# FINITE DYNAMIC RESPONSE OF THIN PLATES SUBJECT TO DYNAMIC LOADING

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## SUMMARY

Flat plate-like structural components of reactors are frequently subject to loadings that vary in a random manner with respect to time and are of such intensity that finite deflections of the plate occur. For such large deflections the responses of the plate are described by a pair of coupled nonlinear partial differential equations involving the lateral deflection and a stress (potential) function as unknowns. To date large amplitude responses of plates have been investigated only for the case of white noise type random excitation, i.e. the power spectral density of the loading is constant over all frequencies. Existing analyses are based upon time-dependent finite difference solutions to the coupled equations and involve substantial amounts of digital computer effort to attain the statistical descriptions of structural response, such as mean-square deflection at some prescribed point.

The present investigation treats, for the first time, the large amplitude responses of plates subject to non-white random excitation. In fact, the techniques presented are applicable to arbitrary power spectra of loading. The procedure is to reduce the coupled partial differential equations of motion to a system of nonlinear ordinary differential equations for the generalized coordinates by expanding the deflection into a series of coordinate functions with time-dependent coefficients and then employing variational procedures. These coordinate functions satisfy boundary conditions and in fact may be taken as the eigenfunctions of undamped free vibration if these are known. If they are not known (for unusual boundary conditions) then products of appropriate "beam functions" representing the plate as a collection of strips in the two orthogonal directions may be employed.

The significant point of the investigation is the solution of the nonlinear equations by the method of equivalent linearization. As illustrations of this method, the mean-square response of a simply supported rectangular plate to both white as well as non-white random excitations is determined. It is shown that the response corresponding to the fundamental mode is predominant if the damping of the system is small and if the power spectral density of the excitation does not have any sharp peaks near the higher mode frequencies. In summary, the new approach provides statistical descriptions of structural response to arbitrary power spectral density forces by approximate analytical methods thereby avoiding the tedious numerical computations associated with other approaches.

In the present study the investigation of the response of geometrically nonlinear elastic plates to stationary Gaussian random excitation will be discussed. The simultaneous partial differential equations governing the finite deflections of the plate are reduced to a system of nonlinear ordinary differential equations in the generalized coordinates by expanding the lateral deflection into a series of coordinate functions with time-dependent coefficients and then using Galerkin's variational equation. The mean-square response corresponding to these generalized coordinates is obtained approximately by the equivalent linearization technique [1] in conjunction with the correlation method [2] whose basic theory has been developed by Kolmogorov, Khinchin and Wiener.

Derivation of the Systems Equations

Consider a thin, elastic, isotropic rectangular plate of thickness h, Young's modulus E, and Poisson's ratio $\nu$. The origin o of the rectangular coordinates x-y-z is chosen at that point of the middle surface corresponding to one of the corners of the plate. The dimensions of the plate along the ox and oy axes are denoted by a and b respectively. Also, $w^*$ denotes the normal displacement of a point in the middle surface, and $u^*$ and $v^*$ are the displacements in the x and y directions, respectively. Then, the equation of motion and the compatibility equation governing the finite amplitude motion are

$$L(w^*,F^*) = \rho h\frac{\partial^2 w^*}{\partial \bar{t}^2} + 2c^*\rho h\frac{\partial w^*}{\partial \bar{t}} + D\nabla^4 w^* - h\left(\frac{\partial^2 F^*}{\partial y^2}\frac{\partial^2 w^*}{\partial x^2} - 2\frac{\partial^2 F^*}{\partial x\partial y}\frac{\partial^2 w^*}{\partial x\partial y} + \frac{\partial^2 F^*}{\partial x^2}\frac{\partial^2 w^*}{\partial y^2}\right) - q^* = 0 \qquad (1)$$

$$\nabla^4 F^* = E\left[\left(\frac{\partial^2 w^*}{\partial x\partial y}\right)^2 - \frac{\partial^2 w^*}{\partial x^2}\frac{\partial^2 w^*}{\partial y^2}\right] \qquad (2)$$

where $F^*$ = the Airy stress function, D = flexural rigidity of the plate = $Eh^3/12(1-\nu^2)$, $\rho$ = mass density of the plate material, $c^*$ = viscous damping (assumed constant), $q^*$ = lateral load, $\bar{t}$ = time, and

