DAMAGE-INDUCED TENSILE INSTABILITY

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SUMMARY

The paper presents a unified description of ductile and brittle rupture phenomena in structural components under tensile loading with particular emphasis on creep rupture.

The close connection between ductile instantaneous rupture and ductile creep rupture, first observed by R.L. Carlson (J. Mech. Eng. Sci. 7(1965), 228-229) has been found by H. Broberg (J. Appl. Mech. *41*(1974), 809-811) to have a direct counterpart for brittle failure. The damage concept due to Kachanov and Rabotnov has been extended to describe also instantaneous brittle failure under monotonically increasing load as well as brittle failure under repeated loading.

The damage law is stated in the form

$$d\omega/dt = g'(s)\cdot ds/dt + f(s)$$

where ω denotes damage and s the net stress, defined as load per unit undamaged cross sectional area. In absence of $g'(s)$ this law coincides with the one proposed by Kachanov and Rabotnov.

The paper describes this extended damage law and discusses its potentialities and limitations.

Two structural elements are analyzed in detail: 1) the uniform tensile bar subject to a Heaviside history of tensile force and superimposed such loadings, i.e. staircase histories, and 2) the thinwalled spherical pressure vessel subject to a Heaviside history of internal pressure.

For both these structures the conditions for instantaneous as well as delayed rupture are analysed. It is shown that a state of mechanical instability will be reached at a certain load or after a certain time. The cases of purely ductile rupture and purely brittle fracture are identified as two limiting cases of this general instability phenomenon.

The Kachanov-Rabotnov damage law implies that a structural component will fail in tension only when it has reached a state of complete damage, i.e. zero load carrying capacity. The extended law predicts failure at an earlier stage of the deterioration process and is therefore more compatible with experimental observation. Further experimental support is offered by predictions for staircase loading histories, both step-up and step-down type. The presented damage theory here predicts strain histories which are in closer agreement with test data than predictions based on other phenomenological theories.

1. Introduction

The phenomena of creep deformation and creep rupture have been known for a long time, both to metallurgists and designers. Increasing demands on new materials for severe high temperature applications have led to alloys which are ever more creep resistant. A common tend in this development is towards stronger but also more brittle materials. Hence creep deformation is becoming of less concern to designers, while creep rupture becomes all the more important. This paper discusses some engineering aspects of creep rupture.

The basic physical mechanisms causing creep deformation and creep rupture are becoming increasingly well understood. Phenomena on the microscopic and sub microscopic levels have been indentified, which are relevant to the observed macroscopic behaviour. By tradition, but also for more compelling reasons, most of these basic studies of the creep phenomenon relate to cases of uniaxial stress, constant in time.

In many, if not most, engineering applications neither of these two conditions is fulfilled. Multiaxial stress fields prevail, except for rare cases of bars in pure push, pull or bending. Even if loads are constant in time, relaxation phenomena will cause stresses to vary.

Hence a designer needs to generalize knowledge relating to time independent and uniaxial stresses, to cases of time dependent and multiaxial stresses. In so doing he must rely largely on analogies with similar situations relating to elastic or plastic behaviour. The complexity arising for all but non trivial stress fields must be balanced by simplifying assumptions, or otherwise no design calculations would be feasible. The difficulties are not primarily mathematical, but rather mechanical in nature. What the designer needs is a workable mechanical model, which describes certain essential creep properties but leaves out the rest. Once such a model has been developed, its application to design tasks will only be a matter of applying known principles of structural mechanics and of performing numerical calculations.

The main part of this paper deals with basic properties of a unified mechanical model for creep deformation and creep rupture.

2. Ductile rupture mechanism

In a standard creep rupture test a uniaxial specimen is subjected to a constant load P, and the time t_R to rupture is recorded. With A_0 denoting the cross sectional area before load application, the quantity

$$\sigma_0 = P/A_0 \qquad (1)$$

will be denoted the nominal stress. Results of creep rupture tests are usually presented in log σ_0-log t_R diagrams, and a typical appearance is as shown by Fig. 1. Commonly the rupture behaviour is predominantly ductile at higher stresses and brittle at lower stresses.

