VIBRATION CODE FOR A FUEL PIN WITH HELICALLY WOUND WIRE SPACER

L. CHELLI, G. ORRÙ, G. PAPA
Comitato Nazionale per l'Energia Nucleare, Centro di Calcolo, I-40138 Bologna, Italy

SUMMARY

The code considers fuel pins consisting of bars subdivided lengthwise in segments of different mechanical characteristics corresponding to the various regions of a fast reactor. The fuel element proper is made up by a bundle of such pins. All the pins are fixed at one end by cylindrical hinges to the bundle container. To hold apart the pins from one another for cooling purposes, a steel wire is wound around each of them.

The aim of the code is to evaluate the proper frequencies and modes of vibration of an inner pin of the bundle assuming that the other pins of the bundle are not taking part in the vibration. Therefore a pin surrounded by six similar ones is considered. Its motion in space is constrained by its hinge, its wire spacer making contact with the neighbouring pins and by the pin itself making contact with the six wires of the latter. Thus the pin can be schematized as a beam with one end hinged, the other end free and held along its length by equally spaced rollers allowing the corresponding section of the beam to move only along a perpendicular to the axis and to rotate on the plane of such perpendicular and the axis. The slope of these perpendiculars with respect to any axial plane varies by 60° in passing from a roller to the successive one.

The axial components of the force (tension and torque) and displacements (elongation and torsion) are assumed as negligible. As the code is concerned only with the inception of vibratory phenomena, damping effects also are disregarded. This kinematic model is then handled by the frame theory. Therefore the whole length of the bar is subdivided in a certain number of elements interconnected at nodes, where are defined the coordinates of motion and are concentrated the masses of the elements. Thus the elastic motion of the pin is schematized as that of a string of masses held together by elastic beams.

To reduce for computational efficiency the number of the degrees of freedom of the system, sections of the bar where some displacement components are constrained are chosen as nodes that is the sections where the wires make contact. This leaves four degrees of freedom to each element (two rotations and two linear displacements). A further reduction of degrees of freedom is brought about after the stiffness matrix of the whole structure has been assembled, and is based on the assumption that at the lowest proper frequencies the inertial couples are negligible. In the end the degrees of freedom left for the wole system are only the linear generalized displacements, one for each node.

In computing the stiffness matrices of the single elements only the flexural stiffness coefficients have been considered, disregarding shear effects. The uranium oxide pellets have been considered as contributing only to the mass and not to the stiffness coefficients. The code can also handle elements that bestride the boundary between two segments of the fuel pin and can allocate concentrated masses anywhere along the same. The full capacity of the code is 100 degrees of freedom. The proper frequencies and modes output by the code can be the starting point for a random excitation vibration analysis of the pin.

The code has been made to study, in parallel with the experimental tests, the vibrations induced by fluid pressure fluctuations in the fuel element of the fast reactor PEC of the CNEN.

1. Introduction

The fast reactor fuel element pin here considered consists in a bar subdivided lengthwise in segments of different mechanical characteristics with the same outer diameter. The inner segment corresponds to the core region of the reactor, the two outer segments correspond to the reflector region. Besides these there are segments meant only for structural purposes.

The fuel element proper is made by a bundle of such fuel pins contained in a box with hexagonal section.

All the pins are fixed at one end by cylindrical hinges to a grid at the bottom of the box. To hold apart the pins from one another, thus allowing the coolant flow between them, they are provided with a steel wire wound helically around each of them.

Aim of the present work is to evaluate the proper frequencies of vibration and the corresponding vibration modes of an inner pin of the bundle, considered as self-standing, that is with the other pins of the bundle not taking part in the vibration.

2. Description of the Cinematic Model

We consider therefore a bar surrounded by six similar ones. The motion in space of the bar is constrained not only by its hinge but also by its wire making contact with the neighboring bars and by the bar itself making contact with the six wires of these. All the wires start at the same position relative to their bars and are wound with the same pitch.