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4}$$

The exact solution of Eqs. (1) and (2) with associated boundary conditions and an arbitrary load $q^*(x,y,\bar{t})$ is unknown. We shall seek only approximate solutions of Eqs. (1) and (2) when $q^*(x,y,\bar{t})$ is a stationary Gaussian random process with zero mean value. The problem may be reduced to a system of ordinary differential equations by representing the desired solution as a series of coordinate functions $\psi_k(x,y)$ which satisfy the boundary conditions and are suitable for representing the deflected shape of the plate with time dependent coefficients $f_k(\bar{t})$ such as

$$w^*(x,y,\bar{t}) = \sum_{k=1}^{N} f_k(\bar{t})\psi_k(x,y) \qquad (3)$$

If the eigenfunctions for the undamped free vibration problem are known, these eigenfunctions are suitable for $\psi_k(x,y)$. However, if such eigenfunction are unknown, then products of appropriate "beam functions", that is orthogonal comparison functions , can be taken to represent $\psi_k(x,y)$ [3].

Suppose that we express the deflection $w^*$ in the form of Eq. (3). Substitution of Eq. (3) into Eq. (2) gives an ordinary differential equation for $F^*$. The solution of this ordinary differential equation for $F^*$, can be obtained in terms of $f_k(t)$. Substituting Eq. (3) and $F^*$ into Eq. (1) and applying Galerkin's method, i.e.,

$$\int_0^b\int_0^a L(w^*,F^*)\psi_k(x,y)dxdy = 0 \tag{4}$$

we obtain a system of ordinary differential equations for $f_k(\bar{t})$:

$$[M]\{\ddot{f}\} + 2c^*[M]\{\dot{f}\} + [P]\{f\} = \{Q\} \tag{5}$$

where $\{f\}$ = a column matrix of $f_k$, $[M]$ = a square matrix whose elements are $\{M\}_{ik}$, $[P]$ = a square matrix whose elements are $\{P\}_{ik}$,

$$\{M\}_{ik} = \int_0^b\int_0^a \rho h\psi_i\psi_k dxdy$$

$$\{P\}_{ik} = \int\int \psi_i[D\nabla^4\psi_k - h(\frac{\partial^2F^*}{\partial y^2}\frac{\partial^2}{\partial x^2} - 2\frac{\partial^2F^*}{\partial x\partial y}\frac{\partial^2}{\partial x\partial y} + \frac{\partial^2F^*}{\partial x^2}\frac{\partial^2}{\partial y^2})\psi_k]dxdy \tag{6}$$

$$Q_i = Q(t)\int_0^b\int_0^a \psi_i r(x,y)dxdy$$

Note that the matrix $[P]$ is a function of $f_k$. It has been assumed that the damping coefficient $c^*$ is not a function of x and y, and that the external loading $q^*(x,y,\bar{t})$ is represented by the product of a random function $Q(\bar{t})$ and a function of the coordinate $r(x,y)$. That is,

$$q^*(x,y,\bar{t}) = Q(\bar{t})r(x,y) \tag{7}$$

If we choose $\psi_k$ as orthogonal functions, Eq. (5) yields

$$\ddot{f}_k + 2c^*\dot{f}_i + \omega_i^2 f_i + g_i(f_1,f_2,\cdots\cdots,f_N) = \alpha_i Q(t) \quad i=1,2,\cdots,N \tag{8}$$

where

$$\alpha_i = \frac{\int_0^b\int_0^a \psi_k r(x,y)dxdy}{\int_0^b\int_0^a -\rho h\psi_k^2 dxdy} \tag{9}$$

and $g_k(f_i,f_2,\cdots,f_N)$ = a nonlinear equation of $f_1,f_2,\cdots,f_N$

Our particular interest is to find the mean-square response of the deflection. From Eq. (3)

$$w^{*2}(x,y,\bar{t}) = \sum_{i=1}^{N}\sum_{j=1}^{N} f_i(\bar{t})f_j(\bar{t})\psi_i(x,y)\psi_j(x,y)$$

