# RESPONSE PREDICTIONS FOR RESONANT AND RANDOM VIBRATIONS OF LAST REACTOR FUEL SUB-ASSEMBLIES

D.A. JOBSON
*United Kingdom Atomic Energy Authority*
*Reactor Group, Risley, Warrington, United Kingdom*

## SUMMARY

The sub-assemblies of a fast reactor core are subject to small disturbances, particularly as a result of any unsteadiness in the coolant out-flow. Fluctuations of reactivity may thus result from the associated lateral fuel movements induced at core level. These movements are likely to be complex in character and sensitive to any factors which affect the flow conditions at exit. Response to the latter will depend on the disposition of the fuel and other components, within each subassembly, as well as on the flexibility of the wrapper and the manner of its support.

A simple analytical model is proposed to explore the interactions between the above factors, so enabling comparative predictions to be made between alternative designs. It is based on a consideration of the point of application and likely nature of the excitation. This suggests that only the fundamental mode of the sub-assembly will normally be significantly excited in the above way. Having determined the frequency and modal shape of the latter, its excitation is defined through the use of a Newtonian force coefficient in terms of the dominant flow parameters at exit. Using the diameter of the latter as a normalising dimension, a Strouhal number $S(=ND/V)$ is introduced, based on the undamped natural frequency ($N$), the diameter at exit ($D$) and the associated mean flow velocity ($V$). The root mean square (r.m.s.) of the resonant response is thus obtained and defined in terms of the Strouhal number, for a given sub-assembly and coolant density.

The more realistic dynamic response to broad-band excitation is dependent on the effective cut-off frequency for the latter. Expressions have therefore also been derived for the r.m.s. response to random excitation, based on two alternative assumptions concerning the power spectrum of the latter. In one case cut-off frequencies are related to exit flow parameters; in the other it is assumed that effective band-widths depend on system natural frequencies.

## 1. Introduction

Unsteadiness in the flow at exit from a fuelled fast reactor sub-assembly gives rise to disturbances which cause some oscillatory movement. The particular significance of the latter is that associated fluctuations of reactivity occur, induced by lateral displacements of the fuel at core level. Dynamically the problem is similar in character to wind induced oscillations of large circular stacks, except, in that case, the disturbances normally arise from external transverse flow, see reference [1]. The present paper is concerned with the application of dynamic methods to the consequent resonant and random response of sub-assemblies. The results so obtained enable assessments to be made of the likely effect of design changes, such as repositioning of the core and breeder fuel, or changes in the spike, the wrapper and its support positions. By assuming that the disturbances arise dynamically from the flow at exit, account is also taken of such factors as changes in coolant velocity and the effect of outlet diameter.

## 2. Coupled equations for a lumped representation

Fig 1 shows a typical sub-assembly, which is restrained at each end of the spike and at the support pads just below core level.

The distributed inertia of each sub-assembly is lumped as a number of discrete masses $m_0$, $m_1$ etc along their axes; the latter are assumed to flex as a beam of varying cross-section, as may the leaning post around which each cluster of sub-assemblies is centred, see fig 2. Static analysis enables the lateral deflection of any node to be related to the forces applied at each of the other nodes. Conversely, it is possible to deduce the elastic force which arises at each node due to its distortion into any given shape, as defined by the component deflections $u_0$, $u_1$ etc. An array of stiffness expressions is thus obtained which are of the type:

$$k_{r_0} u_0 + k_{r_1} u_1 + \dots \qquad (1)$$

for the elastic forces acting on any node such as r. This array may be expressed in matrix terms as:

$$\underset{\sim}{K}\,\underset{\sim}{u} \qquad (2)$$

in which $\underset{\sim}{K}$ is, by the reciprocal theorem, symmetric and $\underset{\sim}{u}$ is a column of the component deflections $(u_0\ u_1\ \dots)^T$. The elastic restoring forces, together with the externally applied forces $\underset{\sim}{f} = (f_0\ f_1\ \dots)^T$ determine the mass accelerations, so that the coupled set of equations for the undamped motion are of the form:

$$\underset{\sim}{M}\,\ddot{\underset{\sim}{u}} + \underset{\sim}{K}\,\underset{\sim}{u} = \underset{\sim}{f} \qquad (3)$$

in which $\underset{\sim}{M}$ is a diagonal matrix of the lumped masses.

