THEORETICAL ANALYSIS OF STRESS, STRAIN AND FRACTURE FOR FAST REACTOR FUEL CLADDING UNDER CONSTANT POWER AND POWER-CYCLING CONDITIONS:

VISCO-ELASTIC MODEL OF NUCLEAR FUEL-PIN BEHAVIOUR

J. H. GITTUS

United Kingdom Atomic Energy Authority,
The Reactor Group, Reactor Fuel Element Laboratories,
Springfields, Salwick, Preston, Lancashire, United Kingdom

SUMMARY

In the model adopted for analysis, the strength of the pellet is entirely dictated by that of an outer crust—the zone within which irradiation-creep is dominant. The inner bound of the outer crust is the isotherm at which irradiation-creep and thermal creep become equal. The crust is assumed free from open radial cracks and in it, swelling due to fission-gas precipitation is negligible: solid-fission-products are the only source of swelling. Irradiation-creep entirely dominates the plastic deformation of the cladding; like the pellet, the cladding swells even at zero stress.

The stress (S) in the cladding at constant rating is calculated as a function of burn-up; it rises towards an asymptote. Square-wave rating variations produce a new equilibrium in which S alternates between levels that depend on the fraction (f) of burn-up accumulated at high rating.

At constant rating, the equilibrium value of $S\ (=S_\infty)$ is about 3.5 MPa (500 psi) and the effects of irradiation dominate the deformation-processes in both pellet and cladding. The equilibrium tensile creep-rate of the cladding is about 1.25×10^{-3} per atom % burnup. The approach to S_∞ occurs exponentially, with a time-constant (τ) of about 5 h (during which a burnup of about 0.02 atom % occurs).

If a crack is propagating through the clad by the Tomkins mechanism then, theoretically, there will be a critical value of f at which propagation-rate (r) is a maximum. f-values that significantly increase over its constant rating value are unlikely to occur in practice.

The conditions which will lead to the formation, during square-wave rating variations of a gap between pellet and clad are defined and the cyclic stress-states for the cases where a gap forms or does not form are calculated. In both cases the value of f is a determining factor, provided that the cycle-period is much less than τ.

1. Introduction

The general features of visco-elastic models of the kind that are suitable to the analysis of nuclear fuel pin behaviour have been laid down [1]: they were developed from earlier studies of the relaxation behaviour of materials [2,3] and have been applied to the calculation of non-metallic fuel behaviour [4]. In the present paper some results of applying this technique to the behaviour of metal clad nuclear fuel pins are given.

2. Model Adopted for Analysis

The differential equation for the change in state of strain in a model fuel pin is as follows:

$$d\varepsilon_s + d\varepsilon_u = (S_u - S_s)\, dB + \alpha_u d\theta_u - \alpha_s d\theta_s - (K_u\varepsilon_u + K_s\varepsilon_s)dB \qquad (1)$$

In equation (1), ε_s is the elastic (tensile) strain in the clad

ε_u is the elastic (compressive) strain in the pellet

dB is a small increment of burnup, at%

α_u is the thermal expansion coefficient of the pellet's outer crust, $^{o}C^{-1}$

α_s is the thermal expansion coefficient of the cladding, $^{o}C^{-1}$

$d\theta_u$ is the rise in effective temperature of the pellet's crust, due to a power increase, ^{o}C

$d\theta_s$ is the associated rise in clad temperature, ^{o}C.

Now the compressive force in the outer crust of the UO_2 always equals the tensile force in the cladding

So
$$\varepsilon_s Y_s Z_s = \varepsilon_u Y_u Z_u \qquad (2)$$

In equation (2), Y is an elastic modulus and Z is the thickness of clad or pellet-crust. From equation (2), then,

$$\varepsilon_u = \frac{Y_s Z_s}{Y_u Z_u} \cdot \varepsilon_s \qquad (3)$$

Substituting equation (3) and its differential into equation (1):

$$M d\varepsilon_s = (S_u - S_s)\, dB + \alpha_u d\theta_u - \alpha_s d\theta_s - K\varepsilon_s dB \qquad (4)$$