Taking the ensemble average over $w^{*2}(x,y,\bar{t})$, we find

$$E[w^{*2}(x,y,\bar{t})] = \sum_{i=1}^{N}\sum_{j=1}^{N} E[f_i(\bar{t})f_j(\bar{t})]\psi_i(x,y)\psi_j(x,y) \tag{10}$$

By definition,

$$E[f_i(\bar{t}_1)f_j(\bar{t}_2)] = R_{f_if_j}(\bar{t}_1,\bar{t}_2) \tag{11}$$

where $R_{f_if_j}(\bar{t}_1,\bar{t}_2)$ is the cross-correlation function of $f_i(t_1)$ and $f_j(\bar{t}_2)$. Since $f_i(\bar{t})$ and $f_j(\bar{t})$ are stationary random process,

$$R_{f_if_j}(\bar{t}_1,\bar{t}_2) = R_{f_if_j}(\bar{t}_1-\bar{t}_2) = R_{f_if_j}(\tau) \tag{12}$$

where $\tau = \bar{t}_1-\bar{t}_2$. Therefore, it is necessary to calculate the correlation function $R_{f_if_j}(\tau)$ at least for $\tau=0$ in order to compute $E[w^*(x,y,\bar{t})]$.

Equivalent Linearization Technique

Assume that the nonlinear equations for the generalized coordinates $f_i$, Eq. (8) are linearized to the form

$$\ddot{f}_i + 2\beta_{ie}\dot{f}_i + \omega_{ie}^2 f_i = \alpha_i Q_i(\bar{t}) \tag{13}$$

where $\beta_{ie}$ is the equivalent linear damping coefficient and $\omega_{ie}^2$ is the equivalent linear stiffness. The error associated with this linearization is

$$e_i = 2(c^* - \beta_{ie})\dot{f}_i + (\omega_i^2-\omega_{ie}^2)f_i + g_i(f_1,f_2,\cdots,f_N)$$

Taking the ensemble average of $e_i^2$, we have

$$\begin{aligned} E[e_i^2] = {} & 4(c^*-\beta_{ie})^2E[\dot{f}_i^2] + (\omega_i^2-\omega_{ie}^2)^2E[f_i^2] + E[g_i^2(f_1,\cdots,f_N)] \\ & +4(c^*-\beta_{ie})(\omega_i^2-\omega_{ie}^2)E[f_i\dot{f}_i] \\ & +4(c^*-\beta_{ie})E[\dot{f}_i g_i(f_1,f_2,\cdots,f_N)] \\ & +2(\omega_i^2-\omega_{ie}^2)E[f_i g_i(f_1,\cdots,f_N)] \end{aligned} \tag{14}$$

The solution of Eq. (13) would be an exact solution if $e_i = 0$. In general we cannot choose $\beta_{ie}$ and $\omega_{ie}$ so that $e_i=0$, but we can choose $\beta_{ie}$ and $\omega_{ie}^2$ in such a manner that $E[e_i^2]$ is as small as possible. The equivalent linearization technique consists of selecting $\beta_{ie}$ and $\omega_{ie}^2$ so that $E[e_i^2]$ is minimum. Minimization of $E[e_i^2]$ is accomplished by requiring that

$$\frac{\partial}{\partial\beta_{ie}} E[e_i^2] = 0 \tag{15}$$

$$\frac{\partial}{\partial(\omega_{ie}^2)} E[e_i^2] = 0 \tag{16}$$

Noting that for a stationary process $E[f_i\dot{f}_i]=0$, and solving for $\beta_{ie}$ and $\omega_{ie}^2$ from Eqs. (15) and (16), we obtain

$$2\beta_{ie} = 2c^* + \frac{E[\dot{f}_i g_i(f_1,\cdots,f_N)]}{E[\dot{f}_i^2]} \tag{17}$$

$$\omega_{ie}^2 = \omega_i^2 + \frac{E[f_i g_i(f_1,\cdots,f_N)]}{E[f_i^2]} \tag{18}$$

