

## FREE VIBRATION OF A CYLINDRICAL SHELL WITH A HEMISPHERICAL HEAD

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### SUMMARY

In this paper the mutual interference of shell structures is studied in the example of free vibration of a cylindrical shell with a hemispherical head.

The differential equations governing the deformation of thin elastic shells can be obtained from P.M. Naghdi: *Quart. Appl. Math.*, Vol. XIV, No. 4, pp. 369-380. Contrary to Naghdi, in this investigation the effects of transverse normal stress, transverse shear deformation, as well as rotary inertia are neglected. Assuming that the time dependence of all shell variables is of the form  $\sin(\omega \cdot t)$ , where  $\omega$  is the circular frequency, the solutions of the spherical shell equations are of the form

$$w = \sum_{m=0}^{\infty} \sum_{l=1}^3 A_l^m P_{n_l}^m(\cos \varphi) \cos m\vartheta \sin \omega t \quad \text{and} \quad \psi = \sum_{m=0}^{\infty} A_4^m P_{n_4}^m(\cos \varphi) \cos m\vartheta \sin \omega t \quad (1)$$

where  $w$  is the normal deflection of the shell and  $\psi$  is an auxiliary function, introduced in order to uncouple the differential equations governing the motion of the spherical shell (see: A. Kalnins: *J. Acoust. Soc. of Am.*, Vol. 33, No. 6, pp. 1102-1107). The  $P(\cos \varphi)$  are the associated Legendre functions of order  $m$  and degree  $n_l$ .

In the cases of a rigidly clamped and an absolutely free hemispherical shell with thickness to radius  $h/R = 1/50$  and Poisson's ratio  $\nu = 0.2$ , the referred frequencies  $\bar{\omega}$  were numerically calculated. For a complete cylinder, the general solution for free vibration can be written in the following form:

$$\begin{aligned} u &= \sum_{m=0}^{\infty} \sum_{s=1}^8 \alpha_{s \sin} W_{s \sin} e^{\lambda \sin X/R} \cos m\vartheta \sin \omega t \\ v &= \sum_{m=1}^{\infty} \sum_{s=1}^8 \beta_{s \sin} W_{s \sin} e^{\lambda \sin X/R} \sin m\vartheta \sin \omega t \\ w &= \sum_{m=0}^{\infty} \sum_{s=1}^8 W_{s \sin} e^{\lambda \sin X/R} \cos m\vartheta \sin \omega t. \end{aligned} \quad (2)$$

For a clamped-clamped and a clamped-free shell having the same properties  $h/R$  and  $\nu$  as the spherical shell discussed above and  $L/R = 1$ , the lowest frequencies are calculated.

The problem of free vibration of a cylindrical shell with a hemispherical head can be solved by the application of the solutions of the free vibrations of a spherical shell and of a cylindrical shell, discussed above.

Four boundary conditions at the edge of the cylindrical shell and eight junction conditions (four continuity conditions and four equilibrium conditions at the junction) yield a set of twelve equations in the twelve unknown constants  $A_l^m$  and  $W_{s \sin}$ . Since these conditions are homogeneous, the determinant of these equations must be zero for nontrivial solutions leading to the corresponding frequency  $\omega$ .

The results of parametric calculations (parameters  $h/R$  and  $L/R$ ) are discussed. Considering e.g. the 1.mode shapes, it is shown that from  $m = 0.2$  primarily the hemispherical shell is vibrating while the influence of the cylindrical shell gets more and more apparent for higher values of  $m$ . Whether the lowest frequency occurs for  $m = 1$  or higher values of  $m$ , strongly depends upon the parameters  $h/R$  and  $L/R$ .

1. Introduction

In the field of modern engineering very often shell-structures are used. For the proof of dynamical loading an investigation of the natural frequencies and mode shapes of vibration of these shells is necessary. In how far it is possible to estimate the natural frequencies and forms of the connected shells from the vibration of the single shells, shall be discussed in this paper.

