underline{M}_{ss} \underline{\phi}_n$$

If $\hat{\underline{r}}$ can be adequately described by a combination of the first J mode shapes then

$$\hat{\underline{r}}(\omega) \simeq \sum_{n=1}^{J} \hat{Y}_n(\omega) \underline{\phi}_n = \underline{\Phi} \hat{Y}(\omega) \tag{9}$$

Combining eqs. 8 and 9:

$$\begin{Bmatrix} \hat{\underline{r}}^t_s \\ \hat{\underline{r}}^t_b \end{Bmatrix} = \begin{Bmatrix} \underline{1}_s \\ \underline{1}_b \end{Bmatrix} \hat{u}_g + \underline{A} \begin{Bmatrix} \hat{\underline{Y}} \\ \hat{\underline{r}}_b \end{Bmatrix} \tag{10}$$

where

$$\underline{A} \equiv \begin{bmatrix} \underline{\Phi} & \underline{L} \\ \underline{0} & \underline{I} \end{bmatrix} \qquad (11)$$

Substituting eq. (10) in eq. (3), premultiplying by $\underline{A}^T$ and utilizing the orthogonality property of mode shapes leads to

$$\left(-\omega^2 \begin{bmatrix} \underline{I} & \underline{\Phi}^T \underline{M}_{ss} \underline{L} \\ \underline{L}^T \underline{M}_{ss} \underline{\Phi} & [\underline{M}_{bb} + \underline{L}^T \underline{M}_{ss} \underline{L}] \end{bmatrix} + (1 + i\gamma) \begin{bmatrix} \underline{\Omega} & \underline{0} \\ 0 & [\underline{K}_{bb} + \underline{K}_{bs} \underline{L} + (1 + i\gamma)^{-1} \underline{X}_f(\omega)] \end{bmatrix}\right)$$

$$\begin{Bmatrix} \hat{\underline{Y}}(\omega) \\ \hat{\underline{r}}_b(\omega) \end{Bmatrix} = \omega^2 \hat{u}_g(\omega) \begin{Bmatrix} \underline{\Phi}^T \underline{M}_{ss} \underline{1}_s \\ \underline{L}^T \underline{M}_{ss} \underline{1}_s + \underline{M}_{bb} \underline{1}_b \end{Bmatrix} \qquad (12)$$

Eq. (12) presents two very important advantages over eq. (3). First, because only a few modes generally need to be included in eq. (9) even for structures with several hundred degrees of freedom, the number of equations is drastically reduced. Second, because of the diagonal form of $\underline{I}$ and $\underline{\Omega}$, the computational effort required to solve the algebraic equations by the Gaussian elimination method is considerably reduced. The computational savings due to both of these factors is profound, because solution of equations has to be repeated for many -- several hundred to a few thousand -- values of ω.

The planar structure-foundation system considered in an earlier paper [1] has been reanalyzed using the formulation outlined above. The structure idealized as a finite element idealization had 110 degrees of freedom above and 12 on the structure-foundation interface. Numerical results demonstrated that only 4 modes of vibration need be included in eq. 9 to obtain accurate results, thus requireing for each value of ω solution of only 16 algebraic equations (eq. 12) instead of the 122 original equations (eq. 3), obviously resulting in major computational savings.

Further reduction of the number of degrees of freedom can be achieved in analysis of a nuclear reactor building due to two considerations: As the concrete base mat is several feet thick, it is reasonable to assume that it is rigid in its own plane. In the frequency range of interest and for the lower modes of vibration of the structure, the deformations of the base mat can be most effectively described by the functions shown in Fig. 4 -- the first few terms of a Taylor series expansion. Thus

$$\hat{\underline{r}}_b = \underline{L}_b \hat{\underline{Z}} \qquad (13)$$

where $\hat{\underline{Z}}$ is a vector of the generalized displacement coefficients Z_1, Z_2, Z_3, etc. (Fig. 4). Introducing the transformation of eq. (13) into eq. (10) and the necessary modifications, an equation similar to eq. (12) but of reduced size is obtained. Numerical results obtained for example systems demonstrated that the three functions shown in Fig. 4 will generally be adequate.

2.4 Time Response

Once eq. (12), or its modified version where transformation of eq. (13) has been incorporated, is solved for a large number of suitable spaced excitation frequencies, covering an appropriate frequency range, the response as a function of time is obtained by computing the inverse Fourier transform of each unknown generalized displacement. This can be carried out very efficiently by the Fast Fourier Transform algorithm. The nodal point displacements are then obtained from eqs. (4), (9) and (13), and stresses from finite element stress transformation matrices.

3. Advantages of the Substructure Method

The substructure method described above presents a number of advantages over the procedures currently in use in the nuclear industry for earthquake analysis of structures including foundation interaction: (1) Considering the flexibility of the base mat, halfspace foundations can be handled. This capability would lead to better results when the soil conditions at the site are such that there is no obvious "rigid" boundary. (2) Structural response to the design ground motion specified at the structure-foundation interface is determined directly without deconvolution calculations. (3) The substructure method can take advantage of the important feature that response of the structure to earthquake ground motion is essentially contained in the first few modes of vibration. This is possible because the system is considered to be composed of two substructures -- the structure and the foundation-- and the modes of vibration of the structure on rigid foundation can therefore be introduced. (4) Damping in the structure and foundation can be defined separately with no restrictions on their relationship and without any increase in computational effort.

4. Concluding Remarks

The substructure method presented above can analyze any structure-foundation system that the standard finite element method can, and more. Both methods can analyze systems with the foundation as well as the structure idealized as an assemblage of finite elements. It can be shown theoretically that the two methods will lead to identical results; from the computational point of view, however, the substructure method is expected to be more reliable of the two. The substructure method can handle halfspace foundations whereas the standard finite element method can not.

