DYNAMIC PLASTIC BUCKLING OF RINGS AND CYLINDRICAL SHELLS

N. JONES, D.M. OKAWA

Department of Ocean Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, U.S.A.

SUMMARY

A theoretical analysis is developed to predict the dynamic plastic buckling of a long, impulsively loaded cylindrical shell. This theoretical work is used to examine various features of plastic buckling and to assess the importance of several approximations which previous authors have introduced in dynamic plastic buckling studies. In particular, the influence of a time-dependent circumferential membrane force, the sharpness of the peaks in the displacement and velocity amplification functions, the restrictions which are implicit when employing the Prandtl-Reuss equations in this class of problems, and the limitations due to elastic unloading are examined in some detail.

A summary of all previously published theoretical investigations known to the authors is undertaken for the dynamic plastic behavior of cylindrical shells and rings which are made from rigid-plastic, rigid-viscoplastic, elastic-plastic and elastic-viscoplastic materials and subjected to initial axisymmetric impulsive velocity fields. The theoretical predictions of the dominant motions, critical mode numbers, and threshold impulses are compared and critically reviewed.

An experimental investigation was also undertaken into the dynamic plastic buckling of circular rings ($L/R \simeq 0.21$) subjected to uniformly distributed external impulsive velocities. It appears that no experiments have been reported previously on mild steel cylindrical shells with an axial length (L) less than four times the shell radius (R). The experimental values of the average final radial deflections, critical mode numbers and dimensions of the permanent wrinkles in the mild steel and some aluminum 6061 T6 specimens are compared with all the previously published theoretical predictions and experimental results on cylindrical shells with various axial lengths.

It was found that the theoretical predictions of Perrone and Florence for the permanent dominant radial displacements of viscoplastic cylindrical shells or rings gave reasonable agreement with the experimental values of the average final radial displacements which were recorded on the hot rolled mild steel rings. Other theoretical results, which were derived using a strain rate insensitive material, were not suitable for predicting the response of the highly strain rate sensitive mild steels rings, as expected. The predictions of Perrone were also found to give good agreement with the aluminum 6061 T6 specimens when using the Cowper-Symonds viscoplastic material constants.

It appears that the experimental values of the critical mode numbers of the aluminum 6061 T6 rings are similar to the corresponding experimental results observed on the mild steel rings. It turns out that the simple theoretical prediction of Florence and Vaughan provides a reasonable engineering estimate of the critical mode numbers of all the mild steel and aluminum rings that were examined in the current experimental investigation. Furthermore, the threshold impulses predicted by Florence and Vaughan ensure that the permanent wrinkles of all the rings remain small.

A number of other results are presented which explore the influence of various parameters (including imperfections) and compare the predictions of all previously published theoretical analyses and experimental results from a number of laboratories.

1. Introduction

The stable dynamic plastic response of various basic structural members has been studied systematically over the past two decades so that today a reasonable level of understanding has been achieved which permits engineers to design a small class of structures with a fair degree of confidence [e.g. 1-4]. However, far less is known about the phenomenon of dynamic plastic buckling which may occur in a number of structures which are used frequently in engineering design. For example, large external dynamic pressures can produce dynamic plastic instability of cylindrical shells or rings. The wrinkled profiles of buckled cylindrical shells exhibit characteristic wavelengths which were found to be reproducible in experimental investigations. Abrahamson and Goodier [5] developed a theoretical procedure which predicted qualitative agreement with the corresponding experimental results. This analytical method assumed that buckling stemmed from the growth of small imperfections in the otherwise uniform initial displacement and velocity fields. A number of authors have extended Abrahamson and Goodier's work and the corresponding articles which are relevant to this particular paper are discussed later.

The theoretical behavior of a long cylindrical shell subjected to an axisymmetric impulse is considered in some detail in order to explore various limitations of the theoretical procedures. A review of relevant previous theoretical investigations is then undertaken. Some experimental results on aluminum 6061 T6 and hot rolled mild steel circular rings with $L/R \simeq 0.21$ which were subjected to axisymmetric impulsive velocities are then presented. This experimental study was undertaken because most previously published experimental results had been recorded on long cylindrical shells. No experiments have been reported previously on mild steel cylindrical shells with L/R less than four, although a few tests have been conducted on aluminum shells with $L/R \simeq 0.68$[1]. The results of these tests were thought to be useful because Vaughan and Florence [7] observed in their theoretical work that dynamic plastic instability is sensitive to the axial length of cylindrical shells.

This paper concludes with comparisons between the theoretical predictions of various authors and the experimental results recorded on impulsively loaded cylindrical shells and rings which are reported herein and elsewhere.

