VELOCITY INFLUENCE ON THE FREE VIBRATIONS IN A COUPLED SYSTEM FLUID-STRUCTURE

F. CESARI, S. CURIONI

Istituto di Meccanica Applicata alle Macchine, Facoltà di Ingegneria, Università di Bologna, Bologna, Italy

SUMMARY

The PEC reactor (a sodium—cooled fast reactor) has in the core central zone a cavity within which some test loops are inserted to carry out experiments on fuel elements. These loops are fuelled by an independent circuit from the reactor primary circuit so that the experimental fuel element may be subjected to particular load conditions. The realization of these loops raises remarkable problems for the reactor safety, and both the static and dynamic response of these structures must be carefully evaluated.

In the present stage of design two different solutions exist: the first foresees the construction of a **U** test loop in which the inlet and the outlet pipe of the coolant are uncoupled, while a second solution foresees a more compact system constituted by two coaxial tubes. In this second case, the coolant flows down in the interspace between the pipes and flows up again in the internal one, this determining an influence of both velocities on the system dynamic response.

The present work has the aim to study the dynamic behaviour of the latter solution, determining the natural frequencies of the coupled fluid—structure system as function of the two fluid velocities (in the internal tubes and in the interface).

Recent studies on this subject are reported by S. S. Chen, "Free vibration of a coupled fluid structural system" *J. of Sound and Vibration* (1972), Vol. 21, 387-398. In these investigations the effect of the fluid velocity in the interface and in the piping internal part is neglected. These assumptions hold for not too high velocities, while in our conditions a considerable growth in the coolant amount as compared to the usual operating conditions is foreseen; it is therefore not possible to neglect the effect of the fluid velocity and in this characteristic the present is different from the previous ones.

After defining the problem by writing the system of motion differential equations relatively to the internal and external fluid, the pressure acting on the two tubes is derived from the velocity potential assuming an irrotational flux and imposing the conditions to the outline.

The calculation of the natural vibration frequency is made by means of the model analysis. The natural frequencies have been obtained by solving a polynomial equation of 2n degrees corresponding to n in-phase frequencies and n out-of-phase frequencies. It is to be noted that the natural frequency is highly affected above a given critical velocity of the fluid in the interface and that this critical velocity depends on the fluid velocity in the internal tube.

1. Introduction

The aim of this paper is to study the dynamic behaviour of two coaxial tubes with coolant in the annular region and in the internal tube.The natural frequencies of the coupled fluid-structure system have been determined as a function of the two fluid velocity (in the internal tube and in the annular region).

These tubes constitute a loop inserted in the core central area of a sodium - cooled fast reactor (PEC); the loop is utilize to test fuel elements (fig. 1).

The design and construction of these loops (it is possible to put in the reactor three loops) raises remarkable problems for the reactor safety;therefore the static and dynamic response of these structures must be carefully evaluated.

The problem is formulated in terms of the beam modal functions of the inner and outer shells and a fluid velocity potential.

The pressure acting on the two tubes is derived from the velocity potential assuming an irrotational flux and from the boundary conditions.

The calculation of the natural vibration frequency is made by means of the modal analysis. The natural frequencies have been obtained by solving a polynomial equation of 2.n degrees corresponding to n in-phase and n out-of-phase frequencies. A IBM 360/75 digital computer has been used for numerical results.

The simply-supported end conditions has been used to illustrate the method of analysis; however the results can be applied to other end conditions.

From numerical results, it may be observed that the natural frequency is highly affected above a given critical fluid annular velocity and that this critical velocity depends on the fluid velocity in the internal tube.

In |1| a reference is given of the most recent research carried out on this subject; in this work the effect of the two fluid velocity are neglected.