Guided by the example of a cylindrical shell with a hemispherical head, the vibration of a shell-structure shall be studied.

The linear bending theory for thin, elastic, homogeneous, isotropic shells shall be valid. The influences of shear deformation as well as those of rotary inertia are of no importance on the lowest natural frequencies. Therefore these effects are not taken into consideration.

2. Vibrations of the Cylindrical Shell

Considering motion of an element of the shell, with the denotations in Fig. 2.1, one obtains 5 equations (2.1) for the 8 stress resultants and couples  $N_x, N_{\phi}, N_{x\phi}, M_x, M_{\phi}, M_{x\phi}, Q_x$  and  $Q_{\phi}$ :

$$\begin{aligned}
 N_x' + N_{x\phi}^{\circ} &= R \rho h \frac{\partial^2 u}{\partial t^2} \\
 N_{x\phi}' + N_{\phi}^{\circ} + Q_{\phi} &= R \rho h \frac{\partial^2 v}{\partial t^2} \\
 Q_x' + Q_{\phi}^{\circ} - N_{\phi} &= R \rho h \frac{\partial^2 w}{\partial t^2} \\
 M_x' + M_{x\phi}^{\circ} - R Q_x &= 0 \\
 M_{x\phi}' + M_{\phi}^{\circ} - R Q_{\phi} &= 0
 \end{aligned}
 \tag{2.1}$$

with  $(\ )' = \frac{\partial(\ )}{\partial x}$   
 $(\ )^{\circ} = \frac{\partial(\ )}{\partial \phi}$

where  $\rho$  is density of shell material.

The stress resultants  $N_x, N_{\phi}, N_{x\phi}$ , and the couples  $M_x, M_{\phi}, M_{x\phi}$ , are associated by the law of elasticity with extensions and changes of curvatures or derivations of displacements respectively.

$$\begin{aligned}
 N_x &= \frac{D}{R} (u' + \nu(v' + w)) \\
 N_{\phi} &= \frac{D}{R} (v' + w + \nu u') \\
 N_{x\phi} &= \frac{D(1-\nu)}{2R} (v' + u^{\circ})
 \end{aligned}
 \tag{2.2a}$$

$$\begin{aligned}
 M_x &= -\frac{K}{R^2} (w'' + \nu(\ddot{w} - \dot{v})) \\
 M_{\vartheta} &= -\frac{K}{R^2} (\ddot{w} - \dot{v} + \nu w'') \\
 M_{x\vartheta} &= -\frac{K(1-\nu)}{R^2} (\dot{w}' - \frac{v'}{2})
 \end{aligned}
 \tag{2.2b}$$

where  $D = \frac{Eh}{1-\nu^2}$  is the extensional rigidity

and  $K = \frac{Eh^3}{12(1-\nu^2)}$  is the bending rigidity of the shell.

Inserting the equations (2.2 a,b) into (2.1) and separating the time by (2.3)

$$\begin{aligned}
 u &= U(x, \vartheta) \cdot \sin \omega t \\
 v &= V(x, \vartheta) \cdot \sin \omega t \\
 w &= W(x, \vartheta) \cdot \sin \omega t
 \end{aligned}
 \tag{2.3}$$

one obtains three partial differential equations for the displacements  $u$ ,  $v$  and  $w$ .

$$\begin{aligned}
 u'' + \bar{\omega}^2 u + \frac{1-\nu}{2} \ddot{u} + \frac{1+\nu}{2} \dot{v}' + \nu w' &= 0 \\
 \frac{1+\nu}{2} \dot{u}' + \ddot{v} + \bar{\omega}^2 v + \frac{1-\nu}{2} v'' + \epsilon(\ddot{v} + (1-\nu)\frac{v''}{x}) + \dot{w} - \epsilon(\dot{w}'' - \ddot{w}') &= 0 \\
 \nu u' + \dot{v} - (\ddot{v} + \dot{v}') + w - \bar{\omega}^2 w + \epsilon \nabla_x^2 \nabla_z^2 w &= 0
 \end{aligned}
 \tag{2.4}$$

where  $\epsilon = \frac{h^2}{12R^2}$  is a shell parameter,  $\bar{\omega}$  is the dimensionless frequency