## 3. Corresponding modal equations

It is convenient to express the resulting motion as a superposition of the separate modal responses. These latter arise, not only from the fundamental and higher flexural modes of the sub-assemblies, see fig 3, but also from their behaviour as a group round each leaning post. In addition to the symmetric modes, for which the leaning post remains at rest, there are asymmetric modes in which the leaning post participates, see fig 4. If $\underset{\sim}{\phi} = (\phi_0\ \phi_1\ \dots)^T$, defines one of these modal shapes, then the corresponding $\underset{\sim}{u}$ is related to the amplitude $\alpha$ of that mode by:

$$\underset{\sim}{u} = \alpha \underset{\sim}{\phi} \tag{4}$$

Premultiplication of the equation of motion by the transpose $\underset{\sim}{\phi}^T$ gives the corresponding uncoupled modal equation:

$$(\underset{\sim}{\phi}^T \underset{\sim}{M} \underset{\sim}{\phi}) \ddot{\alpha} + (\underset{\sim}{\phi}^T \underset{\sim}{K} \underset{\sim}{\phi}) \alpha = \underset{\sim}{\phi}^T \underset{\sim}{f} \tag{5}$$

in which $\underset{\sim}{\phi}^T \underset{\sim}{M} \underset{\sim}{\phi}$ is the quadratic expression which occurs in the denominator of Rayleigh's quotient [2] for the corresponding undamped natural frequency:

$$\omega_n^2 = (\underset{\sim}{\phi}^T \underset{\sim}{K} \underset{\sim}{\phi})/(\underset{\sim}{\phi}^T \underset{\sim}{M} \underset{\sim}{\phi}) \tag{6}$$

We are primarily interested in the response of the system to excitation at 0, which is taken to be at the top of one of the sub-assemblies; for that case:

$$\underset{\sim}{\phi}^T \underset{\sim}{f} = \phi_0 f_0 = f_0 \tag{7}$$

if the mode shape is normalized to $\phi_0 = 1$. The corresponding equivalent mass in the equation of motion is then:

$$m_e = \underset{\sim}{\phi}^T \underset{\sim}{M} \underset{\sim}{\phi} = m_0 + m_1\phi_1{}^2 + m_2\phi_2{}^2 + \ldots \tag{8}$$

and the modal equation can be written as:

$$\ddot{\alpha} + \omega_n^2 \alpha = f_0/m_e \tag{9}$$

The corresponding equivalent stiffness is thus:

$$k_e = \omega_n^2 m_e \tag{10}$$

and the equivalent static deflection in response to a force of magnitude $f_0$ is, in these terms:

$$\alpha = f_0/m_e\omega_n^2 = f_0/k_e \tag{11}$$

## 4. The effect of damping

If an equivalent linear damping is introduced into the modal equation, through the use of a factor $\zeta$ (which takes the value unity if natural oscillations are dead-beat) the modal equation becomes:

$$\ddot{\alpha} + 2\zeta\omega_n \dot{\alpha} + \omega_n^2 \alpha = f_0/m_e \tag{12}$$

For light damping, the amplitudes of the successive swings when the system is not being excited occur at intervals of $2\pi/\omega_n$ in the mode considered; they are such that:

$$\zeta = (X_p - X_{p+1})/2\pi X_p. \tag{13}$$

It is known however that the slight asymmetries which are characteristic of a real support system can give rise to beats, which result from the superposition of damped modal oscillations which differ only slightly in frequency. This results in a consequent wandering of energy between the components of the cluster as the oscillations decay.

## 5. Harmonic and resonant response

The steady-state response to harmonic excitation, corresponding to the real part of $f_0 = F_0 \exp i\omega t$, may similarly be written as $\alpha = A \exp i\omega t$ in which:

$$A = (F_0/k_e) \left[1 - (\omega/\omega_n)^2 + i\, 2\zeta\omega/\omega_n\right]^{-1} = (F_0/k_e)\, H(\omega) \tag{14}$$

see for example ref [3].

The corresponding mean-square value of $\alpha$, defined by:

$$<\alpha^2> = \int_0^T \alpha^2 \, dt/T \tag{15}$$

is thus related to $<f_o^2>$ by:

$$<\alpha^2> = HH^* <f_o^2>/k_e^2 = |H|^2 <f_o^2>/k_e^2 \tag{16}$$

in which $H^*$ is the complex conjugate of H and $|H|$ is its modulus, so that:

$$|H|^2 = \left[\{1 - (\omega/\omega_n)^2\}^2 + \{2\zeta\omega/\omega_n\}^2\right]^{-1} \tag{17}$$