2. Problem statement

Consider a fluid/structural system which consists of two cylindrical shells, with incompressible frictionless fluid in the internal and annular regions (fig. 2):

The governing equations of motion are:

$$E_2 J_2 \frac{\partial^4 y_2}{\partial x^4} + M_2\left(\frac{\partial}{\partial t} + U_2 \frac{\partial}{\partial x}\right)^2 y_2 + m_2 \frac{\partial^2 y_2}{\partial t^2} = P_2(x,t) \tag{1}$$

$$E_1 J_1 \frac{\partial^4 y_1}{\partial x^4} + M_1\left(\frac{\partial}{\partial t} \pm U_1 \frac{\partial}{\partial x}\right)^2 y_1 + m_1 \frac{\partial^2 y_1}{\partial t^2} = P_1(x,t) \tag{2}$$

A list of nomenclature is given in Appendix.
As an example, simply-supported conditions are considered; thus

$$y_2 = \frac{\partial^2 y_2}{\partial x^2} = 0 \qquad \text{for} \quad x = 0, L \tag{3}$$

$$y_1 = \frac{\partial^2 y_1}{\partial x^2} = 0 \qquad \text{for} \quad x = 0, L \tag{4}$$

The forces P_1 and P_2 are attributed to fluid motion and they can be obtained as a function of the fluid pressure; thus

$$P_2(x,t) = R_2 \int_0^{2\pi} \left| p_2(x,r,\theta,t) - p_1(x,r,\theta,t) \right|_{r=R_2} \cos\theta \, d\theta \tag{5}$$

$$P_1(x,t) = -R_1 \int_0^{2\pi} p_1(x,r,\theta,t) \Big|_{r=R_1} \cos\theta \, d\theta \tag{6}$$

the fluid pressure is given by

$$p = -\rho \left(\frac{\partial \Psi}{\partial t} + U \frac{\partial \Psi}{\partial x} \right) \tag{7}$$

Assuming an irrotational flow, we can define a velocity potential Ψ so that

$$\vec{v} = \nabla \Psi \tag{8}$$

From the continuity equation for an incompressible fluid, Laplace's equation is obtained:

$$\nabla^2 \Psi = 0 \tag{9}$$

the fluid boundaries conditions are:

$$v_r = \frac{\partial y_1}{\partial t} \cos\theta \qquad r = R_1 \tag{10}$$

$$v_r = \frac{\partial y_2}{\partial t} \cos\theta \qquad r = R_2 \tag{11}$$

At the two ends, we can have two conditions:

zero velocity: $v_x = 0$ at $x = 0, L$ (12)

Constant pressure: $p = 0$ at $x = 0, L$ (13)

3. Uncoupled frequency

For free vibration, let

$$\begin{cases} y_1(x,t) = \phi_1(x)\, e^{i\Omega t} \\ y_2(x,t) = \phi_2(x)\, e^{i\Omega t} \end{cases} \tag{14}$$

$$\begin{cases} P_1(x,t) = Q_1(x)\, e^{i\Omega t} \\ P_2(x,t) = Q_2(x)\, e^{i\Omega t} \end{cases} \tag{15}$$

substitution of eq. (14) and (15) into eq. (1), (2), (3), (4) yields.

$$E_1 J_1 \frac{d^4\phi_1}{dx^4} - (M_1 + m_1)\Omega^2 \phi_1 + M_1 U_1^2 \frac{d^2\phi_1}{dx^2} = Q_1 \tag{16}$$

$$E_2 J_2 \frac{d^4\phi_2}{dx^4} - (M_2 + m_2)\Omega^2 \phi_2 + M_2 U_2^2 \frac{d^2\phi_2}{dx^2} = Q_2 \tag{17}$$

$$\phi_1 = \frac{d^2\phi_1}{dx^2} = 0 \qquad \text{at} \qquad x = 0, L \tag{18}$$

$$\phi_2 = \frac{d^2\phi_2}{dx^2} = 0 \qquad \text{at} \qquad x = 0, L \tag{19}$$

$\emptyset_1$ and $\emptyset_2$ are modal functions of the two shells with fluid coupling and can be represented in series form as superpositions of the uncoupled modal. functions.