2. Theoretical Behavior of a Long Cylindrical Shell

The dynamic response of a thin-walled circular cylindrical shell which is made from a rigid-linear strain hardening material and subjected to an almost axisymmetric external impulsive velocity field is considered in reference [8]. It transpired that simple analytical solutions were obtained for the particular case of a long cylindrical shell even when a two-dimensional constitutive equation was used. This permitted an examination of various features of plastic buckling and allowed the importance of several approximations normally used in dynamic plastic buckling analyses to be assessed.

It was indicated that the velocity amplication function is in general of order H/R times the displacement amplification function. Thus, the dynamic plastic buckling of a cylindrical shell is more sensitive to initial imperfections in the profile than to imperfections in the initial velocity field.

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[1] Al-Hassani [6] has recently reported some experimental results on aluminum alloy tubes with $L/R \simeq 0.25$.

An examination of the sharpness of the peak of the displacement amplification function revealed that some scatter might be anticipated in the values of the critical mode numbers observed in experiments. Generally speaking, it appears that the greatest amount of scatter might be expected to occur in the experimental critical mode numbers of long cylindrical shells with large values of R/H and/or small values of the material parameter β_1.

Local unloading of a cylindrical shell is not catered for in the analyses presented in reference [8] and in references [5,7 etc]. The circumstances under which this condition occurs are examined in reference [8] and the simple expression

$$\frac{\overline{w}_f}{R} \leq \frac{w^*H}{4R^2}\left(n^{c^2} - 1\right) \tag{1}$$

is suggested when the initial imperfections are unknown. Eq. (1) was derived from the inequality $H K_\theta^1/2 \geq \overline{\varepsilon}_\theta$. It appears when the expression for the critical mode number (n^c) is substituted in eq. (1) that unloading is more likely to occur for larger values of R/H and smaller values of β_1.

3. Summary of Previous Theoretical Studies on Cylindrical Shells and Rings

This section contains a brief summary of some previously published theoretical investigations on the dynamic plastic behavior of cylindrical shells and rings which were subjected to uniformly distributed external initial velocity fields. A more comprehensive summary is presented in reference [8], while discussed here are only those analyses which predict substantial agreement with the experimental results obtained by the writers.

3.1 Dominant Behavior

The dominant behavior is obtained by assuming that axisymmetric motion persists throughout the entire response of a cylindrical shell or ring.

The permanent radial displacement of an impulsively loaded rigid perfectly plastic cylindrical shell or ring is

$$\frac{\overline{w}_f}{H} = \frac{\lambda H}{2R} \tag{2}$$

when using a simple one-dimensional idealisation.

Perrone [9] showed that an analysis of a circular ring with a time-independent yield stress evaluated at the initial strain rate was considerably simpler than a more exact analysis with a time - dependent flow stress and entailed little sacrifice in accuracy. In this circumstance eq. (2) for a rigid viscoplastic material becomes

$$\frac{\overline{w}_f}{H} = \frac{\lambda' H}{2R} \tag{3}$$

according to Perrone's observation, where

$$\lambda' = \frac{\rho V_o^2 R^2}{\sigma_o' H^2} \tag{4}$$

and the dynamic yield stress σ_o' is given by the Cowper-Symonds constitutive equation [e.g. 10],

$$\sigma_o' = \sigma_o \left\{ 1 + \left(\frac{\dot{\varepsilon}}{D}\right)^{1/p} \right\} \tag{5}$$

with $\dot{\varepsilon} = V_o/R$ and where p and D are material constants which must be determined experimentally. Perrone [9] used his observations to simplify the finite-deflection analysis of an impulsively loaded circular ring which was made from a rigid viscoplastic material and described with a one-dimensional constitutive equation. The permanent radial displacement for this particular case is

$$\frac{\bar{w}_f}{H} = \frac{R}{H}\left\{ e^{\frac{\rho V_o^2}{2\sigma_o}} - 1 \right\} \qquad (6)$$

which reduces to the corresponding infinitesimal prediction of eq. (2) when $\frac{\lambda'}{4}\left(\frac{H}{R}\right)^2 \ll 1$.

3.2 Critical Mode Number

The critical mode number characterises the deformed profile of a buckled cylindrical shell or ring.