$\bar{\omega}^2 = \frac{\rho R^2 (1-\nu^2)}{E} \omega^2$  and  $\nabla_z^2$  is the Laplace operator in cylindrical coordinates.

$$\nabla_z^2 ( ) = \frac{\partial^2 ( )}{\partial (\frac{x}{R})^2} + \frac{\partial^2 ( )}{\partial \vartheta^2}
 \tag{2.5}$$

The solutions of the differential equation system (2.4) are the functions:

$$\begin{aligned}
 w &= \sum_{m=0}^{\infty} \sum_{s=1}^8 W_{sm} e^{\lambda_{sm} \frac{x}{R}} \cos m\vartheta \sin \omega t \\
 u &= \sum_{m=0}^{\infty} \sum_{s=1}^8 \alpha_{sm} W_{sm} e^{\lambda_{sm} \frac{x}{R}} \cos m\vartheta \sin \omega t \\
 v &= \sum_{m=0}^{\infty} \sum_{s=1}^8 \beta_{sm} W_{sm} e^{\lambda_{sm} \frac{x}{R}} \sin m\vartheta \sin \omega t
 \end{aligned}
 \tag{2.6}$$

The  $\lambda_{sm}$  are the roots of the characteristic equation.

The coefficients  $\alpha_{sm}$  and  $\beta_{sm}$  are determinable from the differential equation system (2.4).

For every wave number  $m$ , from the 8 boundary conditions - 4 at the edge  $x = 0$

and 4 at the edge  $x = l$  -, results a homogeneous system of equations by means of which the natural frequency and the associated amplitudes may be calculated.

For a clamped-clamped and a clamped-free cylindrical shell with  $h/R = 1/50$ ,  $l/R = 1$  and Poisson's ratio  $\nu = 0.2$ , the lowest natural frequencies have been calculated. The results of the numerical evaluation have been entered in table 1. The lowest frequency of the clamped-clamped shell lies at  $m = 6$  ( $\bar{\omega}_6 = 0.4081$ ), those of the clamped-free shell lies at  $m = 4$  ( $\bar{\omega}_4 = 0.1873$ ) (see Fig. 2.2).

### 3. Free Vibrations of a Hemispherical Shell

Referring to spherical coordinates (Fig. 3.1), one obtains the equations of motion of the spherical shell when considering an element of the shell in the form of (3.1)

$$\begin{aligned}
 N_y' + N_{y,\vartheta} \cos \vartheta \sec \vartheta + (N_y - N_{y,\vartheta}) \cot \vartheta + Q_y &= \rho R h (1 + \varepsilon) \frac{\partial^2 u}{\partial t^2} \\
 N_{y,\vartheta}' + N_{y,\vartheta} \cos \vartheta \sec \vartheta + 2 N_{y,\vartheta} \cot \vartheta + Q_{y,\vartheta} &= \rho R h (1 + \varepsilon) \frac{\partial^2 v}{\partial t^2} \\
 Q_y' + Q_{y,\vartheta} \cos \vartheta \sec \vartheta + Q_y \cot \vartheta - (N_y + N_{y,\vartheta}) &= \rho R h (1 + \varepsilon) \frac{\partial^2 w}{\partial t^2} \quad (3.1) \\
 M_y' + M_{y,\vartheta} \cos \vartheta \sec \vartheta + (M_y - M_{y,\vartheta}) \cot \vartheta - R Q_y &= 0 \\
 M_{y,\vartheta}' + M_{y,\vartheta} \cos \vartheta \sec \vartheta + 2 M_{y,\vartheta} \cot \vartheta - R Q_{y,\vartheta} &= 0
 \end{aligned}$$

with  $(\quad)' = \frac{\partial(\quad)}{\partial \vartheta}$   
 $(\quad)'' = \frac{\partial^2(\quad)}{\partial \vartheta^2}$