$$\begin{cases} \phi_1(x) = \sum_{i=1}^{\infty} a_i\, \phi_i(x) & \frac{1}{L}\int_0^L \phi_i \phi_j\, dx = \delta_{ij} \\ \phi_2(x) = \sum_{i=1}^{\infty} \bar{a}_i\, \bar{\phi}_i(x) & \frac{1}{L}\int_0^L \bar{\phi}_i \bar{\phi}_j\, dx = \delta_{ij} \end{cases} \tag{20}$$

the modal functions, $\emptyset_i$ and $\emptyset_i$ are given by eq. (16)and (17) without Q_1 and Q_2.
Let

$$\emptyset_i(x) = \bar{\emptyset}_i(x) = \text{sen}\ \frac{i\pi x}{L}$$

and put in eq. (16) and eq. (17), without Q_1 and Q_2, the natural frequencies are given by

$$\begin{aligned} \omega_i^2 &= \left[E_1 J_1 (i\pi/L)^4 - M_1 U_1^2 (i\pi/L)^2\right] / (M_1 + m_1) \\ \bar{\omega}_i^2 &= \left[E_2 J_2 (i\pi/L)^4 - M_2 U_2^2 (i\pi/L)^2\right] / (M_2 + m_2) \end{aligned} \tag{21}$$

4. Coupled frequency

We must consider the fluid field; the solution of eq. (9) is taken as

$$\Psi = X(x)\, Y(r)\, \Theta(\theta)\, e^{i\Omega t} \tag{22}$$

$$\begin{cases} X(x) = C \sin(sx) + D \cos(sx) \\ Y(r) = A\, I_q(sr) + B\, K_q(sr) \\ \Theta(\theta) = E \sin(q\theta) + F \cos(q\theta) \end{cases} \tag{23}$$

4.1 - Annular region

We have

$$v_r = \frac{\partial \Psi}{\partial r} \tag{24}$$

Introducing in eq. (24) the eq. (22), (10),(11), we have

$$\begin{aligned} &\Theta(\theta) = \cos\theta \\ &X(x)\, \dot{Y}(R_1) = i\Omega\, \phi_1(x) \\ &X(x)\, \dot{Y}(R_2) = i\Omega\, \phi_2(x) \end{aligned} \tag{25}$$

and introducing in eq. (25) the eq. (20) yields

$$\begin{aligned} &\sum_n X_n \dot{Y}_n(R_1) = i\Omega \sum_i a_i\, \phi_i(x) \\ &\sum_n X_n \dot{Y}_n(R_2) = i\Omega \sum_i \bar{a}_i\, \bar{\phi}_i(x) \end{aligned} \tag{26}$$

From eq. (26) we can have

$$\begin{aligned} &\sum_n \dot{Y}_n(R_2) \frac{1}{L} \int_0^L X_n X_k\, dx = i\Omega \sum_i \bar{a}_i \frac{1}{L} \int_0^L \bar{\phi}_i(x)\, X_k(x)\, dx \\ &\sum_n \dot{Y}_n(R_1) \frac{1}{L} \int_0^L X_n X_k\, dx = i\Omega \sum_i a_i \frac{1}{L} \int_0^L \phi_i(x)\, X_k(x)\, dx \end{aligned} \tag{27}$$

Let

$$\frac{1}{L}\int_0^L X_n X_k \, dx = \delta_{ij} M$$

$$\frac{1}{L}\int_0^L \phi_i(x) X_k(x) \, dx = \alpha_{kj} \tag{28}$$

$$\frac{1}{L}\int_0^L \bar{\phi}_i(x) X_k(x) \, dx = \bar{\alpha}_{kj}$$

we have

$$A_n = \frac{i\Omega}{M} \sum_{j=1}^{\infty} \left[\alpha_{nj}\, a_j\, K_1'(s_n R_2) - \bar{\alpha}_{nj}\, \bar{a}_j\, K_1'(s_n R_1) \right] / \Delta_n$$