Florence and Vaughan [11] examined the behavior of short cylindrical shells and observed that the critical mode numbers of aluminum 6061 T6 shells with $E_t/\sigma_o \simeq 3$ and $15 < R/H < 23$ is approximately

$$n^c = (74)^{\frac{1}{4}}\left(\frac{R}{H}\right)^{\frac{1}{2}} \qquad (7)$$

3.3 Threshold Impulse

The concept of a threshold or critical impulse has been used by a number of authors to estimate the smallest external impulse that a structure can tolerate without excessive permanent deformation. This definition is clearly somewhat arbitrary but is nevertheless an important concept from a design viewpoint.

The theoretical procedure of Florence and Vaughan [11] when specialized to short aluminum 6061 T6 cylindrical shells with $E_t/\sigma_o = 3$ indicated that at the cessation of dominant motion an impulse

$$I^T = 2.5R(\rho\sigma_o)^{1/2}(H/R)^{3/2} \qquad (8)$$

would have amplified by a factor of one hundred any initial displacement imperfections having the same mode number as the associated critical mode number (n^c). However, Florence and Vaughan's experimental results showed that

$$I^T = 1.3R(\rho\sigma_o)^{1/2}(H/R)^{3/2} \qquad (9)$$

produced wrinkles with amplitudes having a small fraction of the wall thickness. The initial displacement imperfections in the critical mode would be amplified approximately six times for an impulse given by equation (9) according to Florence and Vaughan's theoretical work. Unfortunately, neither the initial imperfections nor the final deformed profiles of the experimental models were reported in reference [11]. Equations (8) and (9) can be written in the non-dimensional form

$$\lambda^T \frac{H}{R} = 6.25, \quad \text{and} \quad \lambda^T \frac{H}{R} = 1.69 \qquad (10),(11)$$

respectively, where

$$\lambda^T = \frac{\rho R^2 (V_o^T)^2}{\sigma_o H^2} \qquad (12)$$

4. Experimental Results

The general experimental technique which was used to impulsively load the circular rings examined herein is similar to that employed by several authors for various structural geometries and is ciscussed in some detail in reference [12] to which an interested reader is referred.

The circular rings (R $\simeq$ 1.75 in.) had small axial lengths (L $\simeq$ 0.375 in.) and were subjected to uniformly distributed axisymmetric initial velocity fields with the aid of sheet explosive. The rings were made from either hot rolled mild steel or aluminum 6061 T6 and the wall thicknesses ranged from 0.04 in. to 0.175 in.. Further details of the experimental techniques, dimensions of the specimens and material properties are presented in reference[8].

5. Discussion

The experimental results for W_f/H which were reported in reference [8] are plotted in Figure 1 with the non-dimensional parameter $\lambda H/R$ as an abscissa and compared with various theoretical predictions. The theoretical predictions[2] of equations (3), (6) and Florence[13] for a viscoplastic material with $D = 40.4$ sec^{-1} and $p = 5$ are quite similar and, except for the thinnest specimens, give reasonable agreement with the experimental results which were recorded on the hot rolled mild steel rings. Eq. (2) and the analyses in references [5, 7 and 14] were developed for strain rate insensitive materials and therefore are not suitable for predicting the response of the highly strain rate sensitive mild steel rings. Nevertheless, for illustrative purposes, the theoretical predictions of Vaughan and Florence [7] for the behavior of the mild steel specimens are included in Figure 1. These theoretical values are larger than the experimental results for the mild steel rings and the various viscoplastic predictions as expected. However, they indicate a sensitivity to the ring thickness (H) which is also present but to a lesser degree in the experimental values. Furthermore, the results of Vaughan and Florence exhibit a marked sensitivity to the magnitude of the tangent modulus (E_t), a parameter which does not enter into any of the viscoplastic predictions.

It is evident from Figure 2 that eq. (2) and the predictions of references [5, 7, 14] do not agree with the corresponding experimentsl results on the aluminum 6061 T6 rings. The discrepancy also remains large when using the theoretical predictions of eqs. (3), (6) and Florence [13] for a viscoplastic material with the material constants $D = 1288000$ sec^{-1} and $p = 4$ which are suggested in reference [15] for aluminum 6061 T6. It appears, therefore, that the aluminum 6061 T6 rings tested herein have a significant strain rate sensitivity though less than that for hot rolled mild steel. However, it is emphasised in reference [15] that insufficient and contradictory experimental data exists on the strain rate sensitive properties of aluminum 6061 T6 which may also be sensitive to various quantities which have not been monitored in many experimental programs. Nevertheless, it is evident from Figure 2 that eq. (3) gives much better agreement with the experimental results when using the material constants $D = 6500$ sec^{-1} and $p = 4$ which were suggested by Cowper and Symonds [e.g. 10] nearly two decades ago.