For a lightly damped system the peak value of $|H|$ occurs at $\omega = \omega_n$; the r.m.s. values of $\alpha$ and $f_o$ are then related by:

$$\sqrt{<\alpha^2>} = \sqrt{<f_o^2>}/2\zeta k_e = \sqrt{<f_o^2>}/2\zeta m_e \omega_n^2 \tag{18}$$

The disturbing forces arise dynamically from momentum fluctuations in the exit jet and are thus expressible as:

$$f_o = \tfrac{1}{2} C_o \rho A V^2 \tag{19}$$

in which $\rho$ is the density of the coolant, A is the exit area $\pi D^2/4$ and V is the mean coolant velocity through that area. There are currently insufficient data to specify the force coefficient $C_o$ or its variation with the various non-dimensional coefficients, such as Reynolds' number, on which its value may depend. Introducing the Strouhal number S, based on the undamped natural frequency, N $(= \omega_n/2\pi)$:

$$S = ND/V \tag{20}$$

we thus finally obtain for the resonant response:

$$\sqrt{<\alpha^2>} = C_o \rho D^4/64\pi S^2 m_e \zeta \tag{21}$$

Resonant response, although unlikely, could infrequently occur. Even if it does not, such an expression enables comparative assessments to be made between alternative designs for this hypothetical condition. It highlights some of the factors which influence the relative response, particularly the predicted sensitivity of the resonant response to the value of S.

6. Response to white noise

We next suppose that the fluid induced excitation at the outlet from a sub-assembly is not essentially repetitive in character. It is probably more realistic to assume that the forces will be basically random and spread over a range of frequencies with no fixed phase relationship between them. Because of this lack of coherence in the relative phase relationships, and the large number of exciting frequencies which may be present, it becomes more convenient to work spectrally in terms of power. The power spectral density of $f_o$, $W_f(\omega)$ say, is related to the mean square of $f_o$ by:

$$<f_o^2> = \frac{1}{2\pi}\int_0^\infty W_f(\omega)\, d\omega \tag{22}$$

and the corresponding power spectral density of the response is such that:

$$<\alpha^2> = \frac{1}{2\pi}\int_0^\infty W_\alpha(\omega)\, d\omega \tag{23}$$

It can be shown that $W_f$ and $W_\alpha$ are related in the same way as the mean-square values of the harmonic response [3], that is:

$$W_\alpha = |H|^2 \, W_f/k_e^2 \tag{24}$$

see fig 5.

Thus if the power spectrum of $f_o$ is essentially flat over the range of frequencies at which the sub-assembly responds significantly, so that:

$$W_f \; (\omega) = W_o \tag{25}$$

then:

$$\langle\alpha^2\rangle = \frac{W_o}{2\pi k_e^2}\int_0^\infty \frac{d\omega}{\left[1 - (\omega/\omega_n)^2\right]^2 + \left[2\zeta\omega/\omega_n\right]^2} \tag{26}$$

This may be integrated by the method of residues [4] to obtain:

$$\langle\alpha^2\rangle = \omega_n \, W_o/8\zeta k_e^2 = W_o/8\zeta m_e^2\omega_n^3 \tag{27}$$

for light damping. The latter implies an essentially narrow-band random modal response, in which the system vibrates essentially at each of its natural frequencies, with a time dependent amplitude that has an essentially Gaussian distribution. It is not strictly meaningful to define a power spectrum of uniform density up to infinite frequencies; it would, for example imply an infinite mean square value for the excitation. If we again assume that $f_o$ is proportional to $\rho AV^2$, dimensional arguments require that the cut-off frequency for the power spectrum be proportional to $V/\sqrt{A}$, see fig 5, so that:

$$\langle f_o^2\rangle = \frac{1}{2\pi}\int_0^\infty W_f \; (\omega) \; d\omega$$

$$\propto W_o \; V/\sqrt{A} \tag{28}$$

We thus infer that:

$$W_o \propto (\rho AV^2)^2/(V/\sqrt{A})$$

$$\text{ie} \qquad W_o = K_o\rho^2 A^{5/2} V^3 \tag{29}$$

With this assumption:

$$\langle\alpha^2\rangle = K_o\rho^2 A^{\frac{5}{2}} V^3/8\zeta m_e^2\omega_n^3 \tag{30}$$

Introducing the Strouhal number $S = \omega_n \; D/2\pi \; V$ as before, we thus obtain

$$\sqrt{\langle\alpha^2\rangle} = \sqrt{(2 \; K_o)} \; \rho D^4/64\pi^{\frac{1}{4}} \; S^{\frac{3}{2}} \; m_e \; \zeta^{\frac{1}{2}} \tag{31}$$