$$B_n = \frac{i\Omega}{M} \sum_{j=1}^{\infty} \left[-\alpha_{nj}\, a_j\, I_1'(s_n R_2) + \bar{\alpha}_{nj}\, \bar{a}_j\, I_1'(s_n R_1) \right] / \Delta_n \tag{29}$$

$$\Delta_n = I_1'(s_n R_1) K_1'(s_n R_2) - I_1'(s_n R_2) K_1'(s_n R_1)$$

From eq. (12) and (13), we obtain

$$X_n(x) = \sqrt{2} \cos(s_n x) \qquad s_n = m\pi/L \tag{30}$$

$$X_n(x) = c_n \left[\sin(s_n x) - \frac{U_1 s_n}{\Omega} \cos(s_n x) \right] \tag{31}$$

$$\varphi = \sum_{n=0}^{\infty} \left[A_n I_1(s_n r) + B_n K_1(s_n r) \right] X_n(x) \cos\theta \, e^{i\Omega t}$$

$$v_x = \sum_{n=0}^{\infty} \left[A_n I_1(s_n r) + B_n K_1(s_n r) \right] X_n'(x) \cos\theta \, e^{i\Omega t} \tag{32}$$

$$p_2 = -i\rho \cos\theta \, e^{i\Omega t} \sum_{n=0}^{\infty} \left[A_n I_1(s_n r) + B_n K_1(s_n r) \right] \left[\Omega X_n(x) + U_1 X_n'(x) \right]$$

4.2 <u>Internal region</u>

From eq. (11), (33), (12), (13) we have

$$\sum_n X_n(x) A_n I_1'(s_n R_2) = i\Omega \phi_2(x) \tag{33}$$

$$A_n = \frac{i\Omega}{M} \sum_j \frac{\bar{a}_j \bar{\alpha}_{nj}}{I_1'(s_n R_2)} \tag{34}$$

$$X_n(x) = \sqrt{2} \cos(s_n x) \qquad X_n(x) = c_n \left[\sin(s_n x) - \frac{U_2 s_n}{\Omega} \cos(s_n x) \right] \qquad s_n = m\pi/L \tag{35}$$

$$\Psi = \sum_{n=0}^{\infty} A_n I_1(s_n r)\, X_n(x) \cos\theta \, e^{i\Omega t}$$

$$v_x = \sum_{n=0}^{\infty} A_n I_1(s_n r)\, X'_n(x) \cos\theta \, e^{i\Omega t} \tag{36}$$

$$p_1 = -i\rho\cos\theta \, e^{i\Omega t} \sum_{n=0}^{\infty} A_n I_1(s_n r)\left[\Omega X_n(x) + U_2 X'_n(x)\right]$$

From eq. (5) and (6), keeping into account eq. (15), (32), (36), we obtain

$$Q_1(x) = \frac{\pi\rho R_1}{M} \sum_n \Big(\sigma_n \sum_j \alpha_{nj} a_j + \bar{\sigma}_n \sum_j \bar{\alpha}_{nj} \bar{a}_j\Big)\Big(\Omega^2 X_n(x) + U_1 \Omega X'_n(x)\Big)$$

$$Q_2(x) = \frac{\pi\rho R_2}{M} \sum_n \Big[\tau_n \sum_j \alpha_{nj} a_j + \Big(\bar{\tau}_n - \frac{I_1(s_n R_2)}{I'_1(s_n R_2)}\Big)\sum_j \bar{a}_j \bar{\alpha}_{nj}\Big]\Big(\Omega^2 X_n(x) + U_2 \Omega X'_n(x)\Big) \tag{37}$$

$$\sigma_n = \left[-I_1(s_n R_1) K'_1(s_n R_2) + I'_1(s_n R_2) K_1(s_n R_1)\right] / \Delta_n$$

$$\bar{\sigma}_n = \left[-I'_1(s_n R_1) K_1(s_n R_1) + I_1(s_n R_1) K'_1(s_n R_1)\right] / \Delta_n$$

$$\tau_n = \left[-I'_1(s_n R_2) K_1(s_n R_2) + I_1(s_n R_2) K'_1(s_n R_2)\right] / \Delta_n \tag{38}$$

$$\bar{\tau}_n = \left[-I_1(s_n R_2) K'_1(s_n R_1) + I'_1(s_n R_1) K_1(s_n R_2)\right] / \Delta_n$$

