CREEP RUPTURE OF STRUCTURES SUBJECTED TO VARIABLE LOAD

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SUMMARY

The effects of variable load and temperature upon the performance of engineering components are becoming of increasing interest to structural designers. In the past considerable emphasis has been placed upon the development of methods for the design of components which operate under steady load and isothermal conditions. Of the different techniques which have been employed two of them have been the most widely used, these are the time-iterative computer procedures and the more simple bounding methods. The iterative computer methods, which have been applied to a wide range of two dimensional situations, tend to require both an accurate representation of material behaviour and considerable computer power. The bounding methods, which require a less precise description of material behaviour, have the advantage of providing a rapid assessment of structural performance which is often useful at the conceptual stages of design. Because of this considerable emphasis has been given to the development of approximate methods.

In this paper an approximate method is derived which enables a lower bound on the rupture life to be obtained for kinematically determinate structures subjected to variable load and isothermal conditions. The bound on the rupture life is expressed in terms of the energy dissipation rates within the structure corresponding to steady-state creep. The method has the advantage that, for a large number of structures, bounds on the rupture lives can be easily obtained from existing steady-state solutions. The effect of multi-axial stress creep-rupture upon the structural performance is examined and bounds are derived for materials which obey maximum tension and octahedral shear stress criteria. For both multi-axial stress rupture laws and structures subjected to variable and reverse load conditions formulae are derived which express the lower bound rupture life in terms of the behaviour of a steady-load uni-axial creep rupture test. The rupture life of the structure can then be determined from the calculated representative rupture stress for the structure and uni-axial deformation and rupture data for the material.

The results of experiments which have been carried out on copper and aluminium beams, tested at $250 \pm 1/2$°C and $210 \pm 1/2$°C respectively, are presented for variable and reverse load conditions. The copper and aluminium alloys were selected to represent materials which obey maximum tension and octahedral shear stress rupture criteria respectively. For both rupture laws the experimental values of the rupture times are closely predicted by the representative rupture stresses and uni-axial data. Statement are made concerning the rupture of materials under uni-axial compressive stresses.

1. Introduction

Of the different techniques used to predict the creep rupture life of structures subjected to steady loads under isothermal conditions two approaches have been most widely used, these are the time-iterative computer procedures and approximate bounding methods. The computer methods [1-2] which have been applied to relatively simple two dimensional problems tend to require both an accurate representation of material behaviour and considerable computer power. Bounding techniques [3-4], which are less sensitive to the description of the material behaviour, have the advantage of providing a rapid assessment of structural behaviour.

Since few structures operate under steady conditions it is the behaviour of components which are subjected to variable loads and temperatures which are of most interest. Little work has been carried out in this area due to the difficulties in the formulation of constitutive relationships. In this paper constitutive relationships are proposed and the rupture behaviour of kinematically determinate structures are studied for variable loading and isothermal conditions. The theoretical results are used to predict the life of a beam of rectangular cross-section subjected to uniform bending moments. The results of experiments carried out on beams subjected to variable and reverse moments are shown to verify the approximate prediction method.

2. Material Representation

The expressions which are used to describe the creep strain rates are those due to Kachanov [5] and Rabotnov [6]. In order to describe tertiary creep behaviour Kachanov introduced the damage parameter ψ which he used in a phenomenological sense to represent the degree of cracking or rupture. Leckie and Hayhurst [4] have presented a more rational justification for the use of the parameter ψ to represent the physical processes which take place during tertiary creep.

2.1 Steady Load

The constitutive and damage relationships are given by

$$\dot{v}_{ij}/\dot{v}_o = \psi^{-n}\,\phi^n(\sigma_{ij}/\sigma_o)\;\partial\phi/\partial(\sigma_{ij}/\sigma_o), \tag{1}$$