It is evident from Figures 1 and 2 that the simple expression of eq. (3), which is based on eq. (2) and Perrone's observation [9] , provides reasonable agreement with the experimental results recorded on all rings[3], except the thinnest ones, and therefore might be particularly useful for design purposes. The theoretical predictions of Perrone [9] (eq.(6)), which includes the influence of finite-deflections, is marginally different from eq. (3) so that

[2] The numerical predictions of eqs. (3) and (6) do not lie exactly on lines 1 and 2 for all specimens but the scatter is very small.

[3] $D = 6500$ sec^{-1} and $p = 4$ are used for the aluminum 6061 T6 rings in this comparison.

the effect of finite-deflections is not significant over the range of deflections encountered in these experiments.

It appears from Figure 3 that the experimental values[4] of the critical mode numbers of the aluminum 6061 T6 rings are somewhat similar to the corresponding experimental results observed on the mild steel rings. The theoretical predictions of Abrahamson and Goodier [5] (E_t = 650,000 lb/in^2) and Florence and Vaughan [11] (eq. (7)) for a strain rate insensitive material provide reasonable engineering agreement with the mild steel experimental results although they do not predict the slight increase in critical mode number with increase in $\lambda H/R$ as suggested by the experimental values. It was observed by Wojewodzki [16] that eq.(7) could be used to predict the critical mode numbers of mild steel as well as aluminum alloy rings. However, for a given $\lambda H/R$, Abrahamson and Goodier [5] predict larger mode numbers for the aluminum 6061 T6 rings than the mild steel ones in contradistinction to the experimental values. Stuiver's [14] strain rate insensitive analysis predicts an increase of critical mode number with $\lambda H/R$ as suggested by the experimental results and the agreement with the thickest steel specimens is quite reasonable but the predictions otherwise are generally larger than the corresponding experimental values, particularly for the $H \simeq 0.06$ in. and $H \simeq 0.04$ in. specimens. Florence's [13] viscoplastic analysis predicts an increase of critical mode number with $\lambda H/R$, but the mode numbers are larger than Stuiver's for the $H \simeq 0.175$ in. and $H \simeq 0.10$ in. specimens and smaller than Stuiver's for the $H \simeq 0.04$ in. and $H \simeq 0.06$ in. rings.

Abrahamson and Goodier's [5] and Stuiver's [14] strain rate insensitive analyses predict critical mode numbers which are significantly larger than the corresponding experimental values when using E_t = 56500 lb/in^2 and E_t = 25300 lb/in^2 for the mild steel and aluminum 6061 T6 rings, respectively[5].

The critical mode numbers observed during the current tests[6] are compared in Figures 4 and 5 with the results of all previous relevant experimental investigations known to the authors[7]. The results appear to be reasonably consistent notwithstanding the differences in yield stresses of the materials, experimental techniques and despite the fact that the buckled profiles of cylindrical shells and rings are irregular as indicated in reference [8] (see also Figure 1 of reference [5] and Figures 8 and 10 of reference [6]). It should also be remarked that a broad band of harmonics is amplified in a long cylindrical shell and therefore possibly in short cylindrical shells and rings as well, which would be responsible for some scatter. The experimental critical mode numbers in Figures 4 and 5 appear to increase with increase in L/R, a phenomenon examined by Vaughan and Florence [7].

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4 The experimental values of Hn^c/R from reference [8] are plotted in Figure 3 with small horizontal bars directly above to indicate Hn^+/R. The mode numbers for the fractured rings marked by (F) were obtained from a visual inspection of the specimens.

5 The final membrane hoop strains of all rings are less than 8.3% and in most cases less than 6%.

6 The experimental values of n^c from reference [8] are plotted in Figures 4 and 5 with small horizontal bars directly above to indicate n^+. The mode numbers for the fractured rings marked by (F) were obtained from a visual inspection of the specimens.

7 The value of $\lambda H/R$ for each specimen is given in parentheses immediately below the experimental results in Figure 5. However, no values are given for the test results reported in reference [6] since they were obtained using an electromagnetic method which imparts a pulse to a specimen rather than an impulse (e.g. Figure 7 of reference [6]).

The largest values of the peak-to-trough radial displacements (w^p) measured on the outside surfaces of the buckled rings and which did not occur at joints in either the sheet explosive or the attenuator material are plotted in Figure 6. It appears from Figure 6 that the threshold impulse predictions of Florence and Vaughan [11] (eq. (8)) ensure that the permanent wrinkles of the thicker buckled rings remain small, but eq. (9) would be more suitable for the thinner specimens. The threshold impulses predicted by Vaughan and Florence [7] are somewhat smaller than those plotted in Figure 6 using eq. (8).