With the law of elasticity and the displacement-strain-relations, the forces and moments may be expressed by the displacements  $u$ ,  $v$  and  $w$ .

$$\begin{aligned}
 N_y &= \frac{D}{R} (u' + w + \nu (\cos \vartheta \dot{v} + \cot \vartheta u + w)) \\
 N_{y,\vartheta} &= \frac{D}{R} (\dot{v} \cos \vartheta \sec \vartheta + u \cot \vartheta + w + \nu (u' + w)) \\
 N_{y,\vartheta,\vartheta} &= \frac{D(1-\nu)}{2R} (v' - v \cot \vartheta + \dot{u} \cos \vartheta \sec \vartheta) \\
 M_y &= -\frac{K}{R^2} (w'' + w + \nu (\dot{w} \cos \vartheta \sec^2 \vartheta + w' \cot \vartheta + w)) + \varepsilon R N_y \\
 M_{y,\vartheta} &= -\frac{K}{R^2} (\dot{w} \cos \vartheta \sec^2 \vartheta + w' \cot \vartheta + w + \nu (w'' + w)) + \varepsilon R N_{y,\vartheta} \\
 M_{y,\vartheta,\vartheta} &= \frac{K(1-\nu)}{R^2} (\dot{w} \cos \vartheta \sec \vartheta \cot \vartheta - \dot{w}' \cos \vartheta \sec \vartheta) + \varepsilon R N_{y,\vartheta,\vartheta}
 \end{aligned} \quad (3.2)$$

Eliminating the shear forces in the equations (3.1) and with the help of the equations (3.2), one obtains three coupled partial differential equations for the displacements  $u$ ,  $v$  and  $w$ .

Under the assumption that the displacements  $u$ ,  $v$  and  $w$  change periodically with time (3.3),

$$\begin{aligned} u &= u(\varphi, \vartheta) \cdot \sin \omega t \\ v &= v(\varphi, \vartheta) \cdot \sin \omega t \\ w &= w(\varphi, \vartheta) \cdot \sin \omega t \end{aligned} \tag{3.3}$$

and replacing the actual shell displacements  $u(\varphi, \vartheta)$  and  $v(\varphi, \vartheta)$  by the new variables  $U$  and  $\Psi$  according to (3.4),

$$\begin{aligned} u &= U' - \Psi \sin \varphi \\ v &= U'' \operatorname{cosec} \varphi \end{aligned} \tag{3.4}$$

one obtains [2; 3] two uncoupled equations for the normal displacement  $w$  and the variable  $\Psi$ .

$$\begin{aligned} \nabla_k^6 w + c_1 \nabla_k^4 w + c_2 \nabla_k^2 w + c_3 w &= 0 \\ \nabla_k^2 \Psi + (2 + \frac{2\bar{\omega}^2}{1-\nu}) \Psi &= 0 \end{aligned} \tag{3.5}$$

where  $\nabla_k^2$  is the Laplace-operator in spherical coordinates

$$\nabla_k^2 ( ) = ( )'' + ( )' \cot \varphi + ( )'' \operatorname{cosec}^2 \varphi \tag{3.6}$$

The coefficients  $c_1$  depend on the shell geometry and on the eigenvalue  $\bar{\omega}$ . They are given explicitly in [1].

$$\begin{aligned} c_1 &= 4 + (1+\varepsilon) \bar{\omega}^2 \\ c_2 &= 4 + \frac{1+\varepsilon}{\varepsilon} (1-\nu^2 - \bar{\omega}^2 (1+\varepsilon\nu)) \\ c_3 &= \frac{1+\varepsilon}{\varepsilon} (1-\nu + \bar{\omega}^2) (2(1+\nu) - (1+\varepsilon) \bar{\omega}^2) \end{aligned} \tag{3.7}$$

The solutions of the differential equations (3.5) are the associated Legendre functions of first and second kind.

$$\begin{aligned} w &= \sum_{m=0}^{\infty} \sum_{l=1}^3 (A_l^m P_{n_l}^m(\cos \varphi) + B_l^m Q_{n_l}^m(\cos \varphi)) \cos m \vartheta \\ \Psi &= \sum_{m=1}^{\infty} (A_4^m P_{n_4}^m(\cos \varphi) + B_4^m Q_{n_4}^m(\cos \varphi)) \cos m \vartheta \end{aligned} \tag{3.8}$$

Because of the singularity of  $Q_{nl}^m(\cos \theta)$  in  $\theta = 0$ , for a shell that is closed in the apex these functions do not apply, that is  $B_i^m = 0$ .