An engineering theory describing ductile creep rupture in a tensile bar in terms of creep deformation parameters was proposed by Hoff [1]. He assumed a creep law in the form

$$d\varepsilon/dt = F(\sigma) \tag{2}$$

where ε denotes the logarithmic or natural strain

$$\varepsilon = \ln L/L_0 \tag{3}$$

and σ denotes the true stress

$$\sigma = P/A \tag{4}$$

Here L_0 and L denote the initial and current lengths of the bar, and A denotes its current cross sectional area. If no volume changes take place during creep deformation, i.e. $AL=A_0L_0$, eq:s (1), (3) and (4) yield

$$\sigma = \sigma_0 \exp \varepsilon \tag{5}$$

This relation holds independent of the mechanical properties of the bar material, i.e. independent of the form of the function $F(\sigma)$.

Taking $F(\sigma)$ as a power function

$$F(\sigma) = B\sigma^n \tag{6}$$

Hoff [1] combined eq:s (2) and (5) to find the stress history $\sigma(t)$ in the bar under constant load P. The stress increases at an accelerating rate, and $d\sigma/dt\rightarrow\infty$, implying rupture, when $t\rightarrow t_R$, where

$$t_R = 1/(nB\sigma_0^n) \tag{7}$$

This relation between the ductile creep rupture time t_R and the nominal stress σ_0 corresponds to a straight line with slope $1/n$ in the diagram of Fig. 1.

Improving on this analysis Odqvist [2] added a term to eq. (2) corresponding to an instantaneous strain response

$$\varepsilon = G(\sigma) \tag{8}$$

resulting in the extended creep law

$$d\varepsilon/dt = (d/dt)\ G(\sigma) + F(\sigma) \tag{9}$$

With a loading history according to Fig. 2 then follows for the phase 0-1 from eq:s (5) and (8)

$$\frac{d\sigma}{d\sigma_0} = \frac{1/\sigma_0}{1/\sigma - G'(\sigma)} \tag{10}$$

and for the phase 1-2 from eq:s (5) and (9)

$$\frac{d\sigma}{dt} = \frac{F(\sigma)}{1/\sigma - G'(\sigma)} \tag{11}$$

Hence ductile rupture will occur during load application ($d\sigma/d\sigma_0\rightarrow\infty$) or during subsequent creep phase ($d\sigma/dt\rightarrow\infty$) when $\sigma\rightarrow\sigma_R$, where

$$1/\sigma_R - G'(\sigma_R) = 0 \tag{12}$$

The resulting relation between σ_0 and σ is shown in Fig. 3. The true rupture stress σ_R, given by eq. (12), is a constant, same for all loading histories $\sigma_0(t)$.

This close connection between ductile instantaneous rupture and ductile creep rupture was observed by Carlson [3], who proposed to denote ductile creep rupture as "creep-induced tensile instability". In the next paragraph a similar unified description of brittle instantaneous rupture and brittle creep rupture will be shown.

3. Brittle rupture mechanism

In order to describe brittle creep rupture in mechanical terms Kachanov [4] and Rabotnov [5] introduced the concepts of damage and net stress. The following redefinitions were proposed by Broberg [6].

Continuing creep deformation causes a progressing deterioration of the material, resulting eventually in accelerating deformation. This deterioration can conveniently be expressed in terms of a decreasing load carrying area or net area A_n. For incompressible deformation eq. (3) defines strain as

$$\varepsilon = \ln A_0/A \tag{13}$$

In analogy to this Broberg [6] defines damage as

$$\omega = \ln A/A_n \tag{14}$$

Likewise, in analogy to eq. (4), the net stress is defined as

$$s = P/A_n \tag{15}$$

From eq:s (1), (13)-(15) follows

$$s = \sigma_0 \exp (\varepsilon+\omega) \tag{16}$$

which is an extension of (5) to include also the effect of damage. It is valid independent of the mechanical properties of the bar material.

Using a slightly different definition of damage Kachanov [4] proposed a law of the form

$$d\omega/dt = f(s) \tag{17}$$

as the governing equation for damage creation. Neglecting creep strain altogether and taking f(s) as a power function

$$f(s) = Cs^{\nu} \tag{18}$$

he derived an expression for the brittle creep rupture time, which corresponds to a straight line with slope $1/\nu$ in the diagram of Fig. 1.

In analogy to the extension made by Odqvist [2], Broberg [6] added a corresponding term

$$\omega = g(s) \tag{19}$$

to eq. (17), resulting in the extended damage law

$$d\omega/dt = (d/dt)\ g(s) + f(s) \tag{20}$$

Replacing σ by s the creep law eq. (9) was restated as

$$d\varepsilon/dt = (d/dt)\ G(s) + F(s) \tag{21}$$

Eq:s (16), (20) and (21) completely define the histories of strain $\varepsilon(t)$ and damage $\omega(t)$ resulting from any prescribed loading history $\sigma_0(t)$.