It can be shown that inequality (1) is satisfied and therefore local unloading has occurred during the dominant motion of all rings examined in reference [8] except for four mild steel specimens. However, the fact that the parameters for these four mild steel rings do not satisfy inequality (1) does not guarantee that no unloading occurred during the dominant motion.

6. Acknowledgments

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7. Notation

n^c	Critical mode number. Number of clearly defined peaks in wrinkled profile.
n^+	Number of clearly defined peaks and pauses in a wrinkled profile.
p	Material constant defined by equation (5).
t	Time.
t_f	Duration of dominant response.
w^*	Maximum peak-to-trough radial displacement on outside surface of a deformed ring.
w^p	Largest peak-to-trough radial displacement on outside surface of a deformed ring which did not occur at attenuator or explosive joints.
$\bar{w}_f$	Dominant radial displacement at $t = t_f$. Average radial displacement measured in experimental tests.
x, θ	Axial and circumferential coordinates in a cylindrical shell.
D	Material constant defined by equation (5).
E_t	Tangent modulus.
H	Average initial wall thickness of cylindrical shell or ring.
I	Total impulse
I^T	Threshold impulse per unit surface area.
L	Average initial axial length of cylindrical shell or ring.
R	Initial mean radius of cylindrical shell or ring.
V_o	Initial axisymmetric external impulsive velocity field.
β_1	$E_t/\bar{\sigma}$.
ε, κ	Strain and curvature, respectively.

λ	$\frac{\rho V_o^2 R^2}{\sigma_o H^2}$.
λ', λ^T	Defined by equations (4) and (12), respectively.
ρ	Density of material.
σ_o	Uniaxial yield stress.
σ_2	Uniaxial stress at 2% strain.
$\bar{\sigma}$	Average flow stress
σ_o'	Dynamic yield stress according to equation (5).
●,▲,■,◆	Hot rolled mild steel rings with H $\simeq$ 0.175 in., 0.10 in., 0.06 in. and 0.04 in., respectively (σ_o = 37400 lb/in^2).
○,△	Aluminum 6061 T6 rings with H $\simeq$ 0.175 in. and 0.10 in., respectively (σ_o = 40300 lb/in^2).

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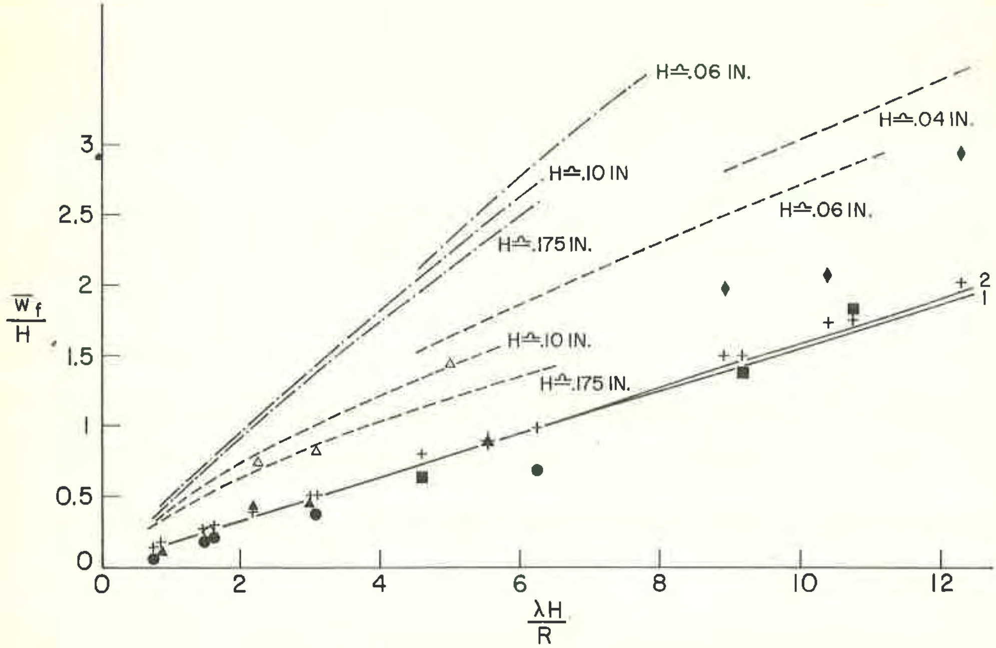


FIGURE 1

Figure 1. Permanent Average Radial Displacements reported in Reference [8] Compared with Theoretical Predictions for Mild Steel Rings.