If the following inequalities can be established, then the conditions (17) and (18) define the true minimum of $E[e_i^2]$:

$$\frac{\partial^2}{\partial\beta_{ie}^2} E[e_i^2] > 0$$

$$\frac{\partial^2}{\partial(\omega_{ie}^2)^2} E[e_i^2] > 0 \tag{19}$$

$$\frac{\partial^2}{\partial\beta_{ie}^2} E[e_{ie}^2] \frac{\partial^2}{\partial(\omega_{ie}^2)^2} E[e_i^2] - \left(\frac{\partial^2 E[e_i^2]}{\partial(\omega_{ie}^2)\partial\beta_{ie}}\right)^2 \quad 0$$

Since

$$\frac{\partial^2}{\partial\beta_{ie}^2} E[e_i^2] = 8E[\dot{f}_i^2] > 0$$

$$\frac{\partial^2}{\partial(\omega_{ie}^2)^2} E[e_i^2] = 2E[f_i^2] > 0$$

$$\frac{\partial^2}{\partial\beta_{ie}\partial(\omega_{ie}^2)} E[e_i^2] = 4E[f_i\dot{f}_i] = 0 \tag{20}$$

the inequalities (19) follow.

To express the right hand sides of Eqs. (17) and (18) in terms of $E[f_i^2]$ and $E[f_i f_j]$, it is assumed that since the input $Q_i(t)$ is a Gaussian random process with zero mean value and the nonlinearity of the system is small, the response of the equivalent linear system (13) is also Gaussian. The mean values of $f_i$ and $\dot{f}_i$ can be evaluated by use of the impulse response function of Eq. (13) defined by

$$h_i(\bar{t}) = \frac{e^{-\beta_{ie}\bar{t}}}{\omega_{id}} \sin\omega_{id}\bar{t} \qquad \bar{t} > 0$$
$$= 0 \qquad \bar{t} < 0 \tag{21}$$

where

$$\omega_{id}^2 = \omega_{ie}^2 - \beta_{ie}^2 \tag{22}$$

Assuming that the initial values of $f_i$ and $f_j$ are zero,

$$f_i(\bar{t}) = \int_0^{\bar{t}} h_i(\bar{t}-\tau)Q_i(\tau)d\tau \tag{23}$$

$$E[f_i(\bar{t})] = \int_0^{\bar{t}} h_i(\bar{t}-\tau)E[Q_i(\tau)]d\tau \tag{24}$$

Since $E[Q_i(\tau)] = 0$, it follows that $E[f_i(t)] = 0$ (25)

and

$$E[\dot{f}_i(\bar{t})] = \frac{d}{dt} E[f_i(\bar{t})] = 0 \tag{26}$$

Then, the probability density function with an N-dimensional Gaussian distribution is given by

$$p(f_1,f_2,\cdots,f_N) = \frac{1}{\sqrt{(2\pi)^N \det(R)}} \exp[-\frac{1}{2}\sum_{i,j=1}^{N} \frac{\mathrm{cof}(R_{f_if_j})}{\det(R)} f_if_j] \tag{27}$$

where R is the correlation matrix of $R_{ij}(\tau)$ defined by

$$R = \begin{bmatrix} R_{f_1f_1}(\tau) & R_{f_1f_2}(\tau) & \cdots & R_{f_1f_N}(\tau) \\ \vdots & \vdots & & \vdots \\ R_{f_Nf_1}(\tau) & R_{f_Nf_2}(\tau) & \cdots & R_{f_Nf_N}(\tau) \end{bmatrix} \tag{28}$$

and det(R) is the determinant of R and $\mathrm{cof}(R_{f_if_j})$ is the cofactor of $R_{f_if_j}(\tau)$ in the determinant det(R). Using the probability density function $p(f_1,f_2,\ldots,f_N)$, $E[f_ig_i(f_1,f_2, \ ,f_N)]$ is calculated by

$$E[f_ig_i(f_1,\cdots,f_N)] = \underbrace{\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}}_{N\text{-fold}} f_ig_i(f_1,\cdots,f_N)p(f_1,\cdots,f_N)\prod_{k=1}^{N} df_k$$

$$= S_i(R_{f_rf_k}(\tau)) \qquad r,k=1,2,\cdots,N \tag{29}$$

where $S_i(R_{f_rf_k}(\tau))$ is a function of the correlation $R_{f_rf_k}(\tau)$. From Eqs. (26) and (27), it follows that

$$E[f_ig_i(f_1,\cdots,f_N)] = 0 \tag{30}$$

Thus, the equivalent linear damping and equivalent linear stiffness become

$$\beta_{ie} = c^* \tag{31}$$

$$\omega_{ie}^2 = \omega_i^2 + \frac{S_i(R_{f_r f_k}(\tau))}{R_{f_i f_i}(\tau)} \tag{32}$$

