ON STRUCTURAL RELIABILITY UNDER TIME-VARYING MULTI-PARAMETER LOADING

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SUMMARY

This paper intends to be a contribution towards the formulation of a procedure for the solution of the title problem that is at the same time correct and not too cumbersome for practical application.

In fact, the loads acting on any structural system would be best described as a multivariate stochastic process: the reliability of any design should be defined and calculated accordingly. However, such an approach would meet not only with great analytical and computational difficulties, but also with lack of data, because the available (or currently collected) statistics on loads are most often expressed in terms of extreme values, return times, etc., rather than as continuing registrations of time-varying processes.

Therefore, with some notable exceptions, alternative, simplified approaches are to be preferred in structural engineering. However, a "correct" (logically sound, not unduly complicated nor oversimplified) formulation for the calculation of reliability under these conditions is still an open problem.

In this paper, after the quotation of some recent papers on similar subjects, the set problem will be examined in detail and a number of possible alternative approaches to the solution discussed.

Special attention will be paid to the superimposition of loads of different origin and characteristics (e.g. long-term loads like the furniture and usual occupancy load in a building, and short-term loads like explosions, earthquakes, storms, etc.): it will be recognized that a single procedure for all cases does not appear practical, and that, within a general framework, special method must be devised according to the type of loads and structural responses. For instance, the superimposition of impulsive loads must be studied with reference to the response time of the structure.

It will be shown that usually, the statistics of extreme values are not sufficient for a correct study of superimposition: the instantaneous probability distributions of the load intensities are also required.

The results here obtained with respect to the loads can be joined with previous results by Augusti and Baratta (see e.g. SMiRT-2 Paper M 7/8) on the structural strength, for the evaluation of the probability of success (i.e. the reliability) of a structural design.

1. RISK UNDER MULTI-PARAMETER LOAD: SINGLE LOAD APPLICATION.

1.1) The risk P_{fail} (*) of a structure for one application of a random multi-parameter load has been expressed in the form (cf. Refs.1,5)

$$P_{fail} = \int_{Rn} f_w(\underline{W})\ P(\underline{W})\ d\underline{W} \qquad (1.1)$$

where $\underline{W}$ is the vector of the load parameters (defined in the real n-dimensional region Rn), $f_w(\underline{W})$ the probability density function (PDF) of the applied loads, and $P(\underline{W})$ the conditional probability of failure (CPF) of the structure when subjected to a specific set of loads $\underline{W}$ (Fig. 1a).

This formulation has the great practical convenience of separating the uncertainties in the load and strenght parameters (described by $f_w(\underline{W})$ and $P(\underline{W})$ respectively), which usually have quite different origins and characteristics. However, the validity of eq.(1.1) (like any other formulation) is subjected to a number of qualifications, often tacitly assumed because at first sight trivial, but worth discussing here.

1.2) Firstly, the very existence of a CPF function $P(\underline{W})$ would imply that the probability of failure under loads $\underline{W}$ does not depend on the path followed to reach these values. As a matter of fact, some limitations on the loading path must be imposed: namely, that along it the value of $P(\underline{W})$ never decreases. This leads naturally to define disequalities between load vectors $\underline{W}$ from the analogous disequalities between the corresponding scalars $P(\underline{W})$, as follows,

$$\begin{aligned} \underline{W}_1 > \underline{W}_2, &\text{ by definition, if } P(\underline{W}_1) > P(\underline{W}_2) \\ \underline{W}_1 < \underline{W}_2, &\text{ " \quad " \quad " } P(\underline{W}_1) < P(\underline{W}_2) \end{aligned} \qquad (1.2)$$

Then, the PDF $f_w(\underline{W})$ is unequivocally defined if the random-valued load $\underline{W}$ is quasi-statically applied (through a series of non-decreasing intermediate steps, according to definition (1.2)) on the structure only once, then is either decreased or removed or kept constant throughout the lifetime of the structure. In the latter case, the independence of $P(\underline{W})$ on time becomes essential.

For convenience, throughout this paper it is also assumed (as usual but inessential) that the probability of failure under nil load is zero:

$$P(\underline{0}) = 0 \qquad (1.3)$$

In other words, the risk P_{fail} considered here is the probability of failure conditional upon failure not occurring before the loads are applied: the corrections necessary to take account of this possibility would not be conceptually difficult.

