THE INFLUENCE OF SINGLE STIFFENERS TO THE SAFETY AGAINST BUCKLING OF CONICAL SHELLS UNDER EXTERNAL PRESSURE

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SUMMARY

This paper deals with the investigation of eigenvalues and the corresponding buckling modes of cylindrical and conical shells subjected to external pressure and stiffened by rings. As a result the "minimum stiffness" of the ring-ribs is evaluated in relation to the shell parameters: increasing of the ring stiffness above the "minimum stiffness" does not influence the eigenvalues within a given tolerance.

For cylindrical shells this problem has been solved in many papers, however for cones very little information exists. Therefore this contribution is directed toward stiffened cones. A relationship is sought to the well-known solutions for cylindrical shells introducing analogous shell parameters (e.g. radius of the osculating cylinder). The investigation is verified numerically in form of a parametric study using a general program for stability and free vibration analysis of shells of revolution assuming linear theory. Imperfections in shell geometry are considered by a subsequent modification of the calculated buckling loads. The study is based on ring-ribs with rectangular cross-section which are commonly used for vessels. The position of the ribs is defined by the angle between the normal of the rib surface and the cone surface. The shell parameters of the cones investigated are selected such that the eigenvalues are practically the same for inside-rings or outside-rings. The ribs are neither smeared out nor represented by local increase of cone-thickness (equivalent cross-section) but are considered as separate shells. Thereby a possible local buckling of the ribs is also included in the investigation.

Experiments on cones subjected to external pressure have shown that the dimensions of the conical shells do not influence noticeably the ratio \( \alpha \) of the actual buckling load versus the buckling load according to linear theory \( (\alpha \sim 0.8) \). Assuming that all the ribs force nodal lines of the buckling mode it can be concluded that the given multiplier \( \alpha \) is to be used also for cones stiffened by rings.

Finally a comparison of the results obtained with the required stiffnesses according to several codes is performed and some hints for designing rings are given.
The stability of monocoque cylindrical shells under external pressure is easily determined according to some codes or by help of well-known formulas. There exists also a lot of publications dealing with the stability of ring-stiffened cylinders and the necessary dimensions of the ribs to prevent not only elastic buckling of the whole system but also a breakdown due to plastic deformations.

The following contribution is based upon the concept of the classical elastic buckling theory. Imperfections in shell geometry are considered by a subsequent modification of the calculated buckling loads. According to the codes the ring-stiffeners for cylinders respectively tubes have to be strong enough in order to prevent a buckling mode whose geometry is independent of the longitudinal coordinate. In this case the problem is reduced to the buckling in the plane of the ring. The required stiffness of the ring \((EJ)_{\text{required}}\) is given by the well-known formula

\[
(EJ)_{\text{required}} = \frac{1}{3} \cdot P_{\text{all}} \cdot L \cdot R^2 \tag{1}
\]

where \(R\) is the radius of the cylinder

\(L\) is a certain length in the axial direction

\(P_{\text{all}}\) is the allowable external pressure.

With regard to the assumption of \(L\) there exist two opinions. In ASME Section VIII and British Standard 1515 it is recommended to use the distance between two ribs, while AD-B 6 requires a stiffness which is derived from the effective width

\[
L = 2 \cdot 0.78 \cdot \sqrt{R \cdot t} \tag{2}
\]

where \(t\) is the thickness of the cylinder.

Peder [1] demonstrated that the whole distance between the ribs should be used for \(L\) and care has to be exercised in using [6].

There exists very little information about the stability of stiffened conical shells under external pressure and especially concerning the disposition of the ribs. Therefore this contribution is directed toward this problem.

The investigation is verified numerically in form of a parametric study using A. Kalmnis' [3] program for stability and free vibration analysis of shells of revolution. The program mentioned calculates eigenvalues of shells of arbitrary shape assuming linear theory, i.e. equilibrium is established with the element in the deformed state and the resulting equations are linearized.
The study is confined to circumferential ribs with rectangular cross-section which are commonly used in pressure vessel applications (e.g. for safety containments). To achieve a favourable stress distribution the thickness of the ribs is mostly the same as that of the shell; that fact is also assumed in the study in hand.

The investigation shows that it is the moment of inertia that effects the results. Therefore the relationships derived will also hold for configurations that have the same moment of inertia. The ribs are neither smeared out nor represented by a local increase of cone thickness (equivalent cross-section) but are considered as separate shells. A possible local buckling of the ribs is also included. The orientation of the ribs is defined by the angle between the normals of the two shell surfaces. The parameters of the cones investigated were selected such that the eigenvalues are practically the same for rings inside or rings outside.

With regard to the orientation of the ribs two cases were considered:

a) the rib is welded perpendicular to the surface of the cone

b) the angle between the rib and the cone is that of the cone (α)

see fig. 1

The results revealed very little difference in the buckling loads associated with the two cases. Subsequently the study was confined to case b) because this type is preferred for manufacturing reasons.