_____ 1: Equation (3) with D = 40. 4 sec^{-1}, p = 5, σ_o = 37400 lb/in^2. _____ 2: Equation (6) with same data as 1. +: Florence [13] with same data as 1. -----: Vaughan and Florence [7] with σ_o = 37400 lb/in^2 and E_t =650000 lb/in^2. __. __. : Vaughan and Florence [7] with σ_o = 37400 lb/in^2 and E_t = 56500 lb/in^2.

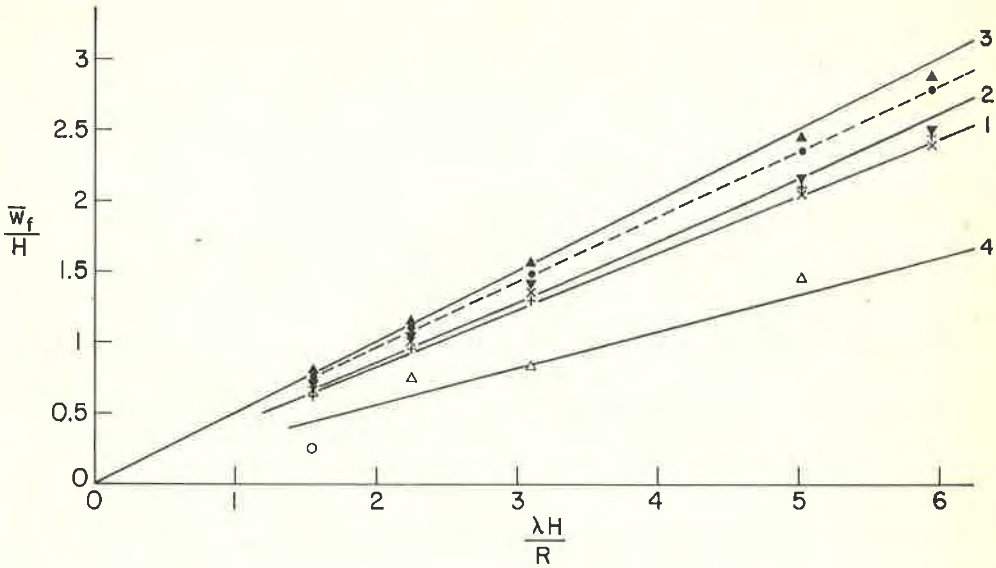


FIGURE 2

Figure 2. Permanent Average Radial Displacements of Aluminium 6061 T6 Rings[8] Compared with Theoretical Predictions.

_____ 1: Equation (3) with D = 1288000 sec^{-1}, p = 4. _____ 2: Equation (6) with D = 1288000 sec^{-1}, p = 4. _____ 3: Equation (2). _____ 4: Equation (3) with D = 6500 sec^{-1}, p = 4.
+: Florence [13] with D = 1288000 sec $^{-1}$, p = 4. x, : Vaughan and Florence [7] with E_t = 130000 lb/in^2 and E_t = 25300 lb/in^2, respectively. ------: Abrahamson and Goodier [5]. ▼▲: Stuiver [14] with E_t = 130000 lb/in^2 and E_t = 25300 lb/in^2, respectively.

