CONDITIONAL COLLAPSE PROBABILITY UNDER GIVEN LOADS OF PLASTIC PLATES WITH RANDOM STRENGTH

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SUMMARY

In the probabilistic context of structural safety, it is commonly accepted that, in order to judge the adequacy of a structure to sustain an assigned set of loads, the strength of the structure be described as a random variable, and that the significant parameter is the probability of failure.

The paper deals with the problem to calculate as small as possible upper bounds on the collapse probability of a perfectly plastic isotropic supported plate subject to uniform loading.

It is assumed that the strength of the plate (expressed by the “yield moment”) in every point is a random variable, and that, possibly, different strengths are realized in different points.

A discrete model of the plate is referred to, in way that a number of “check points”, in which the respect of the yield condition must be checked, are individuated, while the strength parameters (the yield moments) in these points are assumed to be a set of statistical variables whose joint probability distribution is assigned.

A first question rises by observing that it is not realistic to assume statistical independence of these random variables: if it was so, in fact, the collapse probability would approach unity by simply increasing the number of check points. It is necessary, therefore, to introduce the possibility that some degree of statistical correlation runs among the strengths in different points. Moreover, the correlative link between two points probably decreases with increasing their distance. To overcome most of the practical and theoretical difficulties connected with the introduction of the correlation, an approach is proposed leaving the usual concept of the “coefficient of linear correlation”, and introducing an “extinction length” as the minimum distance between two material points whose strengths can be considered statistically independent, and assuming that the correlation varies according to an arbitrary function of the reciprocal distance of the points (but with the purpose to fit with future experimental data), in a way that the corresponding random variables can vary from the absolute identity to the perfect statistical independence.

Once the probabilistic settlement of the data has thus been given, the collapse probability is calculated by the static approach, as formulated in SMiRT-2 Paper M 7/8, by investigating a class of parabolic moment fields of the type

\[ m_x = A(1 - x^2); \quad m_y = B(1 - y^2/\beta^2); \quad m_{xy} = Cx y \]

(1)

where \(A, B\) are arbitrary constants, and \(C\) is defined by the equilibrium condition. The number of couples \(A, B\) to be introduced in the expression of the collapse probability is limited to 10 for computational reasons, but these ten couples are chosen by investigating a much larger number of couples, according to the criterion to minimize the calculated collapse probability.

After discussing the computational detail, the influence of the ratio of the extinction length to the geometrical size of the plate on the collapse probability is investigated.
1. Introduction

In some recent papers \([1, 2, 3, 4, 7]\), it has been proved that an upper bound on the probability of plastic collapse of any elastic-perfectly plastic structure with random strength variations can be calculated for any set of applied loads, by the Static Approach of Probabilistic Limit Analysis.

This approach is a direct consequence of the extension of the classical Static (or Equilibrium) Theorem of deterministic L.A. to the case that the local strengths of the structure are suitably described by a set of random variables, like it may be in problems of structural safety and decision. Essentially the Theorem states that the actual conditional probability of plastic collapse under a given set of applied loads \(W\), say \(P_f(W)\), is not larger than the probability \(P^*_f(W)\) that, assigned a set \((b_1, \ldots, b_n)\) of any stress fields equilibrating the applied loads \(W\), none of them is statically admissible, i.e.,

\[
P^*_f(W) = \text{Prob} \left\{ \exists \ b_i \in (b_1, \ldots, b_n) \text{ stat. adm.} \right\} \geq P_f(W)
\]

(1)

and \(P^*_f(W)\) is referred to as a static approximation of \(P_f(W)\).

It is therefore obvious that any static approximation is a safe bound on the collapse probability \(P_f(W)\).

In the paper, static approximation on the plastic collapse probability of a simply-supported isotropic elastic-perfectly-plastic plate with random strength variations are calculated under uniformly distributed loads, and some basic ideas to keep account of eventual strength-correlations among the various points of the plate are proposed.

2. Basic statements

Consider the rectangular plate of Fig. 1, of constant thickness "s", simply supported along the four edges and subjected to a uniform loading of intensity \(W\).