Introducing the eq. (37) in eq. (16) and (17), multiplying by $\phi_K(x)$ and $\bar{\phi}_K(x)$, integrating with respect to x from 0 to L, assuming zero velocity at the boundaries, we obtain

$$\sum_{l=1}^{\infty}\left[\left(\omega_K^{*2} - \Lambda^2\right)\delta_{Kl} - \beta_1 \Lambda^2 b_{Kl} + \gamma_1 \beta_1 V_1^* \Lambda b'_{Kl}\right] a_l - \beta_1 \sum_{l=1}^{\infty}\left(\Lambda^2 \bar{c}_{Kl} - \gamma_1 V_1^* \bar{c}'_{Kl}\right)\bar{a}_l = 0$$

$$\tag{39}$$

$$\sum_{l=1}^{\infty}\left[\left(\bar{\omega}_K^{*2} - \Lambda^2\mu^2\right)\delta_{Kl} - \beta_2 \Lambda^2 \mu^2 \bar{b}_{Kl} + \beta_2 \mu V_2^* \gamma_2 \Lambda \bar{b}'_{Kl}\right]\bar{a}_l - \beta_2 \sum_{l=1}^{\infty}\left(\mu^2 \Lambda^2 c_{Kl} - \gamma_2 V_2^* \mu \Lambda c'_{Kl}\right) a_l = 0$$

$$\beta_1 = \pi \rho R_1 / (M_1 + m_1)$$

$$\beta_2 = \pi \rho R_2 / (M_2 + m_2)$$

$$b_{k\ell} = \sum_{n=0}^{\infty} \sigma_n \alpha_{nk} \alpha_{n\ell} / R_1$$

$$\bar{b}_{k\ell} = \sum_{n=0}^{\infty} \frac{\bar{\alpha}_{nk} \bar{\alpha}_{n\ell}}{R_2} \left(\bar{\tau}_n - I_1 / I_1' \right)$$

$$b'_{k\ell} = \sum_{n=0}^{\infty} s_n \sigma_n \alpha_{n\ell} \alpha'_{nk} / R_1 \qquad (40)$$

$$\bar{b}'_{k\ell} = \sum_{n=0}^{\infty} s_n \frac{\bar{\alpha}'_{nk} \bar{\alpha}_{n\ell}}{R_2} \left(\bar{\tau}_n - I_1 / I_1' \right)$$

$$c_{k\ell} = \sum_{n=0}^{\infty} \frac{\tau_n \alpha_{n\ell} \bar{\alpha}_{nk}}{R_2}$$

$$c'_{k\ell} = \sum_{n=0}^{\infty} s_n \frac{\tau_n \bar{\alpha}'_{nk} \alpha_{n\ell}}{R_2}$$

$$\bar{c}'_{k\ell} = \sum_{n=0}^{\infty} s_n \frac{\bar{\sigma}_n \bar{\alpha}_{k\ell} \alpha'_{nk}}{R_1}$$

$$\omega_k^* = k\pi \sqrt{\frac{(k\pi)^2 - V_1^{*2}}{1 + \gamma_1^2}}$$

$$\bar{\omega}_k^* = k\pi \sqrt{\frac{(k\pi)^2 - V_2^{*2}}{1 + \gamma_2^2}}$$

$$\gamma_1 = \sqrt{M_1/m_1} \qquad \gamma_2 = \sqrt{M_2/m_2} \qquad (41)$$

$$V_1^* = \sqrt{\frac{M_1 U_1^2 L^2}{E_1 J_1}} \qquad V_2^* = \sqrt{\frac{M_2 U_2^2 L^2}{E_2 J_2}}$$

$$R_1/L = \xi_1 \qquad R_2/L = \xi_2$$

$$\mu = \sqrt{\frac{E_1 J_1 m_2}{E_2 J_2 m_1}} \qquad \Lambda = \Omega L^2 \sqrt{\frac{m_1}{E_1 J_1}}$$