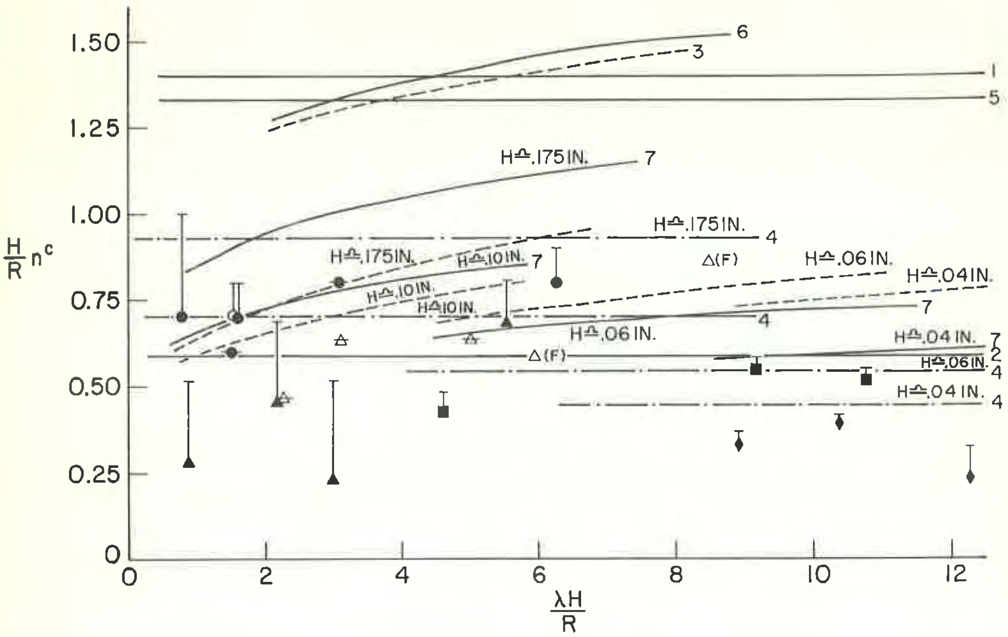


FIGURE 3

Figure 3. Critical Mode Numbers Reported in Reference [8] Compared with Various Theoretical Predictions.

——— 1: Abrahamson and Goodier [5] for aluminium 6061 T6 with σ_2 = 42300 lb/in^2, E_t = 130000 lb/in^2. ——— 2: Abrahamson and Goodier [5] for hot rolled mild steel with σ_2 = 37400 lb/in^2, E_t = 650000 lb/in^2. -----: Stuiver [14] for mild steel with σ_o = 37400 lb/in^2, E_t = 650000 lb/in^2. ---- 3: Stuiver [14] for aluminium 6061 T6 with H ≏ . 10 in., σ_o = 40300 lb/in^2, E_t = 130000 lb/in^2. --. -- 4: Equation (7). ——— 5: Lyons [17] for aluminium 6061 T6, H ≏ 0.10in. ——— 6: Florence [13] for aluminium 6061 T6 with H ≏ . 10 in., D = 1288000 sec^{-1}, p = 4. ——— 7: Florence [13] for hot rolled mild steel with D = 40.4 sec^{-1}, p = 5.

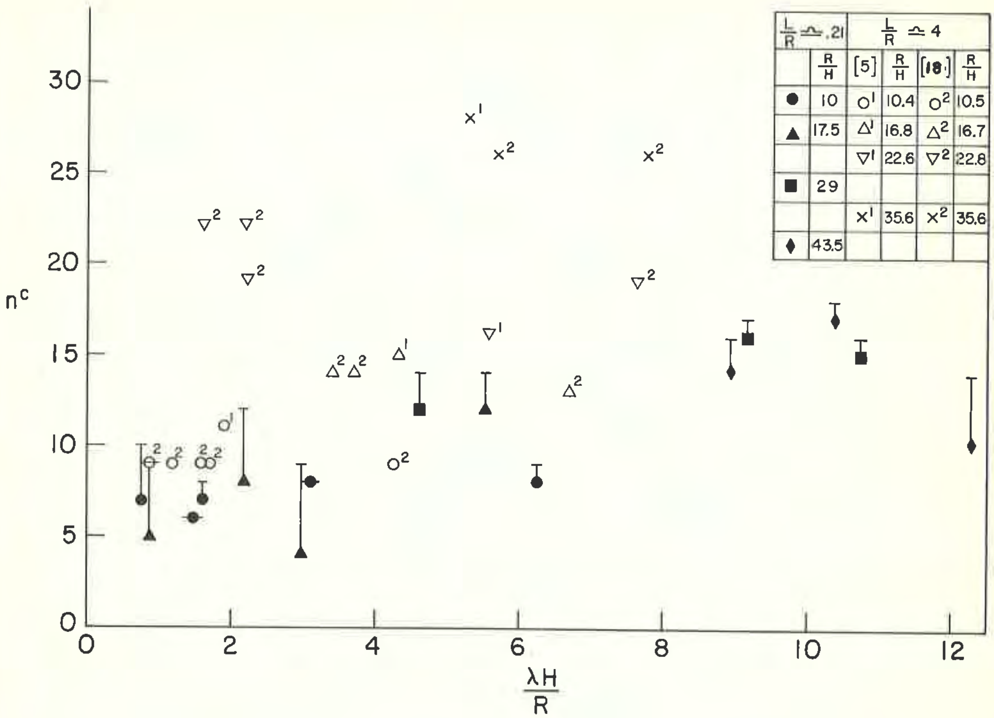
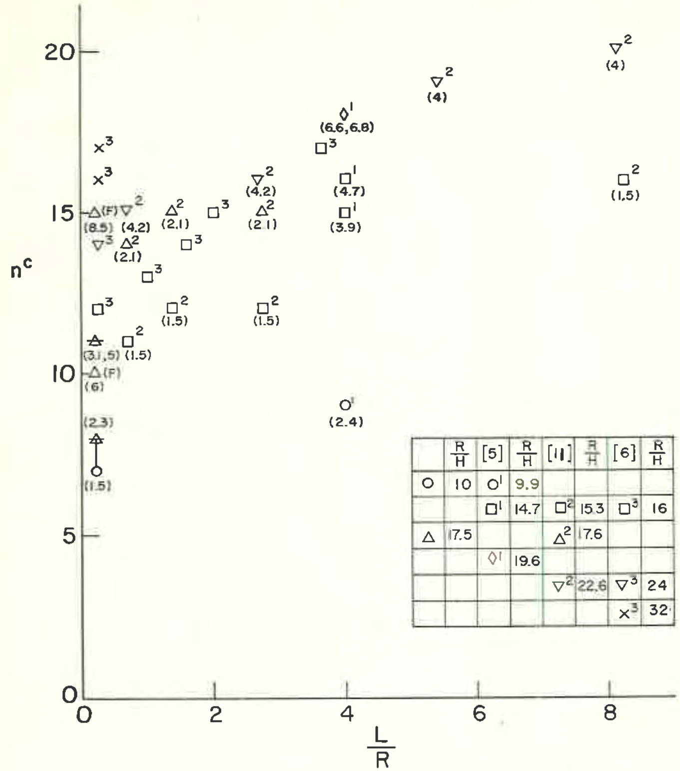