$$\dot{\psi} = -\,A\psi^{-\nu}\Delta^{\nu}\,(\sigma_{ij}/\sigma_o), \tag{2}$$

where ϕ and Δ are homogeneous functions of degree one in (σ_{ij}/σ_o) and A, n, ν, $\dot{v}_o$ and σ_o are material constants. When ψ=1 the material is undamaged and eq.(1) reduces to the steady-state relationship. For uni-axial stress eq.(2) can be integrated for the conditions ψ=1, t=0 and ψ=0 at the failure time t_R to give the following experimentally observed relationship between stress and the rupture life : $t_R = 1/(A(1+\nu))\Delta^{\nu}(\sigma_{ij}/\sigma_o)$. Isochronous rupture surfaces in multi-axial stress space are given by $\Delta(\sigma_{ij}/\sigma_o)$ = constant. The surfaces can take several forms and it has been shown [7] that the extreme types of behaviour are given by $\Delta = \phi$, which represent a maximum octahedral shear stress condition, and Δ = maximum tension stress. Since the deformation behaviour of both types of materials is given by eq.(1) it is convenient to refer to materials which satisfy the maximum shear stress rupture criterion as ϕ-ϕ materials and those which obey the maximum tension stress criterion as ϕ-Δ materials.

2.2 Variable Load

The majority of variable load creep tests have been carried out under uni-axial stresses. For these conditions the creep strain rates can be expressed by the uni-axial steady-state form of eq.(1) multiplied by the time function h(t). Leckie and Ponter [8] have shown that for cyclic applications of stress the time function h(t) is cyclic. The existence of a cyclic state has been shown in the uni-axial tests carried out by Adén et al [9] on creep resistant steels. Some of the tests were carried through to rupture but whilst the results showed an interralation between strain rate and rupture life, the scatter in the rupture data was too great to enable firm conclusions to be made.

The results of variable load uni-axial tests carried out by Goldhoff [10] show that the rupture life can be closely predicted by $\Sigma(t_i/t_{ir}) = 1$, where t_i is the time which the specimen is subjected to the stress σ_i and t_{ir} is the rupture life at the same stress. Values of the rupture life predicted by this method usually show a small error but when these differences are interpreted in terms of an error in the design stress the predictions are remarkably good. The life fraction rule has been shown [11] to correspond to the Kachanov type of expression given by eq.(2).

Eq.(1) can be rewritten to include the time function h(t), but since this does not influence the results obtained later in the paper the relationship given by eq.(1) will be used. In this work the effects of high-temperature fatigue are assumed to be unimportant. This assumption is most likely to be satisfactory provided the peak operating stresses are less than the current high-temperature yield stress of the material.

3. Structural Idealisations

In a structure which is subjected to cyclic loading stress redistribution occurs which is due to two effects. The first is due to the stress distribution changing from the initial elastic value to a stationary or cyclic state of stress and the second is due to the elastic effects which occur during the variable loading. These effects have been examined for a wide range of structures subjected to variable loading [12] and their contribution to the total energy dissipated within the structure has been shown to be small. The same assumptions are made in this paper and in addition it will be assumed that the cyclic state of stress, normally associated with steady-state creep, will be changed from cycle to cycle by the accumulation of the damage parameter ψ. It will be assumed that the cyclic state of stress, in the absence of rupture, is given by the steady-state stress distribution which is in equilibrium with the current applied loads. The analysis of a kinematically determinate structure subjected to cyclic applied loads is now examined for a body which undergoes material deterioration.

4. Rupture life of a kinematically determinate structure

Consider a structure in which the displacement rates are zero over the part A_u of the surface and over the remaining area A_p the loads are given by $P_i\, g(t)$, where g(t) is a function of time used to describe the loading cycle. With the passing of time deterioration takes place and at the time t_I local rupture occurs, a rupture zone [1] then spreads throughout the structure until complete failure occurs at the time t_R. The behaviour of a kinematically determinate structure is now examined for materials which are of the ϕ-ϕ and ϕ-Δ types.