There is a well-known relationship between the input power spectral density function and the output power spectral density function [4]:

$$\Phi_{f_i f_i}(\omega) = |H_i(\omega)|^2 \Phi_{Q_i Q_i}(\omega) \tag{33}$$

$$\Phi_{f_i f_j}(\omega) = H_i(\omega)\overline{H}_j(\omega)\Phi_{Q_i Q_j}(\omega) \tag{34}$$

where $H_i(\omega) = 1/(\omega_{ie}^2 - \omega^2 + 2ic^*\omega)$ = a transfer function of the equivalent linear system associated with i th-mode described by Eq. (13). $\overline{H}_j(\omega) = 1/(\omega_{je}^2 - \omega^2 - 2ic^*\omega)$ = a complex conjugate of the transfer function $H_j(\omega)$, $\Phi_{Q_i Q_j}(\omega)$ = cross power spectral density of the generalized forces $Q_i$ and $Q_j$, $\Phi_{f_i f_j}(\omega)$ = cross power spectral density of the generalized coordinates $f_i$ and $f_j$. Also, $R_{Q_i Q_j}(\tau)$ and $R_{f_i f_j}(\tau)$ are Fourier inverse transforms of $\Phi_{Q_i Q_j}(\omega)$ and $\Phi_{f_i f_j}(\omega)$, respectively. That is,

$$R_{Q_i Q_j}(\tau) = \frac{1}{2}\int_{\infty}^{\infty} \Phi_{Q_i Q_j}(\omega) e^{i\omega\tau} d\omega \tag{35}$$

$$R_{f_i f_j}(\tau) = \frac{1}{2}\int_{\infty}^{\infty} \Phi_{f_i f_j}(\omega) e^{i\omega\tau} d\omega \tag{36}$$

Therefore,

$$E[f_i^2] = R_{f_i f_i}(0) = \int_{o}^{\infty} \Phi_{Q_i Q_i}(\omega)|H_i(\omega)|^2 d\omega \tag{37}$$

$$E[f_i f_j] = R_{f_i f_j}(0)$$

$$= \frac{1}{2}\int_{\infty}^{\infty} \Phi_{Q_i Q_j}(\omega) H_i(\omega)\overline{H}_j(\omega) d\omega \tag{38}$$

If $\Phi_{Q_i Q_i}(\omega)$ and $\Phi_{Q_i Q_j}(\omega)$ are analytic functions of $\omega$, the integrals of Eqs. (37) and (38) can be evaluated by the residue theorem:

$$E[f_i^2] = N_{ii}(\ {}^{2}_{ie}) \tag{39}$$

$$E[f_i f_j] = N_{ij}(\omega_{ie}, \omega_{je}) \tag{40}$$

where $N_{ii}$ and $N_{ij}$ are functions of $\omega_{ie}$ and $\omega_{je}$. From Eq. (32), since the equivalent linear stiffness $\omega_{ie}^2$ is a function of $E[f_i f_j]$ (i=1,2,$\cdots$,N. j=1,2,$\cdots$,N), Eqs. (39) and (40) are the simultaneous nonlinear algebraic equations for $E[f_i f_j]$. Thus, the porblem reduces to solving a system of nonlinear algebraic equations.

Application to Simply Supported Rectangular Plate

Let us consider the stationary response of a simply supported rectangular plate to a random excitation distributed uniformly over the plate. Thus, the coordinate function $r(x,y) = 1$ in Eq. (7), and for convenience the following dimensionless quantities are introduced: $\xi = x/b$, $\eta = y/b$, $t = \bar{t}(D/\rho hb^4)^{1/2}$, $c = c^*(\rho hb4/D)^{1/2}$, $w = w^*/h$, $F = F^*/Eh^2$, $Q = Q^*b^4/Eh^4$ and $\lambda = a/b$.