With the loading history given in Fig. 2 there follows for phase 0-1

$$\frac{ds}{d\sigma_0} = \frac{1/\sigma_0}{1/s-G'(s)-d'(s)} \tag{22}$$

and for phase 1-2

$$\frac{ds}{dt} = \frac{F(s) + f(s)}{1/s-G'(s)-g'(s)} \tag{23}$$

These expressions are analogies to eq:s (10) and (11). They imply that rupture, of a mixed ductile and brittle nature, will occur during load application ($ds/d\sigma_0 \to \infty$) or during a subsequent creep phase ($ds/dt \to \infty$) when $s \to s_R$, where

$$1/s_R - G'(s_R) - g'(s_R) = 0 \qquad (24)$$

The resulting relation between σ_0 and s is shown in Fig. 4. The net rupture stress s_R, given by eq. (24), is a constant, same for all loading histories $\sigma_0(t)$. If no damage occurs, i.e. $\omega \equiv 0$, the relations of the preceding paragraph are regained. If g'(s) is a non decreasing function, it follows from eq:s (12) and (24) that $s_R < \sigma_R$, implying also that rupture occurs at a smaller strain in the case of deteriorating material. The strain history predicted by this unified theory contains a terminating phase of tertiary creep, which is in agreement with most creep test results.

If no strain occurs, i.e. $\varepsilon \equiv 0$, relations describing purely brittle rupture result. With $G(s) \equiv 0$ and $g(s) \equiv 0$ eq:s (22) and (23) become

$$\frac{ds}{d\sigma_0} = \frac{1/\sigma_0}{1/s - g'(s)} \qquad (25)$$

$$\frac{ds}{dt} = \frac{f(s)}{1/s - g'(s)} \qquad (26)$$

in complete similarity with eq:s (10) and (11) respectively. Hence brittle creep rupture may be termed "damage-induced tensile instability" to mark its close resemblance to ductile creep rupture. The mixed instability modes predicted by eq:s (22) and (23) respectively include the purely ductile and purely brittle modes as limiting cases.

4. Rupture under time variable load

It is of interest to study the predictions of time to rupture given by eq:s (22) and (23) for various loading histories. To simplify the discussion the effects of instantaneous strain and damage will be neglected, i.e. $G(s) \equiv 0$, $g(s) \equiv 0$. Then eq. (22) yields

$$ds/d\sigma_0 = s/\sigma_0 \qquad (27)$$

for instantaneous load changes. If a constant load $\bar{P}$ is applied instantaneously, corresponding to a nominal stress $\bar{\sigma}_0$, eq. (27) yields the initial net stress $s(0) = \bar{\sigma}_0$. The net rupture stress, according to eq. (24) is $s_R = \infty$. Hence eq. (23) yields the corresponding rupture time

$$\bar{t}_R = \int_{\bar{\sigma}_0}^{\infty} \frac{ds}{s[F(s)+f(s)]} \qquad (29)$$

To each load $\bar{P}$ eq. (29) relates a rupture time $\bar{t}_R$. If $\bar{P} = \bar{P}(t)$ then, formally, $\bar{t}_R = \bar{t}_R(t)$. The corresponding rupture time is denoted t_R^*. It is of interest, then, to calculate the magnitude of the integral

$$J = \int_0^{t_R^*} dt/t_R(t) \qquad (30)$$

Such an analysis was performed by Hult [7] for purely ductile deformation, i.e. with $f(s)\equiv 0$. Following the same procedure, assuming the forms (6) and (18) for $F(s)$ and $f(s)$ respectively, we find for the loading histories of Fig. 5a and b

	step up	step down
$B=0$ or $C=0$ or $\nu=n$	$J = 1$	$J = 1$
$B\neq 0$ and $C\neq 0$ and $\nu\neq n$	$J < 1$	$J > 1$

Since $B\neq 0$ and $C\neq 0$ and $\nu\neq n$ for most alloys of engineering interest, the life fraction rule $J=1$, postulated by Robinson [8], is usually not fulfilled. The prediction $J<1$ for step up loading, and $J>1$ for step down loading, is in full agreement with experimental results by Marriott & Penny [9].