Assume that the plate is made by perfectly-plastic material, and that the yield locus in any point of the plate obeys the Mises' law

\[
\varphi = \sigma_x^2 - \sigma_y^2 + 3 \tau_{xy}^2 = \sigma_o^2
\]

(2)

If the simple theory of plastic bending of thin plates holds \([5, 6, 7]\), eq. (2) can be better expressed in terms of generalized stress components (bending moments \(M_X, M_Y\) and twisting moment \(M_{XY}\))

\[
\varphi = M_X^2 - M_X M_Y + M_Y^2 + 3 M_{XY}^2 = M_o^2
\]

(3)

\(M_o\) being the yield moment in pure bending of the plate.

If a moment field \(b = (M_X(X, Y), M_Y(X, Y), M_{XY}(X, Y))\) is in equilibrium with the applied load, the following well known differential equation must be satisfied

\[
\frac{\partial^2 M_X}{\partial X^2} + 2 \frac{\partial^2 M_{XY}}{\partial X \partial Y} + \frac{\partial^2 M_Y}{\partial Y^2} = -W
\]

(4)

with the boundary conditions

\[
M_X = 0 \quad \text{for} \quad X = \pm A
\]

\[
M_Y = 0 \quad \text{for} \quad Y = \pm B
\]

(5)

(1) A stress field "b" is said to be statically admissible if it does not violate the material limit yield condition in any point of the structure.
\[
\begin{align*}
\text{With the positions } & \begin{bmatrix} 5, 6 \end{bmatrix} \\
m_x &= \frac{M_X}{M_0}; \quad m_y = \frac{M_Y}{M_0}; \quad m_{xy} = \frac{M_{XY}}{M_0} \\
x &= \frac{X}{A}; \quad y = \frac{Y}{A}; \quad \beta = \frac{W}{A}; \quad w = \frac{W A^2}{M_0}
\end{align*}
\]
\(\text{the yield condition, eq. (3), can be written in terms of non-dimensional variables} \)
\[
\varphi = m_x^2 - m_x m_y + m_y^2 + 3 m_{xy} = 1
\]
and the equilibrium eqs. (4) - (5)
\[
\frac{\partial^2 m_x}{\partial x^2} + 2 \frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} = -w
\]
\(\text{m}_x = 0 \quad \text{for} \quad x = \pm 1; \quad m_y = 0 \quad \text{for} \quad y = \pm \beta
\]
\(\text{Let } \text{"B" be the set of all generalized stress fields equilibrating the applied uniform load } w. \quad \text{Clearly, B is the general solution of eq. (9) and boundary conditions, eqs. (10). Unfortunately, this solution is not available in closed form. Leaving out the interesting possibility to get finite-element solutions of eqs. (9)-(10), only some restricted class of equilibrated stress fields can be individuated and let it be the subset } B_1 \text{ of } B \text{ defined by assuming the following expressions for bending and twisting moments} \)
\[
\begin{align*}
m_x &= C_x (1-x^2); \quad m_y = C_y (1-y^2/\beta^2); \quad m_{xy} = C_{xy} x y
\end{align*}
\]
\(\text{C}_x \text{ and } C_y \text{ being arbitrary constants, while } C_{xy} \text{ is related to } C_x, C_y \text{ and to the load intensity } w \text{ by the eq. (9) which yields} \)
\[
C_{xy} = C_x + C_y / \beta^2 - w/2
\]
\(\text{Thus, any choose of } C_x \text{ and } C_y \text{ yields an equilibrated stress field of } B_1. \text{ In particular, } n \text{ couples } (C_{x_1}, C_{y_1}) \text{ of values of } C_x, C_y \text{ can be considered, obtaining correspondingly } n \text{ moment fields } (b_1, \ldots, b_n), \text{ equilibrating } w. \)

\(\text{Assume now that the strength of the plate cannot be deterministically individuated, but, because of various sources of uncertainty (as workmanship, working processes of materials, etc.), can only be described as a random variable, and that the realization of this random variable can also be different in different points.} \)