A somewhat different result is obtained if the cut-off is related to the natural frequency of the mode considered. In that case:

$$W_o = K_1 \; (\rho AV^2)^2/\omega_n \tag{32}$$

so that:

$$\langle\alpha^2\rangle = K_1 \; \rho^2 A^2 V^4/8\zeta m_e^2\omega_n^4 \tag{33}$$

We then obtain:

$$\sqrt{<\alpha^2>} = \sqrt{(2\ K_1)}\ \rho D^4/64\pi S^2 m_e\ \zeta^{\frac{1}{2}} \qquad (34)$$

There are at present insufficient data to discriminate between the assumptions of a flow- or frequency- controlled cut-off for the power spectral density, but the consequences of each assumption are not significantly different.

7. Conclusions

The factors controlling the r.m.s modal responses of sub-assemblies have been examined and expressions obtained to assess the likely effect of each. The results emphasise the importance of a Strouhal number S, based on the outlet diameter (D), the mean fluid velocity at exit (V) and the natural frequency in Hz (N) of the mode considered:

$$S = ND/V \qquad (35)$$

Under resonant conditions the r.m.s. amplitude is found to be proportional to:

$$\rho D^4/S^2 m_e\ \zeta \qquad (36)$$

in which $m_e$ is the effective mass and $\zeta$ is the damping ratio.
The corresponding response to random excitation depends on the assumptions made for the cut-off frequency for the fluid generated noise which is postulated. If there is a controlling cut-off in the fluid spectrum, then the r.m.s. response is proportional to:

$$\rho D^4/S^{3/2}\ m_e \sqrt{\zeta} \qquad (37)$$

If however it is assumed that the sub-assemblies respond to noise up to a frequency which is governed by the natural period of the mode considered, then the r.m.s. response is determined by:

$$\rho D^4/S^2 m_e \sqrt{\zeta} \qquad (38)$$

Methods are available for the prediction of N, and hence S, which correlate well with experimental measurements. The effective vibratory mass ($m_e$) depends on the mode shape considered and the latter can be computed at the design stage [5] , allowance being made for the added mass due to fluid acceleration. Judgement is needed in estimating the effective value of the damping ratio $\zeta$, based on experimental information, such as that obtainable from the behaviour of sub-assemblies and clusters in water test rigs. Fig 6 shows a drawing of such a rig, designed specifically to study the vibration behaviour of sub-assembly clusters. Flow tests in this rig form part of the research, development and endorsement programme for the Dounreay Prototype Fast Reactor. A preliminary core-build of the latter, employing dummy sub-assemblies, has already provided confirmatory information on natural frequencies and modes of vibration. Further measurements will be made under sodium as part of the commissioning programme for that reactor. Subsequent core surveillance will provide the final feedback on sub-assembly vibration and other factors which contribute to reactivity noise.

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References

[1] Wootton, L. R., The Oscillation of Large Circular Stacks in Wind Proc ICE 1969, 43 pp 573-598 (see also communication by Flint, A. R. and Dawe, C. S., 45 p 522)

[2] Southwell, R. V. An introduction to the theory of elasticity, OUP (1941)

[3] ed Crandall, S. H., Random vibration. Technology Press (MIT) and John Wiley (1958)

[4] Churchill, R. V., Introduction to complex variables and applications, McGraw Hill (1948)

[5] Jobson, D. A. and Litherland, J. R., Vibration analysis by computer. UKAEA TRG Report 1919(R) H.M.S.O. (1969)