5. Rupture under multiaxial stresses

General studies of conditions for ductile instability under multiaxial stresses have been made by Storåkers [10] and others. Early applications to ductile creep rupture in pressure vessels are due to Rimrott [11]. Generalizations of the original Kachanov [4] and Rabotnov [5] damage models to multiaxial stress states have been considered by Hayhurst and Leckie [12] and Leckie and Hayhurst [13]. Here a similar generalization of the damage model in the previous two paragraphs will be discussed.

The following assumptions are made:

1) A net stress tensor is defined in analogy to the scalar net stress in the uniaxial case. According to eq:s (5) and (16)

$$s = \sigma \exp \omega \tag{31}$$

In analogy to this the following relation is postulated between the components of the net stress tensor s_{ij} and the true stress tensor σ_{ij}

$$s_{ij} = \sigma_{ij} \exp \omega \tag{32}$$

Hence damage is here assumed to be isotropic, which is in certain disagreement with experimental observation, but which simplifies the modelling to a very large extent.

2) A scalar governing equation for damage creation is postulated, which degenerates to eq. (20) for uniaxial stress. With an effective net stress s_e defined by

$$s_e = \sigma_e \exp \omega \tag{33}$$

where σ_e is an effective true stress, the damage law is assumed as

$$d\omega/dt = (d/dt)\ g(s_e)_t + f(s_e) \tag{34}$$

3) A creep law is postulated, which degenerates to eq. (21) for uniaxial stress, and which satisfies the standard requirements of incompressibility and isotropy. With $\tilde{\sigma}_{ij}$ denoting the true stress deviator, and hence

$$\tilde{s}_{ij} = \tilde{\sigma}_{ij} \exp \omega \tag{35}$$

denoting the net stress deviator, the creep law is assumed as

$$d\varepsilon_{ij}/dt = (3/2)(s_e)d\tilde{s}_{ij}/dt + (3/2)F(s_e)\tilde{s}_{ij}/s_e \qquad (36)$$

The definition of the effective stress σ_e in terms of the components of σ_{ij} may be chosen to fit experimental results, cf. Hayhurst [14]. For a thin-walled spherical pressure vessel, which will be studied here, the distinction between σ_{max}, σ_e (Mises) and σ_e (Tresca) disappears. The creep law (36) may then be stated and used in its scalar form

$$d\varepsilon_e/dt = (d/dt)\ G(s_e) + F(s_e) \qquad (37)$$

Denoting the radius and wall thickness by R and h respectively, the nominal hoop stress is $\sigma_{\phi 0}=pR_0/2h_0$ with p denoting the internal pressure. Incompressibility requires $R_0^2h_0=R^2h$ and hence

$$\sigma_\sigma = pR/2h = \sigma_{\phi 0}(R/R_0)^3 = \sigma_{\phi 0}\ \exp\ 3\varepsilon_\phi \qquad (38)$$

with $\varepsilon_\phi = \ln R/R_0$ denoting the natural hoop strain. From eq:s (31) and (38) follows, cf. eq. (16)

$$s_\phi = \sigma_{\phi 0}\ \exp\ (3\varepsilon_\phi + \omega) \qquad (39)$$

For the sphere we have $\sigma_e=\sigma_\phi$ and $\varepsilon_e=2\varepsilon_\phi$ and hence

$$s_e = \sigma_{\phi 0}\ \exp\ (3\varepsilon_e/2 + \omega) \qquad (40)$$

This combines with eq:s (34) and (37) to give the differential equation

$$(ds_e/dt)(1/s_e - 3G'/2 - g') - 3F/2 - f = (1/p)\ dp/dt \qquad (41)$$

with the rupture criterion

$$1/s_{eR} - 3G'(se_R)/2 - g'(s_{eR}) = 0 \qquad (42)$$

If no instantaneous effects are present, i.e. $G\equiv 0$ and $g\equiv 0$, then $s_{eR}=\infty$, and

$$t_R = \int_{\sigma_{\phi 0}}^{\infty} \frac{ds_e}{s_e[3F(s_e)/2+f(s_e)]} \qquad (43)$$

Comparison with eq. (29) shows that the purely brittle rupture time ($F\equiv 0$) is given by the same form as for uniaxial tension, whereas the purely ductile rupture time ($f\equiv 0$) is shorter by a factor 2/3. Further studies of these results are under way.

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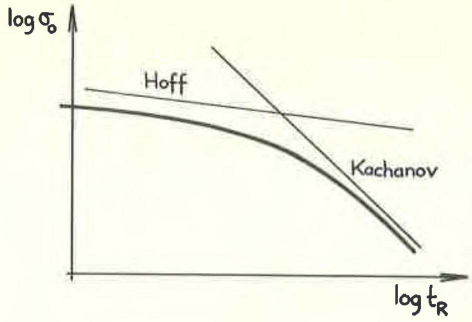

Fig. 1 Creep rupture curve and predictions for purely ductile (Hoff) and purely brittle (Kachanov) rupture.