For the remaining integration constants  $A_i^m$  with the boundary conditions in  $\theta = \theta_0$  a homogeneous system of equations may be formed. The determinant of this system must be zero for non-trivial solutions which leads to the natural frequency  $\bar{\omega}$ . For a hemispherical shell with  $h/R = 1/50$  and  $\nu = 0.2$  the lowest natural frequencies have been calculated for different boundary conditions. The results have been entered in table 2.

In Fig. (3.2)  $\bar{\omega}$  is plotted against the wave number  $m$ . The natural frequencies for a rigidly clamped and a hinged-supported ( $\nu = 0$ ) shell are almost identical. The values for the hinged-supported shell ( $\nu \neq 0$ ) are definitely lower. In all three cases, the lowest eigenvalue occurs for  $m = 1$ , that is when vibrating antimetrically.

Contrary to this, for a shell with free edges the lowest natural frequency occurs when vibrating axisymmetrically ( $m = 0$ ).

Besides these considerably high natural frequencies, for the shell with free edges there is still a branch with extremely low values. These modes of vibration which for  $m = 0$  and  $m = 1$  do not exist, belong to nearly inextensional vibrations [5; 6].

4. Vibrations of the Shell Structure

The problem of free vibration of a cylindrical shell with a hemispherical head can be solved by the application of the solutions of the free vibrations of a spherical shell and of a cylindrical shell, discussed above.

Four boundary conditions at the edge of the cylindrical shell and eight junction conditions (four continuity conditions and four equilibrium conditions at the junction) yield a set of twelve equations in the twelve unknown constants  $A_i^m$  and  $W_{sm}$ . Since these conditions are homogeneous, the determinant of these equations must be zero for nontrivial solutions leading to the corresponding frequency  $\bar{\omega}$ . The numerical results for various shell parameters  $h/R$  and  $l/R$  are entered in table 3.

For a shell with  $h/R = 1/50$ ,  $l/R = 1$  and  $\nu = 0.2$  the frequencies of the first three modes of vibration are plotted against the wave number  $m$  (see Fig. 4.1). The curve has two minima for  $m = 1$  ( $\bar{\omega} = 0.2936$ ) and for  $m = 5$  ( $\bar{\omega}_5 = 0.2808$ ).

Considering the respective mode shapes (Fig. 4.2), you will see that the spherical shell is vibrating considerably for  $m = 0, 1$  and possibly 2.

Contrary to this, the mode shapes of higher wave numbers  $m$  are essentially influenced by the vibrating cylinder.

Whether the lowest natural frequency occurs at  $m = 1$  or at higher  $m$  strongly depends on the parameters  $h/R$  and  $l/R$  (See Fig. 4.3 and Fig. 4.4). The parameter  $h/R$  influences considerably the cylindrical vibrations (Fig. 4.3 rear branch) whereas the vibrations for  $m = 0, 1, 2$  practically are not changed by this parameter.

In contrast, the natural frequencies for all  $m$  decrease steadily with increasing  $l/R$ .

### 5. Conclusion

The natural frequencies of a shell-structure are decisively influenced by the vibrations of the single shells. Guided by the example of the cylindrical shell with hemispherical head, you will see that there exist two branches competing with each other.

For the wave numbers  $m = 0, 1$  and  $2$  the lowest frequencies occur when mainly the hemispherical shell is vibrating. Consequently the natural frequencies for  $m = 0, 1$  and  $2$  are between those of a hinged supported and a free hemispherical shell (see Fig. 5.1).