1.3) Definition (1.2) allows re-writing of eq.(1.1) in a very simple form, useful for further developments. In fact,

$$p = P(\underline{W}) \quad ; \quad 0 \leq p \leq 1 \qquad (1.4)$$

is a measure of $\underline{W}$; its PDF is defined as follows

$$f(p)\ dp = \mathrm{Prob}(p \leq P(\underline{W}) < p + dp) \qquad (1.5)$$

(*) Throughout this paper, for brevity, the terms reliability and risk will indicate respectively the probability of success and failure of the (analyzed or designed) structure during the considered period of time (which, if not specified, is the "design" or "expected" lifetime).

Note that f(p) depends on both the PDF of the load $f_w(\underline{W})$ and the CPF of the structure $P(\underline{W})$: it can be calculated through suitable transformation of variables and integration of $f_w(\underline{W})$ over the (n – 1)dimensional subspaces of Rn defined by

$$P(\underline{W}) = \text{const} = p \quad (1.6)$$

For instance, if n = 2 (Fig. 1), eq.(1.6) defines the curve λ, and

$$f(p)\,dp = \oint_{\lambda} f_w(\underline{W})\,dr\,ds \quad (1.7)$$

where dr is the " distance" between each point of the curve λ, eq.(1.6), and the curve defined by $P(\underline{W}) = p + dp$.

If n = 1 (single-parameter load, W_1), Fig. 1b applies directly and

$$f(p)\,dp = f_w(W_1)dW_{1/A} + f_w(W_1)dW_{1/B} \quad (1.8)$$

Introducing (1.4) (1.5), eq.(1.1) becomes

$$P_{fail} = \int_0^1 f(p)\,p\,dp \quad (1.9)$$

i.e. P_{fail} is equal to the average value of $P(\underline{W})$ corresponding to the PDF f(p).

It is to be remarked that in the above treatment, any other measure of $\underline{W}$ that satifies def.(1.2) can be substituted for p.

Note finally that in case of a deterministic structure with "success region" S , the function f(p) degenerates into two concentrated probabilities that can be represented as Dirac-delta functions (Fig.2).

2. RISK UNDER ITERATED LOADS: GENERAL FORMULATION.

2.1) When the loads vary in time (for instance, are applied several times) the simple treatment of Section 1 is no longer valid, in particular if more than one load parameter, and their superposition, must be considered. Unfortunately the logically straightforward treatment of the load history as a multi-variate stochastic process would meet with the difficulties already underlined in the Introductory Summary: therefore, most engineering-minded researchers have limited themselves to treatments of specific problems: among the most recent works, space does not allow more than a passing quotation of the interesting contributions by Schuëller (Ref.7) and Tichy (Ref.8), besides the fundamental book by Ferry-Borges and Castanheta (Ref.9).

In this paper, a further discussion of these problems is developed, and an effort is made to put it in terms as general as possible through a reformulation of the definition of the PDF of the applied load(s).

2.2) For clarity and brevity of this discussion, the following hypotheses are explicitly introduced:

a) the strength properties of the structure (measured by the CPF function $P(\underline{W})$, Section 1) remain constant in time, irrespective of the number and intensity of load iterations (*);

b) failure occurs because a single load condition, at some time during the life of the structure, violates the " strength condition" ;

(*) The determination of $P(\underline{W})$ is not dealt with in this paper: it has been the object of Refs.1,2,5 and many others.

c) the loads are applied quasi-statically, i.e. dynamic phenomena are not taken into consideration.

Note explicitly that assumptions a) and b) exclude from the present discussion any damage accumulation, hence failures of the fatigue and incremental collapse types (cf. e.g. Refs.3,4,6): therefore the terminology " iterated loads" has been preferred to " repeated loads", usually associated with the latter types of failure.

From the previous assumptions, and recalling definition (1.2), the following corollary is derived:

" If the structure has survived a load $\underline{W}^*$, it does not fail under any load $\underline{W} \leq \underline{W}^*$."

It is then evident that a general formulation of the risk calculation, under the set hypotheses, is possible only through consideration of the largest (in the sense of def.(1.2)) load experienced by the structure: this point is not clear in most previous treatments of similar problems, even in the otherwise excellent book by Ferry-Borges and Castanheta (Ref.9).