\(\text{The admissibility of the stresses in the generic point } Q \text{ of the plate can then be expressed as follows} \)
\[
\varphi \left[ b(Q) \right] = m_x^2 - m_x m_y + m_y^2 + 3 m_{xy} \leq \tilde{\varphi}_0(Q)
\]
\(\tilde{\varphi}_0(Q) \text{ being the realization of the random function } \varphi, \text{ in the point } Q. \)
\(\text{The admissibility of the field } b \text{ over the whole surface } U \text{ of the plate requires then that} \)
\[
\varphi \left[ b(Q) \right] \leq \tilde{\varphi}_0(Q) \quad \forall Q \in U
\]
\(\text{whence it appears clear that } \tilde{\varphi}_0(Q) \text{ expresses the stochastic variations of the strength throughout the plate.} \)
According to the Static Theorem of Probabilistic L. A., recalled in the Introduction, a safe bound on the plastic collapse probability \( P_\psi (w) \) under an assigned intensity of the uniform load \( w \), can be got by calculating the probability \( P_\psi (w) \) that the set \( \{ b_1, \ldots, b_n \} \) does not contain any admissible stress field, i.e. that none of the considered couples \( (C_{xi}, C_{yi}) \) yields, by eqs. (11) a moment field satisfying the ineq. (14).

Obviously, if \( R_\psi (w) \) denotes the probability that, on the contrary, at least one of the quoted couples \( (C_{xi}, C_{yi}) \) does yield an admissible field, \( P_\psi (w) \) is the complement to unity of this latter probability, i.e.

\[
P_\psi (w) = 1 - R_\psi (w)
\]

(15)

Therefore, it is immaterial to calculate \( P_\psi (w) \) or \( R_\psi (w) \).

3. - Probabilistic Limit Analysis of a square plate

3.1. - Consider the square plate ( \( a = 1 \) ) of Fig. 2, and assume that it is sufficient, for technical purposes, to investigate only the check-points \( Q_j \) (\( j = 1, \ldots, m \)) individuated by the mesh shown in Fig. 2, in order to check the admissibility of any moment field \( b_i \).

For any couples \( (C_{xi}, C_{yi}) \), denote by \( \varphi_{ij} \) the value of the yield function (leftward member of eq. (13)) assumes in the point \( Q_{ij} \) in connection with the moments

\[
m_x = C_{xi}(1-x^2); \quad m_y = C_{yi}(1-y^2); \quad m_{xy} = (C_{xi}+C_{yi} - \frac{w}{2}) xy
\]

(16)

Thus

\[
\varphi_{ij} = C_{xi}(1-x_j^2)C_{yi}(1-y_j^2) + C_{yi}(1-y_j^2) + 3(C_{xi}+C_{yi} - \frac{w}{2})x_jy_j
\]

(17)

The admissibility of the \( i \)-th moment field \( b_i \) is therefore expressed by

\[
\varphi_{ij} \leq \varphi_{0j} \quad \forall j \in \{1, \ldots, m\}
\]

(18)

(\( \varphi_{01}, \ldots, \varphi_{0m} \)) being a set of random variables expressing the limit strength of the plate in the check-points.

The probability that the \( i \)-th stress field is admissible is given by

\[
R_{ij}(w) = \text{Prob} \left\{ \varphi_{ij} \leq \varphi_{0j} \quad \forall j \in \{1, \ldots, m\} \right\}
\]

(19)

and the probability that at least one of the investigated couples \( (C_{xi}, C_{yi}) \) yields an admissible field by

\[
R_\psi (w) = \text{Prob} \left\{ \exists i \in \{1, \ldots, n\} : \varphi_{ij} \leq \varphi_{0j} \quad \forall j \in \{1, \ldots, m\} \right\}
\]

(20)

\( R_\psi (w) \) can be furtherly explicitated by putting, for any number \( k \leq n \)

\[
R_{i_1 \ldots i_k} = \text{Prob} \left\{ \left( \max_{i \in \{1, \ldots, k\}} \varphi_{ij} \right) \leq \varphi_{0j} \quad \forall j \in \{1, \ldots, m\} \right\}
\]

(21)

\( (i_1, \ldots, i_k) \) being any combination of class \( k \) of the first \( n \) natural numbers. In fact, \( R_{i_1 \ldots i_k} \) is the probability that the moment fields \( b_{i_1}, \ldots, b_{i_k} \) are all admissible, and by well known formulas of Theory of Probabilities

\[
R_\psi (w) = \sum_{(i_1, \ldots, i_k)} R_{i_1 \ldots i_k}
\]

(22)