For higher wave numbers  $m > 2$  the lowest frequencies occur when primarily the cylindrical shell is vibrating. This frequency-branch for  $m > 2$  consequently lies between the frequencies of a clamped-clamped and a clamped-free cylindrical shell (see Fig. 5.2).

The frequency-curve has two minima, the minimum of the vibration of the spherical shell for  $m = 1$  and the minimum of the vibration of the cylindrical shell for a higher wave number  $m$ . Whether the lowest frequency is at  $m = 1$  or a higher  $m$  strongly depends upon shell geometry, because the parameters  $h/R$  and  $l/R$  influence the shell structure in different ways.

For example, the frequencies for the wave numbers  $m = 0, 1$  and  $2$  hardly change when the parameter  $h/R$  is changing, while the frequencies for  $m > 2$  are changing considerably in that case. Contrary to this, the parameter  $l/R$  influences the frequencies for all wave numbers  $m$ .

References:

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is to appear in Ingenieur-Archiv



Table 1 Frequencies  $\bar{\omega}$  of a clamped-clamped and a clamped-free cylindrical shell

m	clamped-clamped	clamped-free
0	1.0411	1.0405
1	0.8830	0.5886
2	0.6992	0.3633
3	0.5592	0.2411
4	0.4678	0.1873
5	0.4187	0.1890
6	0.4081	0.2309
7	0.4325	0.2970
8	0.4867	0.3897

Table 2 Frequencies  $\bar{\omega}$  of a clamped, a hinged supported ( $\nu = 0$  and  $\nu \neq 0$ ) and a force-free hemispherical shell

m	free edges (inextens.)	free edges	hinged supp. ( $\nu = 0$ )	hinged supp. ( $\nu \neq 0$ )	clamped
0	-	0.8659	0.7564	0.7564	0.7737
1	-	0.8712	0.5707	0.1643	0.5820
2	0.0270	0.9102	0.8970	0.7642	0.8972
3	0.0699	0.9400	0.9437	0.8863	0.9447
4	0.1299	0.9657	0.9716	0.9364	0.9720
5	0.2032	0.9933	0.9977	0.9696	0.9977
6	0.2878	1.0269	1.0269	1.0020	1.0275

Table 3  $\bar{\omega}$  of a cylinder with a hemispherical head for several parameters  $h/R$  and  $\ell/R$

$h/R =$	1/20	1/50	1/100	1/500
m				
0	0.6493	0.6403	0.6363	0.6313
1	0.2972	0.2936	0.2922	0.2897
2	0.4904	0.4861	0.4867	0.4850
3	0.4089	0.3865	0.3839	0.3849
4	0.3820	0.3159	0.3054	0.3033
5	0.4280	0.2808	0.2523	0.2434
6	0.5421	0.2828	0.2223	0.1991
7	0.7105	0.3191	0.2145	0.1662
8	0.8750	0.3827	0.2274	0.1421
9		0.4674	0.2571	0.1255
10		0.5696	0.2996	0.1153
11			0.3519	0.1112
12				0.1124
13				0.1183
14				0.1279
15				0.1404

Table 4  $\bar{\omega}$  against m

(  $h/R = 1/50$  ,  $\nu = 0.2$  )

$\ell/R =$	0.5	1	2
m			
0	0.7074	0.6403	0.4874
1	0.4007	0.2936	0.1744
2	0.6935	0.4861	0.2772
3	0.6218	0.3865	0.1946
4	0.5503	0.3159	0.1585
5	0.5002	0.2808	0.1685
6	0.4750	0.2828	0.2131
7	0.4771	0.3191	0.2791
8	0.5074	0.3827	0.3599
9	0.5652	0.4674	0.4536
10	0.6479	0.5696	0.5602