2.3) To this aim, indicate by

$$\max_{(0,t)} P(\underline{W}) \qquad (2.1)$$

the largest value of $P(\underline{W})$ corresponding to the loads $\underline{W}$ that have acted on the structure between the initial time 0 and the generic time t, and by $h(p/t)$ the PDF defined, at any time t, by

$$h(p/t)\,dp = \mathrm{Prob}(p \leq \max_{(0,t)} P(\underline{W}) < p + dp) \quad ; \quad 0 \leq p \leq 1 \qquad (2.2)$$

Then, in analogy to eq.(1.9), the probability of failure in the time interval (0,t) is the average value of max $P(\underline{W})$:

$$P_{fail}(t) = \int_0^1 h(p/t)\,p\,dp \qquad (2.3)$$

The general problem of risk calculation is thus formally reduced to the determination of the function $h(p/t)$: this in general would be possible if the characteristics of the random loading process $\underline{W} = \underline{W}(t)$ and the CPF of the structure $P(\underline{W})$ were completely known. Otherwise, as discussed below, approximate evaluations and/or different expedients must be accepted.

2.4) Assume firstly that the joint PDF $f_w(\underline{W})$ of the random process $\underline{W} = \underline{W}(t)$ is known at any time t. (In the following developments, $f_w(\underline{W})$ will be considered not dependent on t, as if $\underline{W}(t)$ were a stationary process, but this assumption could be easily removed.) Note that $f_w(\underline{W})$ takes very large values in correspondence of the zeros of the parameters that correspond to short-term loads (explosions, winds, earthquakes, etc.): in these cases, it may be conveniente to introduce function of the Dirac type (i.e. concentrated probabilities) in the definition of $f_w(\underline{W})$ (cf. Tichy, Ref.10).

According to Section 1.3, the multivariate process $\underline{W} = \underline{W}(t)$ can be transformed into a scalar process $p = p(t)$, where p is a suitable positive-definite measure of $\underline{W}$, and its instantaneous PDF $f(p)$ can be calculated from $f_w(\underline{W})$ and $P(\underline{W})$. Then, the function $h(p/t)$ to be introduced into eq.(2.3) is the corresponding PDF of the largest p in the time interval (0,t).

An approximation of $h(p/t)$ that avoids difficulties coonected with time-correlation in the random processes $\underline{W}(t)$ and $p(t)$, can be obtained by dividing (0,t) into N intervals, each not smaller that the correlation extinction interval t_{ext}:

$$N : \qquad \frac{t}{N} \geq t_{ext} \tag{2.4}$$

Then, introducing the instantaneous cumulative distribution function (Fig.3)

$$F(p) = \int_0^p f(p)\, dp = \text{Prob}(P(\underline{W}) \leq p) \tag{2.5}$$

the distribution function of the largest $P(\underline{W})$ in n independent trials is

$$\text{Prob}\,(\max_n P(\underline{W}) \leq p) = F^n(p) \tag{2.6}$$

whence approximately

$$h(p/t) = \frac{dF^n(p)}{dp} = n\, f(p)\, F^{n-1}(p) \tag{2.7}$$

3. RISK UNDER ITERATED LOADS: SPECIAL FORMULATIONS AND EXAMPLES.

3.1) Often, especially when short-term loads are involved, the instantaneous PDF $f_w(\underline{W})$ is not available: instead, the statistics of the corresponding extreme parameters in a given time interval may be known.

To start from the simplest case, let the load be fully described by one scalar parameter W, and assume first that W has a well-definite sign, say $W \geq 0$. The relevant PDF is clearly the PDF of the largest values of W (*):

$$h_w(W)\, dW = \text{Prob}\,(W \leq \max_{(0,t)} W < W + dW) \tag{3.1}$$

With this definition of $h_w(W)$, at any given t,

$$P_{fail} = \int_0^\infty h_w(W)\, P(W)\, dW \qquad ; \qquad 0 < t < \infty \tag{3.2}$$

which is perfectly analogous to eq.(1.1): this is indeed the only case in which the risk under iterated load is formally given by the same expression as for a single load application; but note the different meaning of the PDF of the applied load in the two cases.

Also, since

$$\frac{dP(W)}{dW} \geq 0 \tag{3.3}$$

it is immediate that eqs.(3.1)(3.2) might be put in the form (2.2)(2.3).