3.2. - Assume now that the strengths in the check-points are a set of statistically indepen-


\[ \begin{equation} \Phi_{o j} = \Phi_{o} = 1 ; \quad S_{j} = S_{o} = 0.1 ; \quad \forall j, i, (1, \ldots, m) \end{equation} \]

Because of the assumed independence, eq. (21) can be written

\[ R_{i1 \ldots i_{k}} = \prod_{j=1}^{m} \text{Prob} \left\{ \tilde{\Phi}_{o j} \geq \max_{i \in \{1 \ldots k\}} \Phi_{i j} \right\} \quad (24) \]

An important point to get the best (i.e. the smallest static approximation of \( P_{f}(w) \)) is to choose the \( n \) couples \((C_{x1}, C_{y1})\) that are to be introduced into the expression, eq. (20), of \( R_{\psi}(w) \).

In order to investigate this question, consider the expected plate, i.e. the plate with deterministic strength equal to the expected value, eq. (23), of \( \tilde{\Phi}_{o} \). It is easy to verify that in this case the best lower bound of the collapse load over the subset \( \mathcal{B}_{1} \) is always obtained for \( C_{x} = C_{y} \), and is approximately equal to 5.155. Consider then the plane \( C_{x} = C_{y} \) (Fig. 3), and the line \( C_{x} = C_{y} \). Numerical results show that the maximum individual reliability, eq. (19), of a stress field is got on the line \( C_{x} = C_{y} \). It is therefore possible to individuate the point on this line yielding this maximum (Fig. 3a). The associated field \( b_{1} \) is assumed as the first element of the set to be investigated.

As a matter of fact, and according to the general considerations formulated in \( \int \mathcal{A} \mathcal{J} \), it is reasonable to expect that the largest reliability \( R_{\psi}(w) \) can be got by investigating a suitable neighborhood of \( b_{1} \).

Limiting to 9 the maximum number \( n \) of stress fields that are to be checked for admissibility (1), the best static approximation under this restriction can be found by the following procedure and looking, for instance, only at square neighborhoods of \( b_{1} \) (Fig. 3b).

Consider a square of side \( \mathcal{A} \), with center in the point \( b_{1}(C_{x1}, C_{y1}) \), cover this square by a mesh of, say, 100 nodes, and let \( b_{j} \) be the \( j \)-th node (Fig. 4). Start from the point \( b_{j} \), and calculate, for any \( b_{j} \) the probability that, simultaneously, \( b_{j} \) is admissible and \( b_{1} \) is not. Denote it by

\[ R_{1} = R(b_{j} \cap \mathcal{B}_{1}) \quad (25) \]

and put

\[ R_{1} = R(b_{1}) \quad (26) \]

\[ R_{2} = \max_{j} R(b_{j} \cap \mathcal{B}_{1}) = R(b_{1} \cap \mathcal{B}_{1}) \quad (27) \]

Next, consider the probabilities

\[ R(b_{j} \cap \mathcal{B}_{1} \cap \mathcal{B}_{1}) \quad (28) \]

and put

\[ R_{3} = \max R(b_{j} \cap b_{1} \cap b_{2} \cap \mathcal{B}_{1} \cap \mathcal{B}_{1}) = R(b_{1} \cap b_{1} \cap b_{2} \cap \mathcal{B}_{1} \cap \mathcal{B}_{1}) \quad (29) \]

(1) - Remember that, as can be inferred by eq. (22), the number of operations required to calculate \( R_{\psi}(w) \) grows like \( 2^{n} \).

(2) - Obviously: \( R(b_{1} \cap \mathcal{B}_{1}) = 0 \).
and so on up to get 9 points, and a reliability

$$R_{\psi}(w) = \sum_{i=1}^{9} R_i$$  (30)

This result can be looked at as the best possible approximation, under the condition that \(n\) is not larger than 9, of the probability that the assigned square contains an admissible field, and must be regarded as a function of the square size

$$R_{\psi}(w, \delta) = 1 - P_{\psi}(w, \overline{\delta})$$  (31)

This function ought to have a maximum for \(\delta = \overline{\delta}\). The best bound \(P_{\psi}(w)\) can then be defined by

$$P_{\psi}(w) = 1 - P_{\psi}(w, \overline{\delta})$$  (32)

As an example, consider the case that the collapse probability must be safely bounded under the load intensity \(w = 1.3\). The starting point \(b_{1y}\), i.e., the equilibrated stress field of \(B_1\) yielding the maximum individual reliability, is given \(c_{x1} = c_{y1} = 0.655\).