FIGURE 4

Figure 4. Comparison of Experimental Values of n^c for Mild Steel Rings Reported in Reference[8] with References[5] (Steel 1015, σ_2 = 102000 lb/in^2) and [18] (Steel 1015, σ_0 = 37700 lb/in^2).



	R/H	[5]	R/H	[11]	R/H	[6]	R/H
○	10	○¹	9.9				
		□¹	14.7	□²	15.3	□³	16
△	17.5			△²	17.6		
		◊¹	19.6				
				▽²	22.6	▽³	24
						×³	32

FIGURE 5

Figure 5. Comparison of Experimental Values of n^c for Aluminium 6061 T6 Rings Reported in Reference[8] with Reference[5] (Aluminium 6061 T6, σ_2 = 44000 lb/in^2) [6] (Aluminium Alloy BS 1474, $\sigma_o \simeq$ 12500 lb/in^2) and [11] Aluminium 6061 T6, σ_o = 44000 lb/in^2). The parentheses contain the associated values of λH/R.

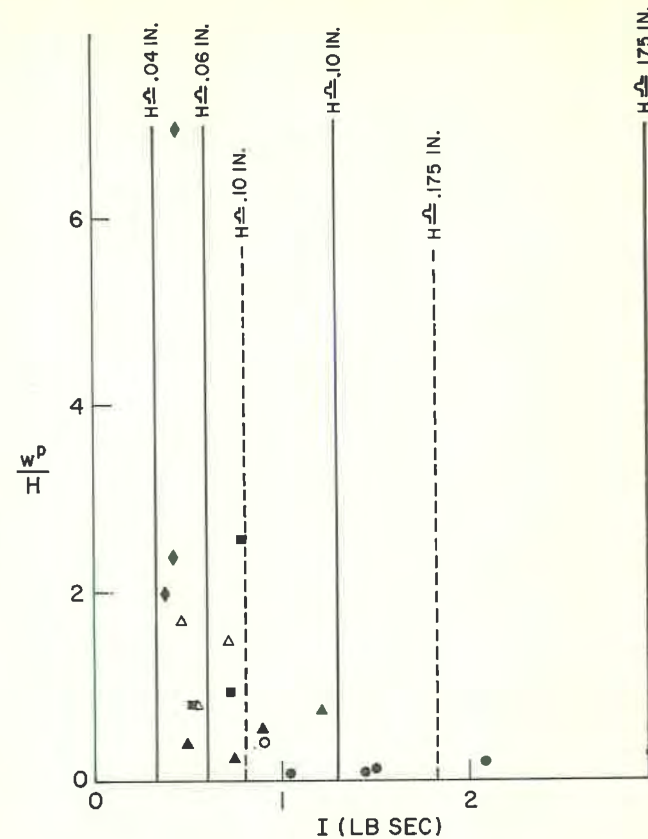


FIGURE 6

Figure 6. Comparison of Experimental Values for w^p from Reference[8] with Equation (8).
———: Equation (8) for hot rolled mild steel. – – –: Equation (8) for aluminium 6061 T6.