3.2) Consider now again a single-parameter load W, but allow values of either sign in the successive applications; such is e.g. the axial load in a diagonal member of a truss-bridge, or the wave bending moment in a ship hull. A common form in which the statistics of this load might be available are the distributions of the maxima and minima (negative maxima), say $h_{w1}(W/t)$ and $h_{w2}(W/t)$ respectively.

From the above statistics, it is impossible (at least to this writer) to derive a general expression of the type (2.3), but the risk P_{fail} can be calculated in the following way.

Define the possible " loading conditions" as the two mutually exclusive and exaustive conditions L_1 and L_2, corresponding respectively to $W > 0$ and $W < 0$. The probability of failure in each condition is (Fig.4)

(*) From now on, the parameter t is not indicated unless strictly necessary to avoid confusion.

$$L_1 : \qquad P_{f1} = \int_0^{\infty} h_{w1}(W)\, P(W)\, dW \qquad (3.4)$$

$$L_2 : \qquad P_{f2} = \int_{-\infty}^{0} h_{w2}(W)\, P(W)\, dW \qquad (3.5)$$

The probability of failure of the structure in either condition L_1 or L_2 (i.e. the total probability of failure P_{fail}) is given by

$$P_{fail} = P_{f1} + (1 - P_{f1})\, P_{f2} = P_{f1} + P_{f2} - P_{f1}\, P_{f2} \qquad (3.6)$$

where $(1 - P_{f1})$ is the probability of success with respect to the first load condition, and success and failure with respect to either condition are considered statistically independent events: since $P(W)$ is a well-defined function, this is true when maxima and minima are statistically independent, i.e. the actual maxium does not affect the likelihood of the minima.

The case considered in Section 3.1, can be regarded as a special case of this latter treatment, with the PDF of the minima $h_{w2}(W)$ coincident with a Dirac-delta function. Then, because of assumption (1.3), eq.(3.5) yields

$$P_{f2} = 0 \qquad (3.7)$$

and eqs.(3.6) and (3.2) coincide.

3.3) To introduce risk calculation under combination of loads of different origins, reference is made to the simple portal frame shown in Fig. 5a. A two component vector $\underline{W}$ (Fig. 5b) defines completely the possible loading, but each component load W_1 and W_2 derives from an independent (random)process the "best" way to calculate P_{fail} depends on the characteristics of these processes and the available statistics.

If both components W_1 and W_2 correspond to long term loads, are statistically independent and the instantaneous PDF of each is known, the instantaneous joint PDF is

$$f_w(\underline{W}) = f_{w1}(W_1)\, f_{w2}(W_2) \qquad (3.8)$$

Then, the procedure of Section 2.4 can be applied: note that t_{ext} in eq.(2.4) must be the larger of the two processes $W_1(t)$ and $W_2(t)$.

3.4) Assume now that W_1 is a " long-term" load, always positive, while W_2 represents a " short-term" load, of indefinite sign and statistically independent on W_1. Three distinct load conditions can be individuated (as diagrammatically indicated in Fig. 6):

L_1 : the downward load W_1 is present alone;

L_2 : while W_1 is present (with a fixed value), $W_2 > 0$ is applied;

L_3 : while W_1 is present (with a fixed value), $W_2 < 0$ is applied;

The probability of failure in condition L_1 is obtained as in Section 3.1:

$$P_{f1} = \int_0^{\infty} h_{w1}(W_1)\, P(W_1, 0)\, dW_1 \qquad (3.9)$$

where $h_{w1}(W_1)$ is the PDF of the largest W_1 in the time interval (0,t).

Considered now each generic value taken by the parameter W_1 at some time in (0,t) without causing failure of the frame. For each given W_1, the probabilities of failure in conditions L_2 and L_3 are respectively

$$L_2 : \qquad P_{f2}/W_1 = \int_0^{\infty} h_{w2}(W_2)\ P(\underline{W})dW_2 \qquad (3.10)$$

$$L_3 : \qquad P_{f3}/W_1 = \int_{-\infty}^{0} h_{w3}(W_2)\ P(\underline{W})dW_2 \qquad (3.11)$$

where $h_{w2}(W_2)$ and $h_{w3}(W_2)$ are respectively the PDF of the maxima and minima (negative maxima) values of W_2 (analogous respectively to $h_{w1}(W)$ and $h_{w2}(W)$ in Section 3.2). In eqs.(3.10)(3.11), it has been assumed that (Fig. 6a)

$$\mathrm{sign}\left[\frac{dP(\underline{W})}{dW_2}\right] = \mathrm{sign}\ (W_2^{\circ} \qquad (3.12)$$

i.e. that along lines parallel to the W_2-axis, $P(\underline{W})$ increases on both sides of the W_1-axis. If (3.12) does not hold, the zero limit of one of the integrals (3.10) or (3.11) must be modified, to take account of the corollary enunciated in Section 2.2 (cf. Fig.6b).