Initially when the structure is undamaged and ψ is everywhere unity the stress distribution is σ^s_{ij} and the strain rate distribution $\dot{v}^s_{ij}$ is that corresponding to the steady-state stresses. Since the structure is kinematically determinate the displacement and strain rates are given by

$$\dot{u} = \dot{u}_s\ f(t) \quad , \qquad \dot{v}_{ij} = \dot{v}^s_{ij}\ f(t) \tag{3}$$

where f(t) is a function of time $\dot{u}_s$ are the steady-state displacement rates. The rate of dissipation of viscous energy per unit volume is given as

$$\sigma_{ij}\ \dot{v}_{ij} = \{\psi\sigma_o\Omega^{1/n}(\dot{v}_{ij}/\dot{v}_o)\partial\Omega/\partial(\dot{v}_{ij}/\dot{v}_o)\}\dot{v}_{ij} = \psi\dot{D}_s\ f^{(n+1)/n}(t), \tag{4}$$

where Ω is a convex homogeneous function of degree one in strain rate $\dot{v}_{ij}/\dot{v}_o$ and $\dot{D}_s$ is the steady-state energy dissipation rate. The rate of external work is given by

$$\int_{A_p} P_i g(t)\dot{u}_i dA = g(t)\ f(t)\int_{A_p} p_i\dot{u}^s_i dA, \tag{5}$$

and by equating internal and external energy rates f(t) can be expressed as

$$f^{1/n}(t) = g(t)\int\dot{D}_s dV/\int\psi\dot{D}_s dV = g(t)/\Psi, \tag{6}$$

where $\Psi = \int\psi\dot{D}_s dV/\int\dot{D}_s dV$ is the global damage. By expressing the stresses in terms of the convex function Ω, used in eq.(4), the following equation can be written for plane stress conditions:

$$\sigma_{ij}/\psi\sigma_o = g(t)\sigma^s_{ij}/\Psi\sigma_o. \tag{7}$$

In the same way the energy dissipation rates can be expressed by

$$\dot{D} = \psi\sigma_o\dot{v}_o\ \phi^{n+1}(\sigma_{ij}/\psi\sigma_o) = \dot{D}_s\ \psi\ g^{(n+1)}(t)/\Psi^{(n+1)}. \tag{8}$$

The function ϕ can be expressed in terms of the global damage by

$$\phi(\sigma_{ij}/\psi\sigma_o) = g(t)\ (\dot{D}_s/\sigma_o\dot{v}_o)^{1/(n+1)}/\Psi, \tag{9}$$

and from eq.(7) the function Δ can be written

$$\Delta(\sigma_{ij}/\psi\sigma_o) = g(t)\ \Delta(\sigma^s_{ij}/\sigma_o)\ /\ \Psi. \tag{10}$$

5. Structural Life for ϕ-ϕ materials

Substitution of eq.(9) into the ϕ-ϕ form of eq.(2) gives an expression which, after multiplication of both sides by $(\dot{D}_s/\sigma_o\dot{v}_o)/\int\dot{D}_s/\sigma_o\dot{v}_o dV$, becomes:

$$d[(\psi\dot{D}_s/\sigma_o\dot{v}_o)/\int(\dot{D}_s/\sigma_o\dot{v}_o)dV]/dt = -Ag^{\nu}(t)\ \frac{(\dot{D}_s/\sigma_o\dot{v}_o)^{(n+1+\nu)/(n+1)}}{\Psi^{\nu}\int(\dot{D}_s/\sigma_o\dot{v}_o)dV}\ . \tag{11}$$

For times less than t_I both sides of this expression are integrated over the volume to give

$$d[\Psi^{\nu+1}/(\nu+1)]/dt = -Ag^{\nu}(t)\int(\dot{D}_s/\sigma_o\dot{v}_o)^{(n+1+\nu)/(n+1)}dV/\int(\dot{D}_s/\sigma_o\dot{v}_o)dV. \tag{12}$$

For times in excess of t_I the first volume integral on the right hand side of the expression is carried out over the volume $\bar{v}$ for which $\psi > 0$. Since the determination of $\bar{v}$ requires a complete solution use will be made of the observation that eq.(12) gives an upper bound on the rate of change of $\Psi^{\nu+1}$ for times greater than t_I. Eq.(12) can then be integrated to obtain a lower bound on the rupture life of the structure. Eq.(12) will now be integrated for the loading cycles shown in Fig. 1.

5.1 Steady load

For steady load conditions, given by g(t) = 1, eq.(12) can be integrated using the initial and final conditions Ψ=1, t=0 and Ψ=0, t=t_R respectively to give the following lower bound on the rupture life:

$$t_R \geq \int(\dot{D}_s/\sigma_o\dot{v}_o)dV/A(1+\nu)\int(\dot{D}_s(\sigma_o\dot{v}_o)^{(n+1+\nu)/(n+1)}dV \,, \qquad (13)$$

which was obtained by Leckie and Hayhurst [4].