In equation (4):

$$M = \left[1 + \frac{Y_s Z_s}{Y_u Z_u}\right] \qquad (5)$$

and
$$K = \left[K_u \cdot \frac{Y_s Z_s}{Y_u Z_u} + K_s\right] \qquad (6)$$

Equation (4) can be integrated as follows:

$$\int_{\varepsilon_{so}}^{\varepsilon_s} \frac{d\varepsilon_s}{(S_u - S_s) + \alpha_u \cdot \frac{d\theta_u}{dB} + \alpha_s \frac{d\theta_s}{dB} - K\varepsilon_s} = \frac{B}{M} \qquad (7)$$

$$\frac{(S_u - S_s) + \alpha_u \cdot \frac{d\theta_u}{dB} + \alpha_s \frac{d\theta_s}{dB} - K\epsilon_s}{(S_u - S_s) + \alpha_u \cdot \frac{d\theta_u}{dB} + \alpha_s \cdot \frac{d\theta_s}{dB} - K\epsilon_{so}} = \exp \left[\frac{-B}{M/K}\right] \tag{8}$$

Equation (8) is the steady state solution for the elastic strain, ϵ_s, produced in fast reactor cladding by swelling, creep and thermal expansion of both pellet and clad. The strain moves to ϵ_s from ϵ_{so} in burnup B. The relaxation burnup (B_e) for the process is given by

$$B_e = M/K \text{ at\%} \tag{9}$$

$$= \text{(typically) } 0.02 \text{ at\%} \tag{10}$$

So the relaxation time will be the time taken to produce 0.02 at% burnup, i.e. some hours.

When $B \rightarrow \infty$ and if $\frac{d\theta_u}{dB} = \frac{d\theta_s}{dB} = 0$, equation (8) gives

$$\epsilon_{s\infty} = \frac{(S_u - S_s)}{K} \tag{11}$$

Here $\epsilon_{s\infty}$ is the equilibrium elastic strain in the cladding. Note that it is independent of flux (because swelling rates and creep rates both have a linear flux dependence). Using the values given below and setting $Y_s Z_s = Y_u Z_u$, $\epsilon_{s\infty} = 0.0025\%$, a strain which will produce a hoop stress of about 500 lb/in^2 (3.5 MP_a) in the cladding.

The equilibrium creep rate of the cladding will be $K_s \epsilon_{s\infty} = 50 \times 0.0025 = 0.125\%$ per at% burnup. If the irradiation creep simply consisted of void-enlargement (see for example ref. [6]) then a volume strain of $0.125 \times 2 = 0.25\%$ would be produced (the stress in the clad is equibiaxial), and the stress due to the UO_2 would double the decrease in density, per unit burnup, of the clad.

3. Rapid Power Cycling: Approximate Equilibrium Solutions

Equation (8) is a generalsolution, applicable to any type of reactor operation, including power cycling. Simpler, approximate solutions can be formulated for certain classes of power cycle, however, and particular interest attaches to cycles whose duration is brief compared to the relaxation burnup, B_e. Such cycles are typified by those which frequency-governed operation will produce and comprise power changes of a few per cent lasting for an hour or so and occurring several times daily. If the contraction of the pellet during one of these minor power reductions exceeds the elastic strain in pellet and clad then a gap will form between pellet and clad. Otherwise (if the conditions are not conducive to gap formation) a small power reduction will simply reduce the elastic strain in pellet and clad. Approximate solutions of equation (1) exist for where a gap forms and for where no gap forms and they are arrived at in the next two sub-sections.

4. Solution if No Gap Forms

The burnup accumulated <u>during</u> a small power change is orders of magnitude lower than the relaxation burnup, B_e. Accordingly, swelling and creep are negligible during the power change and equation (4) can be approximated by

$$d\epsilon_{sc} = \frac{\alpha_u \, d\theta_{uc} - \alpha_s \, d\theta_{sc}}{M} \tag{12}$$

In equation (12) $d\varepsilon_{sc}$ is the tensile strain increment produced in the cladding by an upward power excursion which causes the clad temperature to rise by $d\theta_{sc}$ and the pellet temperature to rise by $d\theta_{uc}$.