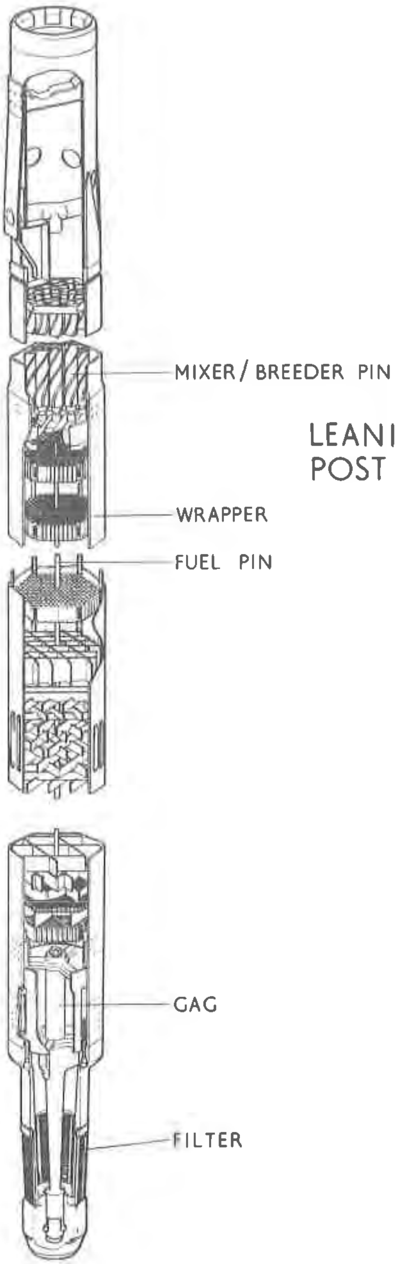


Figure 1 Sub-assembly for UK Prototype Fast Reactor at Dounreay

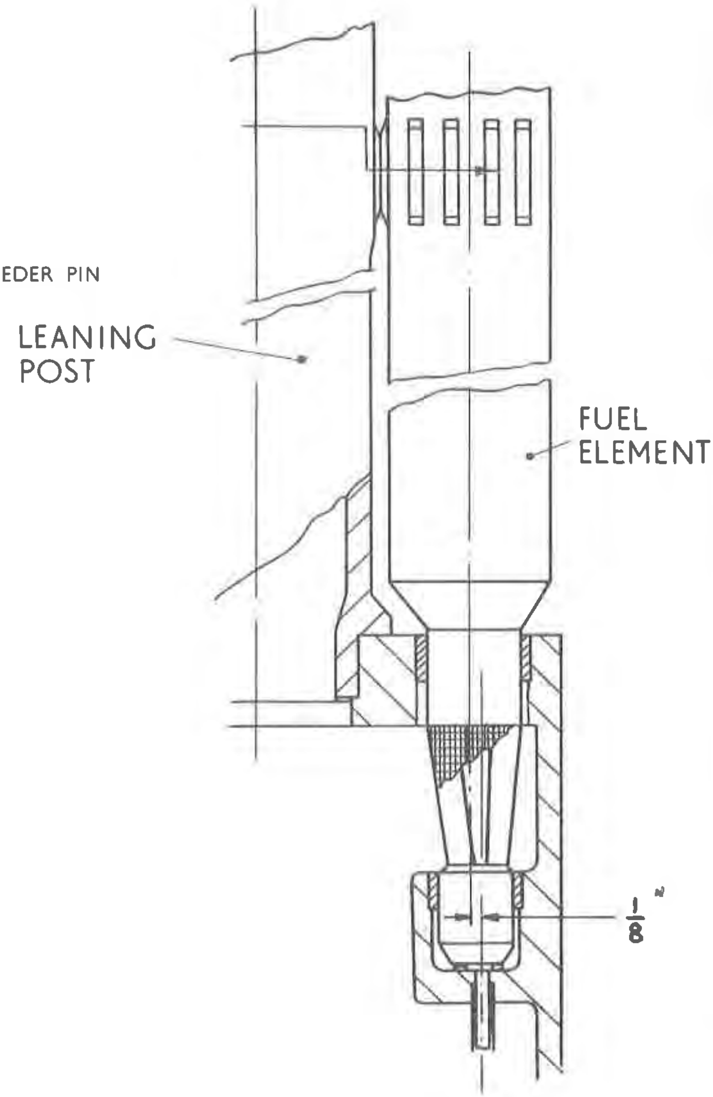


Figure 2 Sub-assembly centering mechanism in leaning post/cluster arrangement

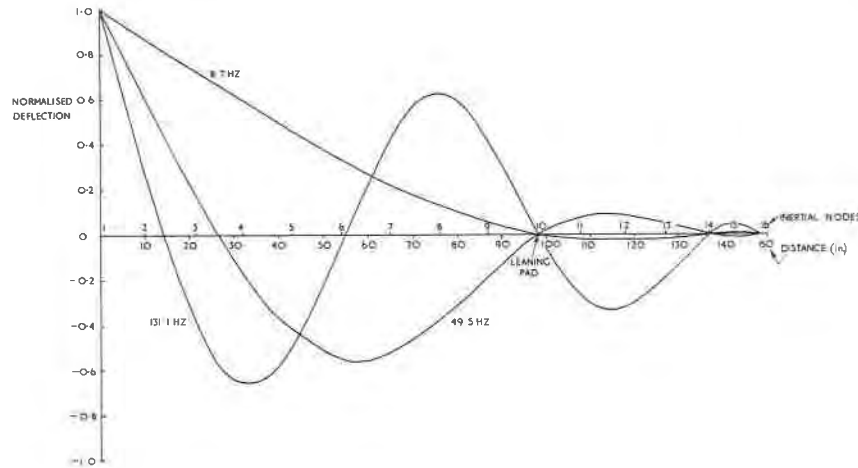


**Figure 3** Computed sub-assembly profiles for symmetric