In the course of the analysis of a system according to fig. 1 it was observed that the point of maximum deflection has a tendency to shift toward the large radius of the cone as the small radius decreases. Therefore this point would be the most effective position to place a rib; however the middle of the slant length was assumed for the position of the stiffener.

First the eigenvalues of frustums without stiffener are evaluated and then a comparison is made with the results obtained by Batdorf's formula 10 for the osculating cylinder.

\[ P_e = \frac{E}{[42(1-\nu^2)]^{1/2}} \cdot \left( \frac{T}{\bar{u}_{av}} \right)^2 \cdot \frac{1}{\frac{1}{t_5} + \left( \frac{n \cdot L}{\pi \cdot \bar{u}_{av}} \right)^2} \]

\[ n = \bar{n} \cdot \cos \alpha \]
Fig. 2 shows that the buckling loads derived from Kalnins' program and Batdorf's approximation (eq. 3) are in good agreement.

If a nodal line of the mode shape is enforced in the middle of the slant length, eq. 3 yields eigenvalues which are accordingly higher. For a given frustum we obtain thus two limiting curves, one corresponding to the unstiffened cone, the other one to an infinitely rigid stiffener. Between these two curves all the eigenvalues \( w = w(n) \) for cones with arbitrary ribs have to lie.

Now the problem is to find the "minimum stiffness" of the ribs in relation to the shell parameters; increasing of the rib dimensions above this "minimum stiffness" does not increase the eigenvalues within a given tolerance. This stiffness is reached when the buckling mode of the lowest eigenvalue shows negligible deflection at the place where the rib branches off. In fig. 3 one can see the eigenvalues determined by Kalnins' program plotted as a function of the wave number; the rib dimensions are introduced as a parameter \( \xi = \frac{H}{T} \). The diagrams are plotted for the cone parameter shown in the figure. In fig. 4 the buckling modes belonging to the critical pressures of fig. 3 are presented.

By plotting the lowest eigenvalues respectively critical pressures as a function of the parameter \( \xi \), one can see the efficiency of a given rib or – on the other hand – one can read off the "minimum stiffness". In this study 27 combinations of the following parameters were evaluated:

\[
\alpha = 30^\circ, 45^\circ, 60^\circ
\]

\[
\frac{L}{T} = 0.5, 1.0, 2.0
\]

\[
\frac{f_{ew}}{T} = 100, 200, 300
\]

These shell dimensions are commonly used in pressure vessel applications. The results of these calculations are summarized in figs. 5 and 6; they are valid for cones with no imperfections. The parameter study showed that the "minimum stiffness" is very insensitive to the angle \( \alpha \).

As a well-known fact the actual critical pressures are smaller than those derived under idealized assumptions. For conical shells under external uniform hydrostatic pressure the value \( P_{cr} \) has been studied theoretically by many investigators \([2]\) \([9]\). The conclusion is that the critical external pressure of a conical frustum is approximately equal to a factor times the critical external pressure of a cylinder having the same wall thickness, a radius equal to the average radius of curvature of the cone, and a length equal to the slant length of the frustum. The factor mentioned is a function only of the ratio of the end radii of the cone.
For completeness the ratio \( \frac{P_{cr}}{P_{ce}} \) as a function of the dimensions of the frustum from Weingarten's very instructive paper is reproduced in Fig. 7. The figure shows also a lot of test data. Since most of the test results fall above \( \frac{P_{cr}}{P_{ce}} = 0.8 \), it is recommended to use this ratio also in the case of stiffened cones.

Summarizing it can be said that the theoretical buckling loads derived in the foregoing agree well with those calculated according to ASME Section VIII and BS 1515 for both cases, the unstiffened cone and the cone with an infinitely rigid stiffener. Code AD-E2 yields theoretical buckling loads that are too great because they are based upon an average radius \( R_m = 0.5 \cdot (R_1 + R_2) \) rather than upon the mean radius of the osculating cylinder.

A further conclusion concerning the recommendations given in the codes for conical shells can be made. The required moments of inertia for circumferential stiffening rings according to all the codes are much greater than those obtained in this study (comparison see Fig. 5,6).

The reason for it is the extension of formulas applicable for cylinders to frustums. While in the case of stiffened cylinder - as mentioned earlier - the problem can be reduced to the buckling of a ring in its plane, application of shell theory is mandatory for ring-stiffened cones.

Finally an example may serve as illustration.
REFERENCES


[7] ASME-Code Section VIII, Division 2: Rules for construction of unfired pressure vessels

[8] British Standard 1500: Fusion welded pressure vessels, Part 1


fig. 1) Structural model for calculations

fig. 2) Comparison with Batdorf's results of computed values
fig. 3) Dependence of eigenvalues w on rod's stiffness

fig. 4) Buckling mode shapes for variable values of $\gamma$
fig. 5) Dependence of the lowest eigenvalues on rib's stiffness
fig. 6) Minimum stiffness according to codes and to parametric study

fig. 7) Comparison with theoretical results of external pressure test data.