Now consider subsequent operation (following the small power increase) at constant power. Swelling and creep will alter the elastic strain in the cladding: let the amount of that alteration be $d\varepsilon_{SH}$ and let the alteration during periods of constant operation at low power be $d\varepsilon_{SL}$. Then at equilibrium:

$$d\varepsilon_{SH} + d\varepsilon_{SL} = 0 \tag{13}$$

From equations (4) and (13):

$$(S_u - S_s)\ (dB_H + dB_L) - K(\varepsilon_{SH} dB_H + \varepsilon_{SL} dB_L) = 0 \tag{14}$$

Now

$$\varepsilon_{SH} - \varepsilon_{SL} = d\varepsilon_{sc} \tag{15}$$

Using equation (15) in equation (14):

$$(S_u - S_s)\ dB_T - K([\varepsilon_{SL} + d\varepsilon_{sc}]\ dB_H + \varepsilon_{SL}\ dB_L) = 0$$

i.e.

$$\left(\frac{S_u - S_s}{K}\right) - \varepsilon_{SL} - d\varepsilon_{sc} \cdot \frac{dB_H}{dB_T} = 0$$

and therefore, using equation (11):

$$\varepsilon_{SL} = \varepsilon_{s\infty} - d\varepsilon_{sc} \cdot \frac{dB_H}{dB_T} \tag{16}$$

Whilst from equation (16) and (15):

$$\varepsilon_{SH} = \varepsilon_{SL} + d\varepsilon_{sc}$$

$$= \varepsilon_{s\infty} + d\varepsilon_{sc} \left(1 - \frac{dB_H}{dB_T}\right)$$

i.e.

$$\varepsilon_{SH} = \varepsilon_{s\infty} + d\varepsilon_{sc} \cdot \frac{dB_L}{dB_T} \tag{17}$$

Equations (16) and (17) are the required approximate solutions for rapid power cycles ($dB_T < B_e$) which do not cause a gap to form between clad and pellet.

5. Condition for Gap Formation

If the differential contraction of pellet, relative to cladding, exceeds the equilibrium elastic strain then a power reduction will cause a gap to form and the condition for this to happen can be derived from equation (16). It is:

$$\frac{dB_H}{dB_T} > \frac{\varepsilon_{s\infty}}{d\varepsilon_{sc}} \tag{18}$$

6. Solution if a Gap Forms

If a gap forms between pellet and clad at low power then creep will cease for, if they are not in contact they cannot stress each other. Accordingly, in equation (14), $\varepsilon_{SL} = 0$ and so

$$(S_u - S_s)\ dB_T - K\varepsilon_{SH}\ dB_H = 0 \tag{19}$$

i.e.

$$\frac{S_u - S_s}{K} \cdot \frac{dB_T}{dB_H} = \epsilon_{SH}$$

i.e.

$$\epsilon_{SH} = \epsilon_{s\infty} \frac{dB_T}{dB_H} \qquad (20)$$

and

$$\epsilon_{SL} = 0 \qquad (21)$$

Equations (20) and (21) apply if inequality (18) is satisfied: otherwise equation (16) and (17) apply.

7. Propagation of Fracture

The class of duty in which cyclic stresses cause plastic strains, generally causes fracture by a process described by Tomkins [7] and examined from the standpoint of tension-tension stress histories by Gittus [8]. According to this fracture model, the fractional extension, $d\ell/\ell$, of a crack is given by:

$$\frac{d\ell}{\ell} \propto \sigma^2 \, d\epsilon \qquad (22)$$

Here σ is the current value of stress and $d\epsilon$ is the small increment of plastic strain.

Providing that the crack does not increase in length inordinately by this process one can regard ℓ in equation (22) as a constant. The extension of a crack in the cladding during a single power cycle is then given by

$$d\ell \propto \epsilon_{SH}^2 \cdot (K_s \, \epsilon_{SH} \, dB_H) + \epsilon_{SL}^2 \cdot (K_s \, \epsilon_{SL} \, dB_L) \qquad (23)$$

and so, for a succession of such cycles:

$$\frac{dB_T}{d\ell} \propto \frac{1}{\epsilon_{SH}^3 \cdot \frac{dB_H}{dB_T} + \epsilon_{SL}^3 \cdot \frac{dB_L}{dB_T}} = G, \text{ say} \qquad (24)$$

and

$$G_o = \frac{1}{\epsilon_{s\infty}^3} \qquad (25)$$

8. Parameter Values and Results

Parameter values used in the analysis are defined as follows:

It is assumed that the strength of the pellet is entirely dictated by that of an outer crust – the zone within which irradiation creep is dominant. Closer to the pellet centre the temperature is high enough to produce ordinary thermal creep in amounts which greatly exceed the irradiation-induced component. The inner bound of the outer crust is the isotherm at which irradiation creep and thermal creep become equal. At a rating of 100 MW/te this is the 1100°C isotherm and it is situated, for example, 0.022 in. from the outer surface of the pellet at the position of maximum rating in a fast reactor fuel pin.