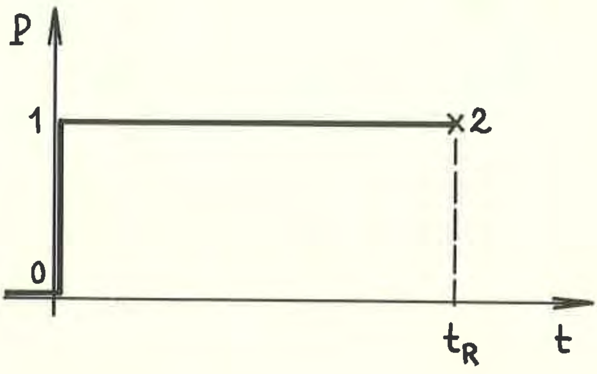

Fig. 2. Instantaneous application of constant load.

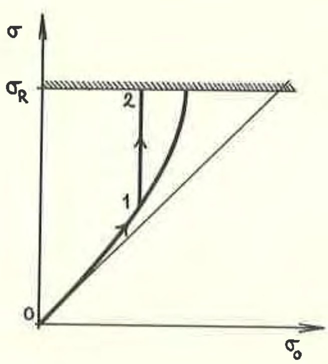

Fig. 3. Real stress σ during instantaneous load application (0-1) and during creep at constant load (1-2).

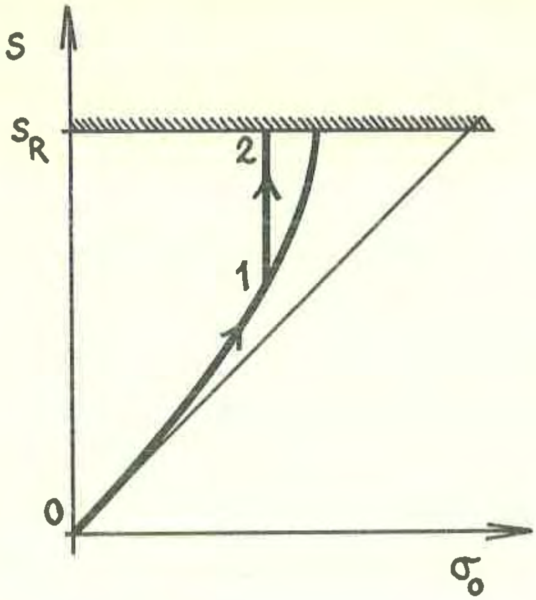

Fig. 4. Net stress s during instantaneous load application (0-1) and during creep at constant load (1-2).

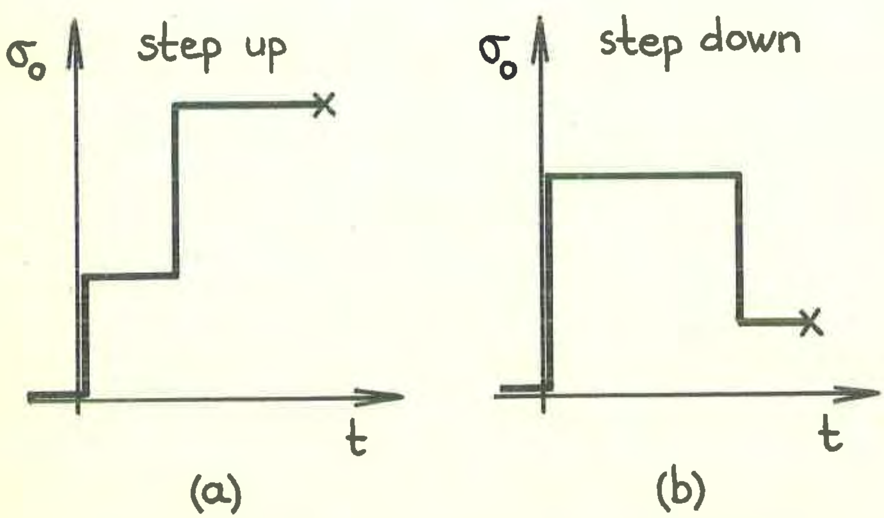

Fig. 5. Step up (a) and step down (b) loading programmes.