5.2 Reverse Load

For a symmetrical reverse load cycle the time function g(t) is defined by:

$$g(t) = +1 \quad \text{for } \tau - \delta\tau < t < \tau$$

$$g(t) = -1 \quad \text{for } \tau < t < \tau + \delta\tau.$$

Eq.(12) can then be integrated over the part of the cycle when g(t) is positive to give:

$$\{\Psi^{\nu+1}(\tau) - \Psi^{\nu+1}(\tau-\Delta\tau)\} = \theta\,\delta\tau, \qquad (14)$$

where $\theta = -A(\nu+1)\int(\dot{D}_s/\sigma_o\dot{v}_o)^{(n+1+\nu)/(n+1)}dV/\int(\dot{D}_s/\sigma_o\dot{v}_o)dV$. A similar expression for the part of the cycle when g(t) is negative can be derived as:

$$\{\Psi^{\nu+1}(\tau+\delta\tau) - \Psi^{\nu}(\tau)\} = \theta\Delta\tau. \qquad (15)$$

Addition of eq.(14) and eq.(15) gives an expression for the accumulation of global damage over the complete cycle. If it is assumed that failure occurs after n complete cycles then:

$$\{-\Psi^{\nu+1}(0) + \Psi^{\nu+1}(2n\delta\tau)\} = 2\theta n\delta\tau = \theta t_R. \qquad (16)$$

On substitution of the condition $\Psi(0) = 1$ and $\Psi(t_R) = 0$ the lower bound on the rupture life is obtained:

$$t_R \geq \int(\dot{D}_s/\sigma_o\dot{v}_o)dV/A(\nu+1)\int(\dot{D}_s/\sigma_o\dot{v}_o)^{(n+1+\nu)/(n+1)}dV. \qquad (17)$$

Hence the lower bound lives for the reverse load cycle and the steady load case are identical.

5.3 Variable positive load

An expression for the lower bound rupture life for the variable loading history given in Fig. 1(b) can be obtained using the procedure outlined for the reverse load case. The lower bound rupture time is given by,

$$t_R \geq \{\int(\dot{D}_s/\sigma_o\dot{v}_o)dV/A(\nu+1)\int(\dot{D}_s/\sigma_o\dot{v}_o)^{(n+1+\nu)/(n+1)}dV\}2/(1+\alpha^{\nu}), \qquad (18)$$

when $\alpha=1$ and $\alpha=-1$ the rupture time given by eqs. (13) and (17) is obtained. When α is a small fraction, and ν has values in the range three to ten, then $(1+\alpha^{\nu})$ is approximately unity, for these conditions the rupture life is almost twice that corresponding to the equivalent steady load test.

6. Structural life for ϕ-Δ materials

Substitution of eq.(10) into eq.(2) gives, after some manipulation, an equation which is similar to eq.(12); the resulting expression can be integrated for the same conditions as eq.(12) to give the following results:

6.1 Steady load

$$t_R \geq \int(\dot{D}_s/\sigma_o\dot{v}_o)dV/A(\nu+1)\int\Delta^{\nu}(\sigma^s_{ij}/\sigma_o)(\dot{D}_s/\sigma_o\dot{v}_o)dV. \qquad (19)$$

6.2 Reverse load

$$t_R \geq 2\int(\dot{D}_s/\sigma_o/\dot{v}_o)dV/A(\nu+1)\int\Delta^{\nu}(\sigma^s_{ij}/\sigma_o)(\dot{D}_s/\sigma_o\dot{v}_o)dV. \qquad (20)$$

Care must be taken in the derivation of this result to ensure that in the volume integration of $\Delta^{\nu}(\sigma^s_{ij}/\sigma_o)$ the physical sense of the damage process is preserved. The rupture life is twice that calculated for the steady load case.

6.3 Variable positive load

$$t_R \geqslant \{\int (D_s/\sigma_o \dot{v}_o)dV/A(1+\nu)\int \Delta^{\nu}(\sigma^s_{ij}/\sigma_o)(D_s/\sigma_o \dot{v}_o)dV\}2/(1+\alpha^{\nu}). \qquad (21)$$

when α is a small fraction the rupture life is twice that of the corresponding steady load result.