The strong outer crust of the pellet is taken to be free from open, radial cracks. It creeps at a rate (K_u) of 50 elastic deflections per at% burnup.

Swelling due to fission-gas-precipitation is assumed negligible in the strong outer crust of the pellet. The crust swells at a rate ($3 \times S_u$) of 1 vol% per at% burnup.

Irradiation-induced creep is the dominant deformation mechanism in the cladding and occurs at a rate (K_s) of 50 elastic deflections per (adjacent) at% burnup. This rate has been deduced from a recent analysis of swelling and creep behaviour [5]: it assumes that irradiation creep continues at a rate which depends on the maximum swelling rate.

The unstressed cladding is assumed to swell isotropically at a rate ($3S_s$) of 0.25 vol% per at% burnup.

Using the parameter-values given above, from equation (5) (with $Y_sZ_s = Y_uZ_u$):

$$M = 2$$

and from equation (6),

$$K = 100$$

So equation (11) gives

$$\epsilon_{s\infty} = 2.5 \times 10^{-5}$$

and from equation (9):

B_e = 0.02 at% which defines the upper bound of dB_T below which this approximate analysis will produce serious errors.

Let $d\epsilon_{sc} = 10^{-4}$, a value approximately corresponding to a 5% power change. Then from equation (18) the condition for a gap to form is

$$\frac{dB_H}{dB_T} > 0.25$$

This means that if the fuel pin accumulates more than 25% of its burnup at high power then enough tensile creep will occur in the cladding during the high power periods to keep it clear of the pellet during the intervening periods at reduced power.

It is quite likely that the reactor will operate for part of the time with $dB_H \div dB_T < 0.25$, for many of the small power changes comprise brief excursions to a slightly higher power level. In Table I, therefore, values for the strains and normalised G-values are given as a function of the burnup ratio, dB_H/dB_T for the full range of values (0 to 1) of that ratio, switching from equations (16) and (17) to equations (20) and (21) at the point where a gap forms (i.e. for $dB_H/dB_T \div dB_T > 0.25$).

In Table I, $G \div G_o$ goes through a minimum at (or near to) the burnup ratio $dB_H \div dB_T = 0.25$, i.e. for cycles in which a gap is on the point of forming. Cycles which comprise brief downward power excursions are less damaging than those which involve brief upward excursions.

The highest stress levels are produced by very brief upward power excursions following long periods at low power. Thus the strain of 1.25×10^{-4} so produced corresponds to a tensile stress of about 2,500 lb/in^2 (17.2 MP_a) in the cladding.

9. Conclusions

An analysis has been made of the elastic and plastic strains produced, at constant power and under power cycling conditions in the cladding on a nuclear fuel pin. The effects of swelling, thermal expansion and irradiation - enhanced creep in both clad and pellet have been taken into account.

At constant power the elastic strain in the clad moves towards an equilibrium value of about 0.25×10^{-4} (i.e. a tensile stress of a few MP_a with an e-folding burnup of around 0.02 at%. This equilibrium stress could be sufficient to double the density decrease due to void formation in the cladding if the irradiation-creep process is void enlargement.

Rapid power cycles of small amplitude cause the stress in the clad to fluctuate assymetrically about its equilibrium value. The highest tensile stresses in the clad are produced by brief upward power excursions. The greater the proportion of the total burnup which accumulates at high power, during a power cycle, the lower the stress in the cladding and the latter falls to zero during the low power parts of the power cycle if more than 25% of the burnup accumulates at high power.

From the stress and plastic strains the rate at which a crack will propagate through the cladding has been calculated. Providing that, during the cycle, the burnup does not increase by more than the e-folding value (and rapid power cycles mostly fulfil this requirement) the number of cycles and the rate of change of power have no effect on propagation rate.

The important parameter is the proportion of the total burnup accumulated at high power.

At high load-factors, rapid cycles which involve upward power excursions are more important than those which involve downward power excursions.

TABLE I

Theoretical values for elastic strains and fracture propagation

$dB_H \div dB_T$	0	0.2	0.6	1.0
$G \div G_o$	1.0	0.08	0.42	1.0
$10^4 \times \epsilon_{SL}$	0.25	0.08	0.00	0.00
$10^4 \times \epsilon_{SH}$	1.25	1.08	0.40	0.24

References

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