Under these premises and supposing, as we have already said, the six neighbouring pins not taking part in the vibration, one can easily see that the bar can be schematized for structural calculations as a beam one end hinged, the other end free and held along its length by a series of frictionless equally spaced rollers allowing the corresponding point of the beam to move only along a perpendicular to the axis of the bar. The slope of these perpendiculars with respect to any axial plane varies by 60° in passing from a roller to the successive one.

Another perhaps more immediate way of rendering the cinematic constraints set by the wires is to consider the bar as going through a succession of equally spaced perpendicular slots, whose axes are rotated by 60° with respect to the preceding one and permitting frictionless relative motion of the corresponding section of the bar (see fig.1). Obviously the basic assumption underlying this cinematic model is that the axial components of the forces (tension and torque) and displacements (elongation and torsion) are negligible, so that any bar section does not vary its position along the axis or rotate around the same. Then the displacement in space of a bar section is sufficiently described by a vector of only four components: linear displacement along the x-direction, rotation around the same, linear displacement and rotation along and around the y-direction.

The same is valid for the force components, applied as well as internal.

3. Mathematical Implementation of the Cinematic Model

The cinematic model discussed in the preceding section can be efficiently handled by the frame theory.

For doing that the whole length of the bar must be subdivided in a certain number of elements, interconnected at points (nodes), in which are defined the coordinates of motion (that is the components of the displacement vector of the corresponding section of the bar) and are concentrated the masses of the bar. These nodes are therefore also the points of application of

the inertial forces. In other words one schematizes the elastic motion of the bar as the motion of a string of masses (nodes) held together by elastic beams (elements). The deformation of the bar at a certain instant is then defined by the displacement vectors of the nodes, whose components constitute then the degrees of freedom of the system.

To reduce the number of the degrees of freedom of the system, that is the number of unknowns in the motion equations, one must choose as nodes those sections of the bar in which some displacement components are constrained. Then the best choice for the nodes is as follows:

node 0 - corresponding to the section of the bar, where the cylindrical hinge reduces the four degrees of freedom of a general section to one: rotation around the hinge.

nodes from 1 to n : corresponding to the sections where the constraint of the contacting wires, that has been rendered by a roller or a slot reduces the degrees of freedom to two: linear displacement along a perpendicular to the axis of the bar and rotation on the plane defined by that perpendicular and the axis.

Thus the degrees of freedom left are $1+2n$, where n is the number of sections where the wires make contact. In other words one for describing the motion of the set of $n+1$ masses (nodes) has adopted a system of $1+2n$ generalized coordinates instead of the system of $4(n+1)$ constrained coordinates.

Therefore one can bring about a further reduction of the number of degrees of freedom. Because if the search for the vibratory nodes is limited to the first few ones, one can disregard the rotations of the sections with the related inertial moments retaining only the linear displacements and the inertial forces.

So in the last the degrees of freedom to consider for all the structure are only n , which is the number of all nodes with the exception of node 0 , which has been excluded because affected only by a rotatory displacement.

Owing to the fact that also in absence of external applied moments (in this case the inertial ones) the internal bending moments, transmitted between the elements, are always present, this last coordinate reduction is not possible at single structural element level, but must be carried out after the $1+2n$ degrees of freedom stiffness matrix of the bar has been assembled.

4. Stiffness and Mass Matrix of a Single Element

We consider only elements of uniform mechanical characteristics. Then disregarding, as we have already said, the axial force and displacement components, between the displacement and force components at the two ends of an element subsists, in a system of constrained coordinates, the relation

$$\left\{\begin{array}{c} p_1 \\ \hdashline p_2 \end{array}\right\} = \left[K_e\right] \left\{\begin{array}{c} d_1 \\ \hdashline d_2 \end{array}\right\}$$

where:

$$\{p_i\} = \left\{\begin{array}{c} p_{ix} \\ p_{iy} \\ m_{ix} \\ m_{iy} \end{array}\right\}$$