The boundary conditions are:

$$\frac{\partial^2 w}{\partial \xi^2} = 0 \text{ and } w = 0 \quad \text{at } \xi = 0 \text{ and } \xi = \lambda \tag{41}$$

$$\frac{\partial^2 w}{\partial \eta^2} = 0 \text{ and } w = 0 \quad \text{at } \eta = 0 \text{ and } \nu = 1 \tag{42}$$

Also, $u = v = 0$ at all four edges. For immovable supports:

$$\int_0^1\int_0^\lambda \left[\frac{\partial^2 F}{\partial \eta^2} - \nu \frac{\partial^2 F}{\partial \xi^2} - \frac{1}{2}\left(\frac{\partial w}{\partial \xi}\right)^2\right] d\xi d\eta = 0 \tag{43}$$

$$\frac{1}{\lambda}\int_0^\lambda\int_0^1 \left[\frac{\partial^2 F}{\partial \xi^2} - \nu \frac{\partial^2 F}{\partial \eta^2} - \frac{1}{2}\left(\frac{\partial w}{\partial \eta}\right)^2\right] d\xi d\eta = 0 \tag{44}$$

We select

$$w(\xi,\eta,t) = \sum_{m=1}^{N}\sum_{n=1}^{N} f_{mn}(t)\sin\left(\frac{m\pi}{\lambda}\xi\right)\sin(n\pi\eta) \tag{45}$$

This expression for $w$ is employed in Eq. (2) and a particular solution is obtained. A complementary solution for $F$ is taken to be of the form $F_c = H\xi^2 + I\eta^2$ where $H$ and $I$ are found from Eqs. (43) and (44). From Eqs. (10) and (45) the mean-square response of central deflection is thus:

$$E\left[w^2\left(\tfrac{1}{2}\lambda,\tfrac{1}{2},t\right)\right] = \sum_{\substack{i=1,3...\\ j=1,3...}}^{N}\ \sum_{\substack{k=1,3...\\ l=1,3...}}^{N} E[f_{ih}f_{kl}]\sin\frac{i\pi}{2}\sin\frac{j\pi}{2}\sin\frac{k\pi}{2}\sin\frac{l\pi}{2} \tag{46}$$

$$\begin{aligned} &= E[f_{11}{}^2] + E[f_{13}{}^2] + E[f_{31}{}^2] + E[f_{33}{}^2] \\ &+ \ldots + E[f_{NN}{}^2] - 2E[f_{11}f_{13}] \\ &- 2E[f_{11}f_{31}] + 2E[f_{11}f_{33}] \end{aligned} \tag{47}$$

When $Q(t)$ is white noise the power spectral density is:

$$\Phi_Q(\omega) = \int_0 = \text{constant} \tag{48}$$

Thus, Eqs. (39) and (40) become

$$E[f_{ij}^2] = \alpha_{ij}S_o \int_0^\infty |H_{ij}(\omega)^2|^2 d\omega \tag{49}$$

$$E[f_{ij}f_{kl}] = \frac{1}{2}\alpha_{ij}\alpha_{kl}S_o \int_{\infty}^{\infty} H_{ij}(\omega)\overline{H}_{kl}(\omega)d\omega \tag{50}$$

where

$$H_{ij}(\omega) = [(\omega_{ije}^2 - \omega^2) + 2i\zeta_{ij}\omega_{ij}\omega]^{-1} \tag{51}$$

$$\overline{H}_{kl}(\omega) = [(\omega_{kle}^2 - \omega)^2 - 2i\zeta_{kl}\omega_{kl}\omega]^{-1} \tag{52}$$

The integrals in (49) and (50) may be evaluated by residues. If $\omega_{ije}^2$ and $\omega_{kle}^2$ are replaced by $\omega_{ij}^2$ and $\omega_{kl}^2$ respectively one obtains the solutions for the corresponding linear system. The probability density function to be used in Eq. (29) may be expressed in the approximate form

$$p(f_{11},\cdots,f_{NN}) = (2\pi)^N \prod_{i,j=1,3,\cdots}^{N} E[f_{ij}^2]^{-1/2} \exp -\frac{1}{2} \sum_{i,j}^{N} \frac{f_{ij}^2}{E[f_{ij}^2]} \tag{53}$$

Finally, Eqs. (39) are formulated and solved by Newton's method.