modes-leaning post stationary

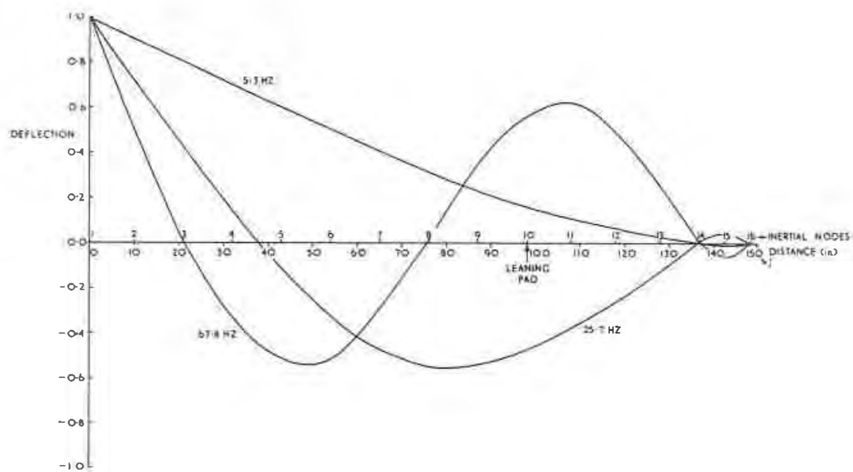


**Figure 4** Computed sub-assembly profiles for swaying cluster

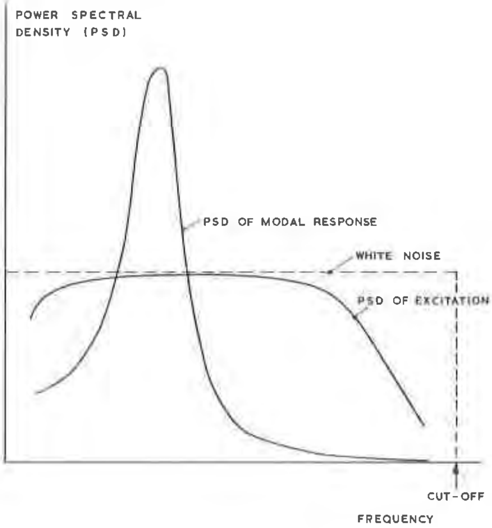


Figure 5    Power spectral densities of excitation and modal response

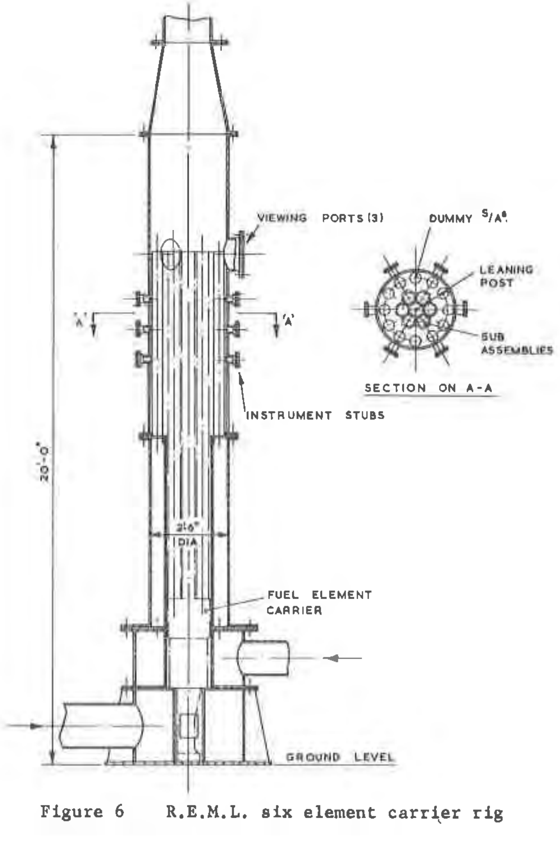


Figure 6    R.E.M.L. six element carrier rig