that is the force vector at the i-th end of the element

$$\{d_i\} = \begin{Bmatrix} d_{ix} \\ d_{iy} \\ \theta_{ix} \\ \theta_{iy} \end{Bmatrix}$$

that is the displacement vector at the same i-th end (see Fig.2)

$$[K_e] = \frac{2EJ}{\ell} \left[\begin{array}{cccc|cccc} \frac{6}{\ell^2} & 0 & 0 & \frac{3}{\ell} & -\frac{6}{\ell^2} & 0 & 0 & \frac{3}{\ell} \\ & \frac{6}{\ell^2} & -\frac{3}{\ell} & 0 & 0 & -\frac{6}{\ell^2} & -\frac{3}{\ell} & 0 \\ & & 2 & 0 & 0 & \frac{3}{\ell} & 1 & 0 \\ & & & 2 & -\frac{3}{\ell} & 0 & 0 & 1 \\ \hline & & & & \frac{6}{\ell^2} & 0 & 0 & -\frac{3}{\ell} \\ & & & & & \frac{6}{\ell^2} & \frac{3}{\ell} & 0 \\ & & \text{Sym} & & & & 2 & 0 \\ & & & & & & & 2 \end{array}\right]$$

that is the stiffness matrix of the element referred to the constrained coordinates.
The corresponding mass matrix is:

$$[M_e] = \left[\begin{array}{cccc|cccc} m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & m_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & I_1 & 0 & 0 & 0 & 0 & 0 \\ & & & I_1 & 0 & 0 & 0 & 0 \\ \hline & & & & m_2 & 0 & 0 & 0 \\ & & & & & m_2 & 0 & 0 \\ & & \text{Sym} & & & & I_2 & 0 \\ & & & & & & & I_2 \end{array}\right]$$

The m_i 's are the masses of the halves of the element with the mass of the pertaining length of wire added. Having considered only elements of uniform mechanical characteristics one has obviously:

$$m_1 = m_2$$

The I 's are the moments of inertia of the halves of the element about the end and one need not bother about them, having in mind to disregard in the last the rotatory components of motion.

In the foregoing derivation of the matrices $[K_e]$ and $[M_e]$ it has been assumed that the uranium oxide pellets contribute only to the masses and not to the stiffness coefficients. This stems from the fact that there must necessarily be some play between the outer diameter of the pellets and the inner diameter of the cladding. One is then justified in that assumption if one is interested only in an analysis of the inception of vibratory fenomena,

which entails a lateral deflection of the cladding very small compared to the sheath-fuel play.

For obtaining the system of generalized coordinates one can assume that in the general bar element the first end is free to move along a slot parallel to the x-direction and the second end along a similar slot rotated by an angle λ (in this case $\lambda=60°$) relative to the first one (see fig.3). The relation between the constrained and the generalized coordinates is then:

$$\begin{Bmatrix} d_{1x} \\ d_{1y} \\ \theta_{1x} \\ \theta_{1y} \\ \hline d_{2x} \\ d_{2y} \\ \theta_{2x} \\ \theta_{2y} \end{Bmatrix} = [C] \begin{Bmatrix} \delta_1 \\ \phi_1 \\ \hline \delta_2 \\ \phi_2 \end{Bmatrix}$$

where:

$$[C] = \left|\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & \cos\lambda & 0 \\ 0 & 0 & \sin\lambda & 0 \\ 0 & 0 & 0 & -\sin\lambda \\ 0 & 0 & 0 & \cos\lambda \end{array}\right|$$

The element stiffness matrix $[K'_e]$, referred to the generalized coordinates, can then be derived from the relation:

$$[K'_e] = [C]^T [K_e] [C]$$

and is, written in full:

$$[K'_e] = \frac{2EJ}{\ell} \left|\begin{array}{cc|cc} \frac{6}{\ell^2} & \frac{3}{\ell} & -\frac{6}{\ell^2}\cos\lambda & \frac{3}{\ell}\cos\lambda \\ & 2 & -\frac{3}{\ell}\cos\lambda & \cos\lambda \\ \hline & & \frac{6}{\ell^2} & -\frac{3}{\ell} \\ \text{Sym} & & & 2 \end{array}\right|$$