The case of a simply supported rectangular plate with $\lambda = 2$ and subject to various magnitudes of white noise $S_o$ with damping $\zeta_{ij} = 0.05$ has been treated numerically. The results are shown in Figure 1. From these results it is evident that the single-degree- of-freedom representation provides a good approximation to the response for such a moderate value of damping. Responses obtained by consideration of four-degrees-of-freedom differ only slightly from that of the single-degree-of-freedom approach. When N = 1 the resulting solution reduces to that found by Lin [5].

For the case of a simply supported rectangular plate with $\lambda = 2$ subject to non-white noise having power spectral density Q(t) of the form:

$$\Phi_Q(\omega) = \frac{S_o[b_1{}^4 + (2h_1b_2\omega)^2]}{(b^2-\omega^2)^2 - (2hb\omega)^2} \tag{54}$$

it is possible to demonstrate that if $\Phi_Q(\omega)$ has no sharp peaks and for small damping, that

$$E[f_{ij}^2] = \alpha_{ij}^2 \, \Phi_Q(\omega_{ije}) \int_0^{\infty} \frac{1}{(\omega_{ije}^2 - \omega^2)^2 + (2\zeta_{ij}\omega_{ij}\omega)^2} d\omega \tag{55}$$

As a numerical example, let us determine the mean-square central response of a plate to jet engine noise in which the parameters in (54) have the values: $h = 0.8$, $h_1 = 0.5$, $b_1 = 0$, $b = b_2 = 1650$ rad/sec, and $S_o = 0.0307$ [6]. Figure 2 shows the variation of mean-square dimensionless central deflection as a function of $S_o$.

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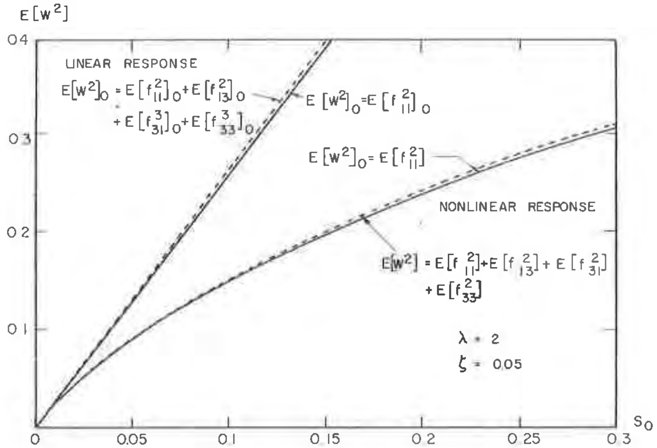


FIGURE 1

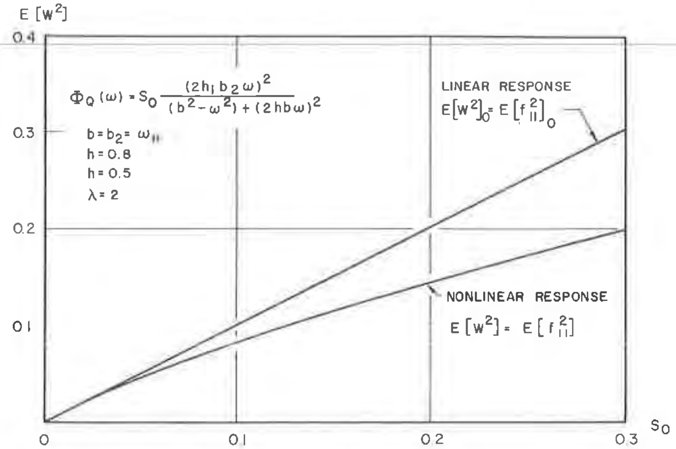


FIGURE 2