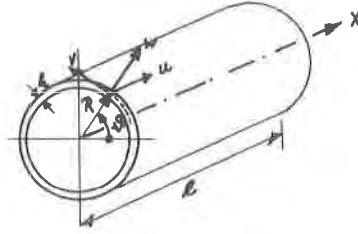


Fig. 2.1 Cylindrical Shell Coordinates

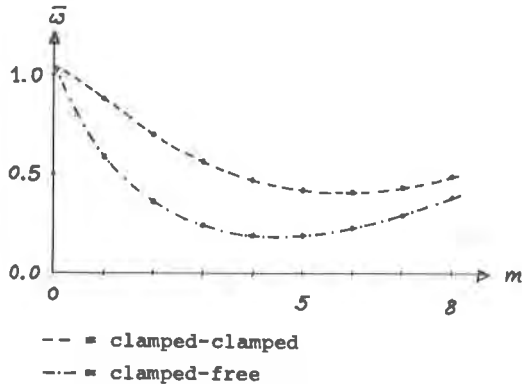


Fig. 2.2  $\bar{w}$  of a cylinder against  $m$  ( $h/R = 1/50$ ,  $l/R = 1$ ,  $\nu = 0.2$ )

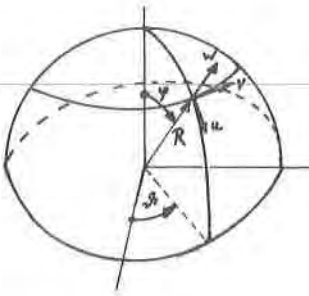


Fig. 3.1 Spherical Shell-Coordinates

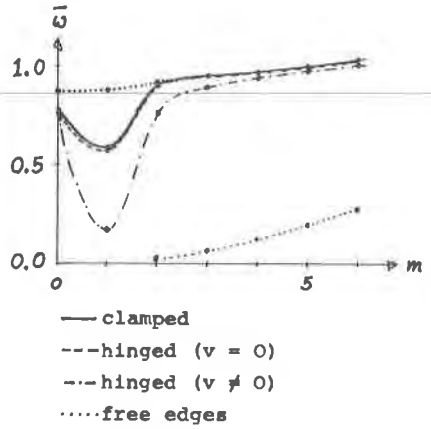


Fig. 3.2  $\bar{w}$  of a hemispherical shell against  $m$  ( $h/R = 1/50$ ,  $\nu = 0.2$ )

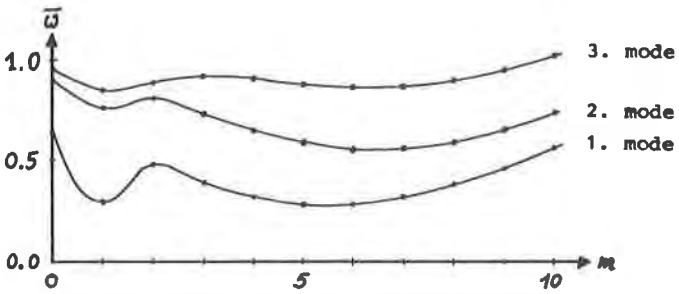


Fig. 4.1 Natural frequencies  $\bar{\omega}$  of the first three mode shapes for a shell with  $h/R = 1/50$ ,  $l/R = 1$ ,  $\nu = 0.2$ )