One can easily see that the constraint conditions for the first element of the bar, cylindrical hinge with axis parallel to the y-direction for the first end and slot an angle relative to the x-direction for the second end, correspond to the transformation matrix:

$$[C] = \left|\begin{array}{c|cc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ \hline 0 & \cos\lambda & 0 \\ 0 & \sin\lambda & 0 \\ 0 & 0 & -\sin\lambda \\ 0 & 0 & \cos\lambda \end{array}\right|$$

with generalized displacements:

$$\left\{\begin{array}{c} \phi_1 \\ \hdashline \delta_2 \\ \phi_2 \end{array}\right\}$$

and generalized stiffness matrix:

$$[K'_e] = \frac{2EJ}{\ell} \left|\begin{array}{c|ccc} 2 & -\frac{3}{\ell}\cos\lambda & \cos\lambda \\ \hline & \frac{6}{\ell^2} & -\frac{3}{\ell} \\ \text{Sym} & & 2 \end{array}\right|$$

The matrices $[K'_e]$ so obtained are not singular.
That signifies that the constraints imposed on the ends of the elements by the transformation $[C]$ block all rigid body movements even for an element disconnected from the structure.
The same transformation must be carried out on the element mass matrices to obtain the generalized mass matrices. Namely:

$$[M'_e] = [C]^T[M_e][C] = \left|\begin{array}{cc|cc} m_1 & 0 & 0 & 0 \\ & I_1 & 0 & 0 \\ \hline & & m_2 & 0 \\ \text{Sym} & & & I_2 \end{array}\right|$$

for the elements numbered from 2 through n , and

$$[M'_e] = \left|\begin{array}{c|cc} I_1 & 0 & 0 \\ \hline & m_2 & 0 \\ & & I_2 \end{array}\right|$$

for the first element.

5. Assemblage of the Stiffness and Mass Matrices for the Whole Bar.

So far the elements have been considered one by one and have been derived their stiffness and mass matrices referred to a system of generalized coordinates, which are the displacements of the ends. Now one must connect the elements together, adopting for the nodes of the structure so obtained a system of generalized node displacement coordinates with the

same directions of the end displacement coordinates of the single elements (see Fig.4). This is equivalent to impose the compatibility condition:

displacement vector of node i = displacement vector of end 2 of element i =

= displacement vector of end 1 of element i+1

that is:

$$\begin{Bmatrix} \delta_i \\ \phi_i \end{Bmatrix} = \begin{Bmatrix} \delta_2 \\ \phi_2 \end{Bmatrix}_i = \begin{Bmatrix} \delta_1 \\ \phi_1 \end{Bmatrix}_{i+1}$$

Moreover it is advisable, for performing the second reduction of degrees of freedom, to express the displacement vector for the whole structure with the components ordered the following way:

$$\left\{ \begin{array}{c} \delta_1 \\ \delta_2 \\ \cdot \\ \cdot \\ \delta_n \\ \hline \phi_0 \\ \phi_1 \\ \phi_2 \\ \cdot \\ \cdot \\ \phi_n \end{array} \right\}$$

The same is valid for the force vector.

To obtain the stiffness matrix $[K]$ of the whole structure, corresponding to displacement and force vectors with their components in the aforesaid order, one must at first assemble a matrix $[\chi]$ obtained juxtaposing along the diagonal the matrices of the elements:

$$[\chi] = \begin{vmatrix} [K_1'] & & & \\ & [K_2'] & & \\ & & [K_3'] & \\ & & & [K_n'] \end{vmatrix}$$

and perform on it the operations of imposing the compatibility conditions and interchanging rows and columns so as to conform to displacement and force vectors expressed in the aforesaid component order. All this is equivalent to the linear transformation:

$$[K] = [\beta]^T [\chi] [\beta]$$

where $[\beta]$ is a $(4n-1)\times(2n+1)$ matrix of the form:

$$[\beta] = \left|\begin{array}{cccc|c} & & & & \downarrow (n+1)\text{-th column} \\ 0 & 0 & 0 & 0 & 1\ 0\ 0\ 0 \\ 1 & 0 & 0 & 0 & 0\ 0\ 0\ 0 \\ 0 & 0 & 0 & 0 & 0\ 1\ 0\ 0 \\ \hline 1 & 0 & 0 & 0 & 0\ 0\ 0\ 0 \\ 0 & 0 & 0 & 0 & 0\ 1\ 0\ 0 \\ 0 & 1 & 0 & 0 & 0\ 0\ 0\ 0 \\ 0 & 0 & 0 & 0 & 0\ 0\ 1\ 0 \\ \hline 0 & 1 & 0 & 0 & 0\ 0\ 0\ 0 \\ 0 & 0 & 0 & 0 & 0\ 0\ 1\ 0 \\ 0 & 0 & 1 & 0 & 0\ 0\ 0\ 0 \\ 0 & 0 & 0 & 0 & 0\ 0\ 0\ 1 \end{array}\right|$$

The same linear transformation must be performed on the $[\mu]$ mass matrix obtained by juxtaposing along the diagonal the mass matrices of the single elements. But, contrarily to what must have been done for the stiffness, one in this case may simplify the transformation by eliminating at once the rotatory displacement components at single element level:

$$[M'_e] = \begin{vmatrix} m_1 & 0 \\ 0 & m_2 \end{vmatrix} \quad \text{with} \quad m_1 = m_2$$

Then juxtaposing and applying the compatibility condition one has at last the mass matrix of the whole structure:

$$[M] = [\gamma]^T[\mu][\gamma]$$

where $[\gamma]$ is a $(2n-1)xn$ matrix of the form:

$$[\gamma] = \left|\begin{array}{ccc} 1 & 0 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right|$$

and therefore:

$$[M] = \begin{vmatrix} (m_1+m_2) & 0 & 0 & & \\ 0 & (m_2+m_3) & 0 & & \\ 0 & 0 & (m_3+m_4) & & \\ & & & (m_{n-1}+m_n) & 0 \\ & & & 0 & m_n \end{vmatrix}$$

where m_i means now the mass of the half of the i-th element. One can now partition $[K]$ this way:

$$[K] = \left|\begin{array}{c|c} [K_{11}] & [K_{12}] \\ \hline [K_{21}] & [K_{22}] \end{array}\right|$$

which corresponds to considering the system of matricial equations:

$$\{F\} = [K_{11}]\{\delta\} + [K_{12}]\{\phi\}$$

$$\{M\} = [K_{21}]\{\delta\} + [K_{22}]\{\phi\}$$

where:

$\{F\}$ is the n-component vector of the external forces applied at the nodes.

$\{M\}$ is the (n+1)-component vector of the external moments applied at the nodes.

Then supposing the external, namely the inertial, moments negligible

$$\{M\} = \{\emptyset\}$$

one can eliminate the unknowns $\{\phi\}$ from the system, reducing it to the form:

$$\{F\} = [K^{**}]\{\delta\}$$

where:

$$[K^{**}] = [K_{11}] - [K_{12}][K_{22}]^{-1}[K_{21}]$$

is just the reduced-coordinate stiffness matrix sought.

6. Eigenvalue Problem

Having to solve a free vibration problem of a discretized structure, one is confronted by the equation:

$$[K^{**}]\{\delta\} = -[M]\{\ddot{\delta}\} \qquad (1)$$

All nodes of the structure, for a given vibratory mode, move in phase and one can write:

$$\{\delta\} = \{\delta_0\} \sin\omega t$$

that is:

$$\{\ddot{\delta}\} = -\omega^2\{\delta_0\} \sin\omega t$$

where ω is the angular frequency for the given mode.

Substituting in eq. (1) :

$$([K] - \omega^2[M])\{\delta_0\} = \{\emptyset\} \qquad (2)$$

This is a typical eigenvalue equation.