The substructure method applies only to linear systems but that is not a serious restriction, for when nonlinear soil properties are considered in practical application, the nonlinear response is usually obtained by an iterative process with a linear analysis required for each iteration.

The progress report present here should provide the reader with the concepts behind the substructure method and its advantages. Results to demonstrate its effectiveness in application will be presented at the conference.

References

[1] VAISH, A. K., CHOPRA, A. K., "Earthquake Finite Element Analysis of Structure-Foundation Systems," Journal of the Engineering Mechanics Division, ASCE, Vol. 100, No. EM6, December, 1974, pp. 1101-1116.

[2] LIAW, C. Y., CHOPRA, A. K., "Earthquake Analysis of Axisymmetric Towers Partially Submerged in Water," International Journal of Earthquake Engineering and Structural Dynamics, Vol. 3, No. 3, January-March, 1975, pp. 233-248.

[3] ZIENKIEWICZ, O. C., The Finite Element Method in Engineering Science, McGraw-Hill, London, 1971.

[4] LUCO, J. E., WESTMANN, R. A., "Dynamic Response of Circular Footings," Journal of the Engineering Mechanics Division, ASCE, Vol. 97, No. EM5, October, 1971, pp 1381-1395.

[5] VELETSOS, A. S., WEI, Y. T., "Lateral and Rocking Vibration of Footings," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 97, No, SM9, September, 1971, pp 1227-1248.

[6] BIELAK, J., "Earthquake Response of Building-Foundation Systems," Report EERL 71-04, Earthquake Engineering Research Laboratory, California Institute of Technology.

[7] KOBORI, T., MINAI, R., SUZUKI, T., "The Dynamical Ground Compliance of a Rectangular Foundation on a Viscoelastic Stratum," Bulletin of the Disaster Prevention Research Institute, Kyoto Univ., Vol. 20, Part 4, No. 183, March, 1971, pp 289-329.

[8] LUCO, J. E., "Impedance Functions for a Rigid Foundation on a Layered Medium," Submitted for Publication to Nuclear Engineering and Design.

[9] VELETSOS, A. S., VERBIC, B., "Vibration of Viscoelastic Foundations," International Journal of Earthquake Engineering and Structural Dynamics, Vol. 2, No. 1, July-September, 1973, pp 87-102.

[10] CHOPRA, A. K., PERUMALSWAMI, P. R., "Dam-Foundation Interaction During Earthquakes," Proceedings, Fourth World Conference on Earthquake Engineering, Santiago, Chile, 1969.

[11] CHOPRA, A. K., GUTIERREZ, J. A., "Earthquake Response Analysis of Multistory Buildings Including Foundation Interaction," International Journal of Earthquake Engineering and Structural Dynamics, Vol. 3, No. 1, July-Sept., 1974, pp 65-77.

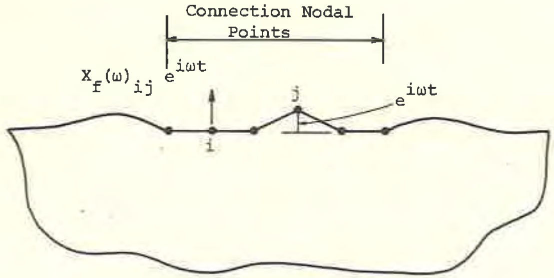

Fig. 1 Physical Interpretation of $\underline{X}_f(\omega)$

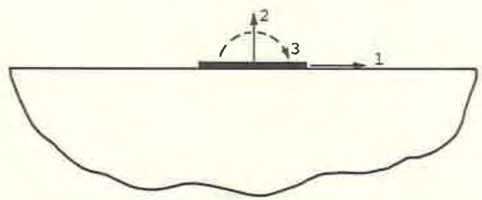

Fig. 2 Foundation Degrees of Freedom -- Rigid Footing

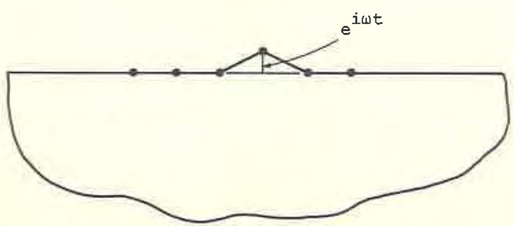

Fig. 3 Displacement Boundary Value Problem

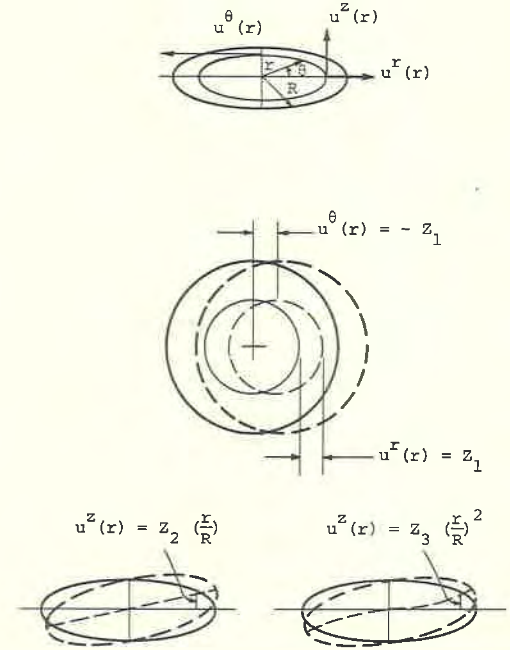

Fig. 4 Generalized Displacements for Base of an Axisymmetric Structure -- Ground Motion Along $\theta = 0$