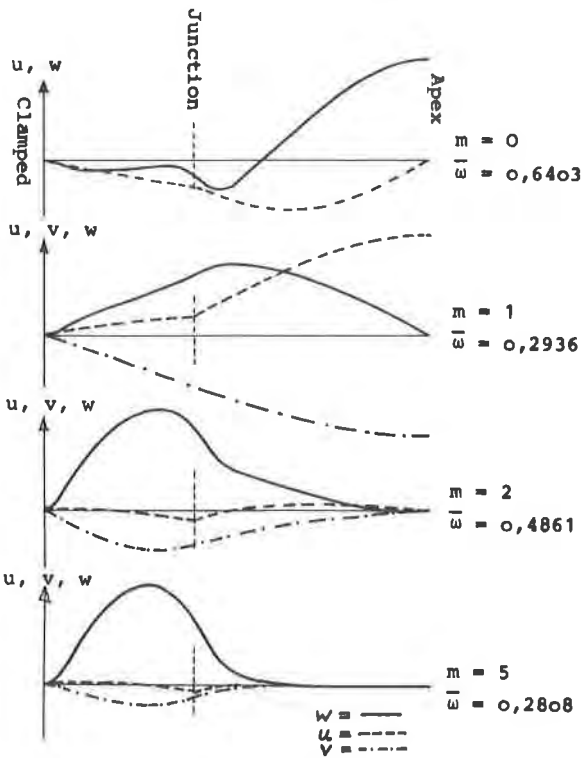


Fig. 4.2 1. mode shapes for  $m = 0, 1, 2, 5$

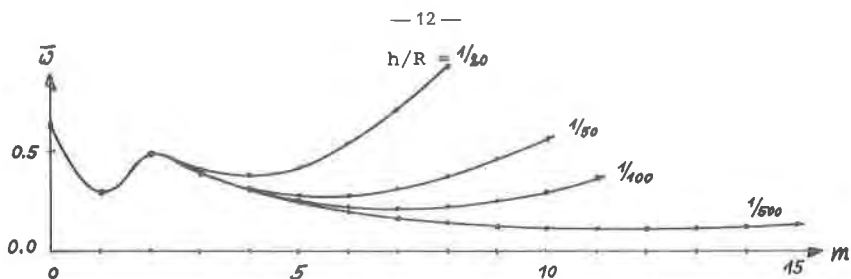


Fig. 4.3  $\bar{\omega}$  plotted against  $m$  for various parameters  $h/R$  ( $l/R = 1$ )

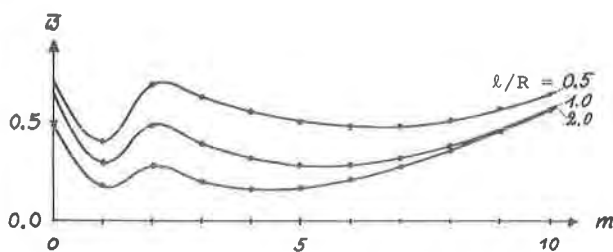


Fig. 4.4  $\bar{\omega}$  plotted against various parameters  $l/R$  ( $h/R = 1/50$ )

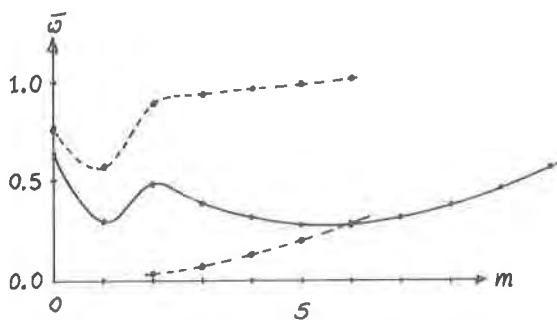


Fig. 5.1 Comparison of the frequencies  $\bar{\omega}$  of the shell structure with those of the hemispherical shell ( $h/R = 1/50$ ,  $l/R = 1$ ,  $\nu = 0.2$ )

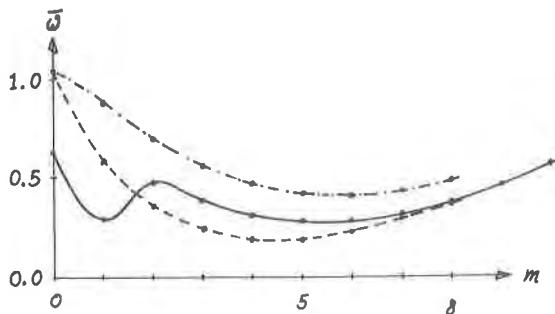


Fig. 5.2 Comparison of the frequencies  $\bar{\omega}$  of the shell structure with those of the cylindrical shell ( $h/R = 1/50$ ,  $l/R = 1$ ,  $\nu = 0.2$ )