Solving it signifies to find the n angular frequencies ω_i of free vibration and the n associated modes $\{\delta_0\}_i$ of the structure.

Setting

$$\lambda = \frac{1}{\omega^2}$$

and factorizing $[K^{**}]$:

$$[K^{**}] = [L][L]^T$$

where $[L]$ is a square matrix with zero elements above the diagonal, eq. (2) can be rewritten in the form:

$$[L]^{-1}[M]\{\delta_0\} = \lambda[L]^T\{\delta_0\}$$

Setting:

$$[L]^T\{\delta_0\} = \{Z\} \qquad (3)$$

and

$$[H] = [L]^{-1}[M][L]^{-T}$$

it derives:

$$[H]\{Z\} = \lambda\{Z\}$$

in which $[H]$ has to be symmetric. This is a form of the eigenvalue problem for which many

computer subroutines are available, permitting to find the largest values of λ , that is the lower vibration frequencies $\frac{1}{2\pi\sqrt{\lambda}}$. From these one derives by eq. (3) the corresponding vibration modes $\{\delta_0\}_i$.

7. Program Output

To output the modes in a readable form one must revert to a system of constrained coordinates . The program assumes for the whole bar the y-direction parallel to the hinge axis and the x-direction perpendicular to it. Then for each node i the generalized displacement amplitude δ_{0i} can be decomposed into its components d_{0ix} and d_{0iy} by the relations:

$$d_{0ix} = \delta_{0i} \cos(60° \cdot i)$$
$$d_{0iy} = \delta_{0i} \sin(60° \cdot i)$$

8. General Considerations about the Program

The program just described presupposes that the boundaries of the segments of the pin, corresponding to the various regions of the reactor, coincide with a section of contact of the wires. The elements of the beam can thus only have uniform mechanical characteristics. Because the elements are as usual very short in comparison with the bar length, this is an acceptable approximation. However in view of a successive refinement of the program have been also devised such modifications of the same as to allow considering elements pertaining to two segments of the pin. With these modifications it is also possible to allocate concentrated masses along the bar.

References

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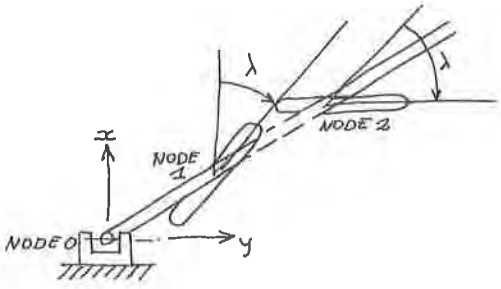

Fig. 1

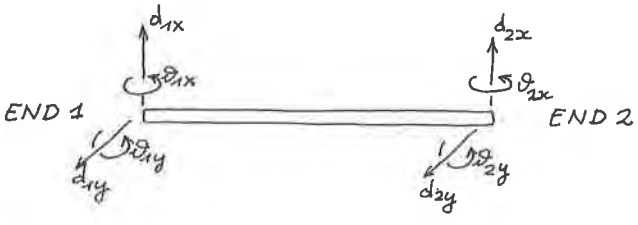

Fig. 2

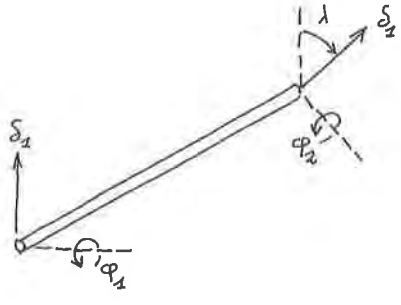

Fig. 3

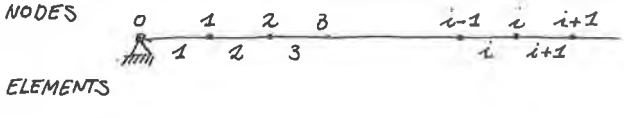

Fig. 4