Integrating to all possible W_1, the probabilities of failure in condition L_2 and L_3 are obtained:

$$L_2 : \qquad P_{f2} = \int_0^{\infty} f_{w1}(W_1)\ dW_1 \int_0^{\infty} h_{w2}(W_2)\ P(\underline{W})\ dW_2 \qquad (3.13)$$

$$L_3 \qquad P_{f3} = \int_0^{\infty} f_{w1}(W_1)\ dW_1 \int_0^{\infty} h_{w3}(W_2)\ P(\underline{W})\ dW_2 \qquad (3.14)$$

where $f_{w1}(W_1)$ is the instantaneous PDF of W_1.

Finally, the total risk is

$$P_{fail} = P_{f1} + P_{f2} + P_{f3} - P_{f1}\ P_{f2} - P_{f2}\ P_{f3} - P_{f3}\ P_{f1} + P_{f1}\ P_{f2}\ P_{f3} \qquad (3.15)$$

Note that, besides the relevent extreme PDF's, also the instantaneous PDF of the long-term load component has been necessary to calculate P_{fail}

3.5) Space does not allow a full discussion of the assumptions that lay behind the treatment of Section 3.4, and even of the definition of " short-term" load: note only that it has been tacitly assumed that each of the three loading conditions occurs with probability 1.

The probability of superposition becomes a main problem when more load components correspond to short term loads, of which very seldom more than extreme statistics are available.

Usually, the possibility of superposition of short-term loads is neglected and the probability of failure for each such load is calculated separately; then the total risk is given by formulae of the type (3.15). However, this is a subject worth much further investigation.

For instance, if each short-term load is considered as a random succession of impulses (concentrated in time), the teoretical probability of contemporary arrivals is zero. On the other hand, any structure has a finite response time: the probability of some superposition of load effectcs is related to the non zero probability of arrival of the second load during the response to the first load. This problem is currently being studied.

4. CONCLUSION.

To make this " compact" text self-contained, only a general discussion of the subject matter of the Summary has been reported. In particular, in Section 2 a novel general formulation for the risk under iterated loads has been presented: however, the usual incomplete knowledge of the loading process statistics often prevents its use and special approaches are requires, as illustrated by some examples in Section 3. Full length paper(s) will develop further details and applications (*)

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(*) The continuing financial support of the C.N.R. (Italian National Research Council) is gratefully acknowledged.

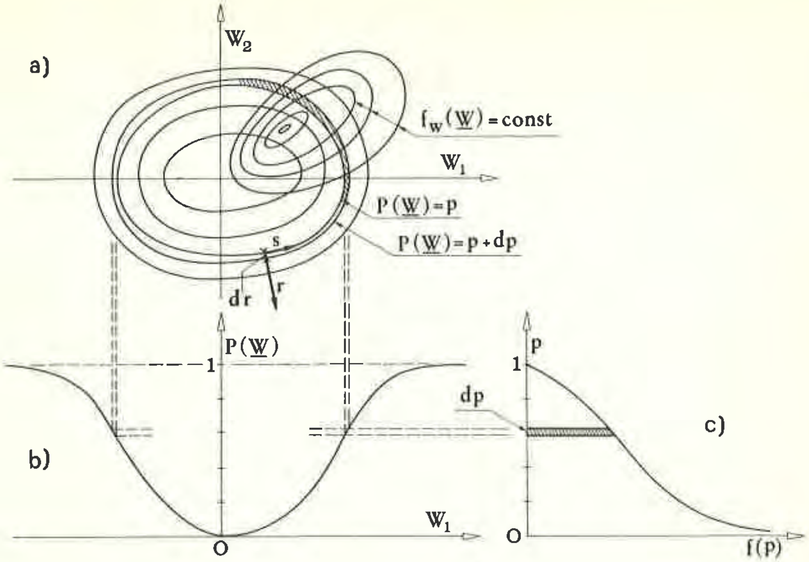

Fig. 1 : Construction of the probability density function f(p) for a structure with a conditional probability of failure $P(\underline{W})$.