7. Representative rupture stress

After comparison of the expressions for the structural rupture times obtained for ϕ-ϕ and ϕ-Δ materials for the different loading conditions with the integrated uni-axial form of the damage eq.(2) a uni-axial stress σ_R can be defined which has the same rupture life. This procedure has been followed for the cases examined and the results are presented in Table 1. It is now possible, with a knowledge of the multi-axial stress rupture criterion of the material and constant load uni-axial rupture data, to predict the life of a structure subjected to variable load from the steady-state stress distribution of the structure.

8. Experimental investigation

The structure selected for this investigation was a beam of rectangular cross-section subjected to uniform bending moments along its length. Aluminium and copper alloys were selected since they can be classified [7] as ϕ-ϕ and ϕ-Δ materials respectively. The constant load uni-axial rupture data for these materials has been presented elsewhere [4]. The tests were carried out at 210 ± 1°C and 250 ± 1°C for the aluminium and copper alloys respectively. The beams were 200mm. long, 12.7mm. wide and 25.4mm. deep and were tested in a large hot box in which the air was continuously circulated. The load changes were effected by means of automatically controlled motor driven screw jacks and deadweights. The average experimental rupture times are presented in Table 2, also included in the Table are the values of the uni-axial representative rupture stresses from which the computed rupture lives were obtained. Close agreement between experimental and predicted values can be observed. Probably the most striking result is that the life of the copper beam which had been subjected to the reverse moment is twice the life of the steady load beam.

9. Discussion

The experimental rupture lives of the aluminium beams which had been subjected to reverse loads were almost identical to the steady load values. This result verifies that the accumulation of damage in tertiary creep occurs almost equally in tensile and compressive states of stress. In the tests carried out on the reverse load copper beams the lives are twice those of the steady load tests. This result indicates that the accumulation of damage in tertiary creep occurs considerably more rapidly in tensile states of stress than in compressive stress states.

10. Conclusions

Lower bounds on the rupture life of kinematically determinate structures have been obtained for variable and reverse load situations for materials which obey maximum tension and octahedral shear stress criteria. The calculated results have been presented in terms of a representative rupture stress so that the structural behaviour can be determined from constant load uni-axial data, a knowledge of the multi-axial stress rupture criterion of the material and the steady-state stress distribution for the structure. The assumptions that the effects of stress redistribution are unimportant and that the constitutive laws are appropriate for variable load situations have been verified. The experimental results verify the assumptions that for the aluminium alloy damage occurs equally under compressive and tensile stress states and that for the copper alloy damage occurs at a significantly higher rate under tensile states of stress than under compressive states of stress.

REFERENCES

[1] HAYHURST, D.R. "Stress redistribution and rupture due to creep in a uniformly stretched thin plate containing a circular hole", Trans. ASME, J.App.Mech. Vol. 40, p.381, 1973.

[2] HAYHURST, D.R., LECKIE, F.A. "The effects of creep constitutive and damage relationships upon the rupture time of a solid circular torsion bar", J.Mech.Phys.Solids, Vol.21, p.431, 1973.

[3] MARTIN, J.B., LECKIE, F.A. "On time dependent failure of cracking structures," J.Mech.Phys. Solids, Vol. 20, p.223, 1972.

[4] LECKIE, F.A., HAYHURST, D.R. "Creep Rupture of Structures", Proc.R.Soc. Lond. A., Vol. 340, p.323, 1974.

[5] KACHANOV, L.M. "Time of the fracture process under creep conditions", Izv.Aknd.Nauk. USSR, Otd. Tekhn. Nauk. No. 8, p.26, 1958

[6] RABOTNOV, Y.N. "Creep Problems in Structural Members", (English Translation edited by Leckie, F.A.) North-Holland, Amsterdam, 1969.

[7] HAYHURST, D.R. "Creep rupture under multi-axial states of stress", J.Mech.Phys. Solids, Vol. 20, p.381, 1972.

[8] LECKIE, F.A., PONTER, A.R.S. "On the state variable description of creeping materials", Leicester University Engineering Department, Report 72-27, 1972.