Next, consider firstly a square neighborhood of \(b_1\) of side \(\delta_0 = 0.3275 = c_{x1}/2\) (Fig. 4a). The points yielding the maximum of \(R_{\psi}(w, \delta_0)\) are marked by \(\star\) in the figure, and show some tendency to be attracted by the center of the square. When the side is reduced, put \(\delta_1 = \delta_0/2\), a new square and a new mesh are obtained, and some of the points yielding \(R_{\psi}(w, \delta_1)\), marked by \(\bigstar\) in the figure, result to be placed on the boundary of the square, but the calculated reliability \((\sim 1.0.64 E=04)\) is not very different from the previous one. When the side is again reduced, put \(\delta_2 = \delta_1/2\) (Fig. 4b), the stress fields \(\bigcirc\) leading to \(R_{\psi}(w, \delta_2)\) are definitively centrifuged, and \(R_{\psi}(w, \delta_2)\) is reduced of about one order of magnitude with respect to \(R_{\psi}(w, \delta_1)\). The plot of \(P_{\psi}(w, \delta)\) versus \(\log_2(\delta_0/\delta)\) shows that the minimum of \(P_{\psi}(w, \delta)\) is attained approximately for \(\delta = \delta_0/\delta\) (Fig. 5), i.e., as soon as centrifugal tendencies are exhibited by the optimal points.

4. Basic ideas for accounting strength-correlation

Consider the case that the chosen check-points exhibit random strengths which cannot be considered statistically independent, but that it is necessary to keep into account some degree of correlation among the strengths in the different points.

Rather than approaching the question by introducing the well known coefficients of linear correlation, a simplified, though not rigorous in principle, approach can be proposed as follows.

The basic step is the calculation of the probability \(R_{1,\ldots,i,k}\), eq. (21), which, if the strengths in the points \(Q_j\), are not statistically independent, cannot be simply expressed by the product at the left side of eq. (24). In order to express it as a product, put

$$\phi_j = \max_{i=1,\ldots,k} \phi_{ij}$$  (33)

then, by well known formulas,

$$R_{1,\ldots,i,k} = \text{Prob}\{Q_1 \leq \tilde{Q}_{01}\} \cdot \text{Prob}\{Q_2 \leq \tilde{Q}_{02}\} \cdot \text{Prob}\{Q_3 \leq \tilde{Q}_{03}\} \ldots \cdot \text{Prob}\{Q_k \leq \tilde{Q}_{0k}\} \leq \tilde{Q}_{01}, \tilde{Q}_{02}, \ldots, \tilde{Q}_{0k}$$  (34)

In this expression, the quantity
\[
\text{Prob} \left\{ \varphi_j \leq \varphi_{ij} \mid \varphi_1 \leq \varphi_{01}, \ldots, \varphi_{j-1} \leq \varphi_{0j-1} \right\} = R_j \quad j-1
\]  
\hspace{1cm} (35)

is the conditional probability that none of the considered fields \((b_1, \ldots, b_n)\) violates the yield condition in the point \(Q_j\), assumed that the same happens in the points \(Q_1, \ldots, Q_{j-1}\) which precede \(Q_j\) in the chosen layout of the check-points.

Put, for any \(j\) and for any \(i\)
\[
R_j = \text{Prob} \left\{ \varphi_j \leq \varphi_{ij} \right\} ; \quad d_{ij} = \text{distance} \ (Q_i, Q_j)
\]
\hspace{1cm} (36)

and consider the ratio
\[
R_j / R_{j-1} = f_j ; \quad f_1 = 1
\]
\hspace{1cm} (37)

This ratio will be, in general, a function of \(\varphi_i, \ldots, \varphi_j\), of the statistical properties of strength, and of the degree of correlation among the strengths in different points. It is hence postulated that this latter influence can be expressed in terms of the reciprocal distances \(1/d_{ij}\) of the check-points
\[
R_j / R_{j-1} = f_j (d_{i1}, \ldots, d_{j-1}, d_{ji})
\]
\hspace{1cm} (38)