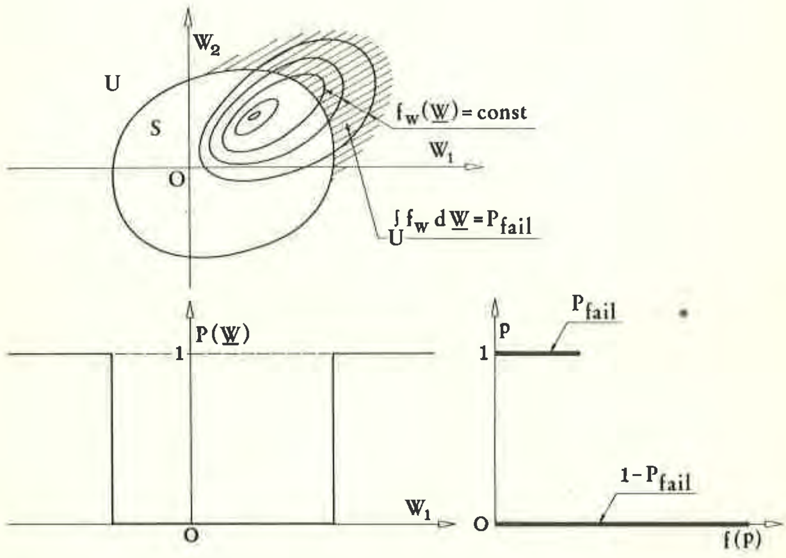

Fig. 2 : Construction of the probability density function f(p) for a deterministic structure with " success region" S.

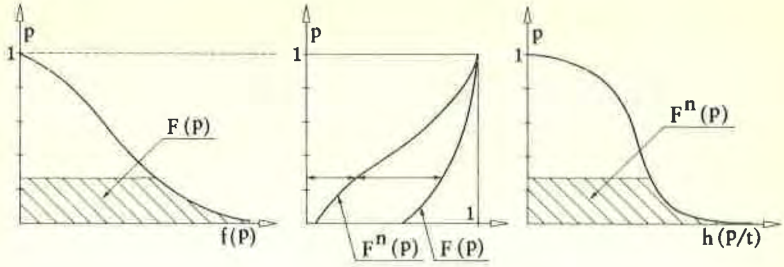

Fig. 3 : Approximate calculation of the extreme distribution h(p/t).

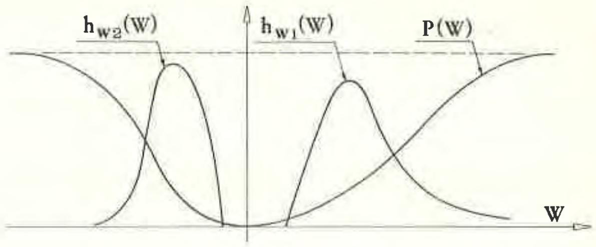

Fig. 4 : Calculation of risk for one-parameter load.

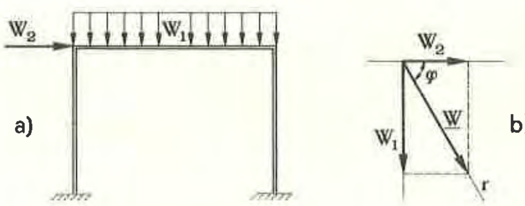

Fig. 5 : Example: portal frame.

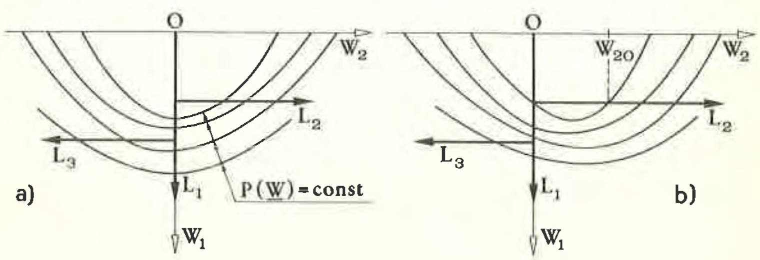

Fig. 6 : Loading conditions for frame in Fig. 5: W_1, long-term load; W_2, short-term load.