[9] ALDÉN, G., ARONSSON, B., ROHLIN, L., "Creep of heat-resistant steels under variable stress", Swedish Inst. for Metals Research. December 1970.

[10] GOLDHOFF, R.M. "Uni-axial creep-rupture behaviour of low-alloy steel under variable loading conditions", Trans. ASME, J.Basic Eng., Vol. 87, p.374, 1965.

[11] ODQVIST, F.K.G., HULT, J., "Some aspects of creep rupture", Ark.Fys., Vol. 19, p.379, 1961.

[12] PONTER, A.R.S., WILLIAMS, J.J. "Work bounds and the associated deformation of cyclically loaded creeping structures", Jnl.App.Mech., Vol. 40, Trans. ASME, Vol. 95, Series E, p.921, 1973.

Table 1. Uni-axial representative rupture stresses for the cyclic loading cases defined in Fig. 1.

Load case	σ_R/σ_o $(\phi-\phi)$	σ_R/σ_o $(\phi-\Delta)$
Steady load	$\theta_*^{1/\nu}$	$\Phi_*^{1/\nu}$
Reverse load	$\theta_*^{1/\nu}$	$\Phi_*^{1/\nu}\{\frac{1}{2}\}^{1/\nu}$
Variable load	$\theta_*^{1/\nu}\{\frac{1+\alpha^\nu}{2}\}^{1/\nu}$	$\Phi_*^{1/\nu}\{\frac{1+\alpha^\nu}{2}\}^{1/\nu}$

$$\theta_* = \int(\dot{D}_s/\sigma_o\dot{v}_o)^{(n+1+\nu)/(n+1)}dV \;/\; \int(\dot{D}_s/\sigma_o\dot{v}_o)dV$$

$$\Phi_* = \int\Delta^\nu(\sigma_{ij}^s/\sigma_o)(\dot{D}_s/\sigma_o\dot{v}_o)dV \;/\; \int(\dot{D}_s/\sigma_o\dot{v}_o)dV$$

Table 2 Computed and experimental results for beams subjected to uniform bending moments.

Table 2(a) Copper Alloy (ϕ-Δ)			
Load cycle	Average Experimental life (hr.)	Computed life (hr.)	uni-axial stress σ_R MPa
Steady load	800	850	33.67
Reverse load 24 hr. cycle	1600	1700	29.74
Variable load 24 hr. cycle	221	245	39.50

Table 2(b) Aluminium Alloy (ϕ-ϕ)			
Load cycle	Average Experimental life (hr.)	Computed life (hr.)	uni-axial stress σ_R MPa
Steady load	730	650	57.41
Reverse load 24 hr. cycle	720	650	57.41
Variable load 24 hr. cycle	360	430	60.44

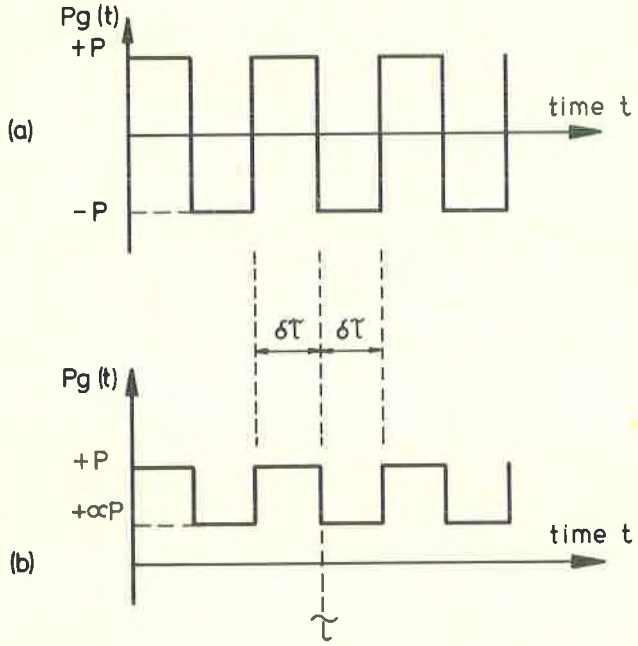

Fig 1. Definition of loading cycles