According to eq. (38), the probability \(R_1 \ldots R_k\) can be expressed by
\[
R_1 \ldots R_k = \left( \prod_{j=1}^m R_j \right) \left( \sum_{j=1}^m f_j (d_{i1}, \ldots, d_{jk}, d_{jk}) \right)
\]
\hspace{1cm} (39)

It is easy to show that eq. (38) cannot be, by itself, a satisfactory estimate of the ratio \(R_j / R_{j-1}\). In fact, put for instance \(m=3\), and consider the probability \(R_1 R_2 R_3\) that
\[
\varphi_{ij} \geq \varphi_j \quad \forall j = (1, 2, 3)
\]
\hspace{1cm} (40)

If the layout of the points \(Q_1, Q_2, Q_3\) is changed, take for instance \(Q_1 Q_2 Q_3\) the same probability would be given by
\[
R_{123} = R_1 R_2 R_3 \cdot f_2 (d_{12}, d_{23}) \cdot f_3 (d_{13}, d_{23})
\]
\hspace{1cm} (41)

which is, in general, different from eq. (40). The postulate (38) can therefore be accepted only if the layout of the check-points is determined; in this sense the dependence of the ratio on \(R_{j-1}, \ldots, R_j\) can be expressed by ordering the points \(Q_1, \ldots, Q_m\) in the direction of increasing \(R_j\).

In this agreement, the probability \(R_1 \ldots R_k\) is uniquely defined, and the postulate (38) is formally consistent.

In order to express function \(f_j\), consider firstly the case \(j=2\)
\[
R_2 / R_1 = f_2 (d_{21})
\]
\hspace{1cm} (42)

Assume the existence of an extinction length "\(d_0\)" , i.e. of a distance beyond which the strengths of two points of the plate can be considered statistically independent, and put
\[
\vartheta_{ij} = \min \{1, d_{ij}/d_0\}
\]
\hspace{1cm} (43)

Thus
\[
R_2 / R_1 = f_2 (\vartheta_{12})
\]
\hspace{1cm} (44)
If \( \varphi_{12} = 0 \), \( Q_1 = Q_2 \), and \( f_2(0) = R_2 = R_1 \). If \( \varphi_{12} = 1 \), \( \tilde{Q}_O1 \) and \( \tilde{Q}_O2 \) are statistically independent, and \( f_2(1) = 1 \). Function \( f_2 \) can be interpolated on the interval \((0, 1)\) between these values.

A possible expression for \( f \) may be of the type (Fig. 6)

\[
f_2(\varphi_{12}) = 1 + (R_2 - 1) (1 - \varphi_{12})^6
\]

\( a \) being any positive real number. This expression of \( f_2 \) can be generalized to the case \( j > 2 \). Put in fact

\[
\varphi_j = \frac{j-1}{j-1} \prod_{\ell=1}^{j-1} \varphi_{1\ell}
\]

\( (46) \)

\[
f_j(\varphi_j) = 1 + (R_j - 1)(1 - \varphi_j)
\]

\( (47) \)

Then

\[
R_j/R_{j-1} |_{j-1} = 1 + (R_j - 1)(1 - \varphi_j)
\]

This position is consistent with the basic requirements. In fact, if \( \varphi_j = 0 \) for at least one value of \( j \), \( R_j |_{j-1} = 1 \), and \( f_j(0) = R_j \) while if \( \varphi_j = 1 \) for any \( j \), \( R_j |_{j-1} = R_j \), and \( f_j(1) = 1 \) (Fig. 6).

Finally, in Fig. 7 are quoted numerical results of the static approximation \( P_\psi (w) \) versus the extinction length \( d_\omega \) still for \( w = 1.3 \).
References


Fig. 1 - Simply supported rectangular plate under uniform loading

Fig. 2 - The investigated plate
Fig. 3 - The plane of the equilibrated stress fields
\[ \bar{C}_x = C_x - C_{x1} \ ; \bar{C}_y = C_y - C_{y1} \]

Fig. 4 - Optimal points for various values of \( A \)
- \( \bullet \) : \( A = A_0 \)
- \( \star \) : \( A = A_1 \)
- \( \odot \) : \( A = A_2 \)
Fig. 5 - The function $P_\Psi (w, d)$

Fig. 6 - Possible approximations of the function $f_j$

Fig. 7 - Static approximation of the collapse probability versus the extinction length $d_o$