From eq. (40), we can obtain

$$b_{kl} = \alpha_{0k}\,\alpha_{0l}\,\frac{1+(\xi_1/\xi_2)^2}{1-(\xi_1/\xi_2)^2} + \sum_{n=1}^{\infty}\frac{2\sigma_n\,\alpha_{nk}\,\alpha_{nl}}{n\pi\,\xi_1}$$

$$b'_{kl} = \sum_{n=1}^{\infty}\frac{2\sigma_n}{\xi_1}\,\alpha_{nl}\,\alpha'_{nk}$$

$$\bar{b}'_{kl} = \sum_{n=1}^{\infty}\frac{2\,\bar{\alpha}'_{nk}\,\bar{\alpha}_{nl}}{\xi_2}\left(\bar{\tau}_n - I_1(n\pi\xi_2)\Big/\left[I_0(n\pi\xi_2)+I_2(n\pi\xi_2)\right]\right)$$

$$\bar{c}'_{kl} = \sum_{n=1}^{\infty}\frac{2\bar{\sigma}_n}{\xi_1}\,\bar{\alpha}_{nl}\,\alpha'_{nk}$$

$$c'_{kl} = \sum_{n=1}^{\infty}\frac{2\tau_n}{\xi_2}\,\bar{\alpha}'_{nk}\,\alpha_{nl} \qquad (42)$$

$$\bar{b}_{kl} = \bar{\alpha}_{0k}\,\bar{\alpha}_{0l}\,\frac{2(\xi_1/\xi_2)^2}{1-(\xi_1/\xi_2)^2} + \sum_{n=1}^{\infty}\bar{\alpha}_{nk}\,\bar{\alpha}_{nl}\,\frac{2}{n\pi\,\xi_2}\left(\bar{\tau}_n - I_1(n\pi\xi_2)\Big/\left[I_0(n\pi\xi_2)+I_2(n\pi\xi_2)\right]\right)$$

$$\bar{c}_{kl} = \bar{\alpha}_{0k}\,\bar{\alpha}_{0l}\,\frac{-2(\xi_1/\xi_2)^2}{1-(\xi_1/\xi_2)^2} + \sum_{n=1}^{\infty}\frac{2\bar{\sigma}_n}{n\pi\,\xi_1}\,\bar{\alpha}_{nl}\,\bar{\alpha}_{nk}$$

$$c_{kl} = -\frac{2(\xi_1/\xi_2)^2}{1-(\xi_1/\xi_2)^2}\,\alpha_{0l}\,\bar{\alpha}_{0k} + \sum_{n=1}^{\infty}\frac{2\tau_n}{n\pi\,\xi_2}\,\alpha_{nl}\,\bar{\alpha}_{nk}$$

From eq. (28), we have:

$$\alpha_{nk} = \begin{cases} 0 & n = k \\ \dfrac{1}{\pi}\left[\dfrac{1-(-1)^{k-n}}{k-n} + \dfrac{1-(-1)^{k+n}}{k+n}\right] & n \neq k \end{cases} \qquad (43)$$

$$\alpha_{nk} = \bar{\alpha}_{nk}$$

$$\alpha'_{nk} = \bar{\alpha}'_{nk} = \delta_{ij}$$

Eq. (39) consist of an infinite number of ordinary equations. The frequency equation is obtained by requiring the determinant of the coefficient matrix of the unknowns a_1 and $\bar{a}_1$ to be equal to zero.

From the eq. (39) we obtain this equation:

$$f(\Lambda, \xi_1, \xi_2, V_1^*, V_2^*, \mu, \gamma_1, \gamma_2, \omega_k^*, \bar{\omega}_k^*) = 0 \qquad (44)$$

5. Numerical results

The frequency equation is solved numerically, using a IBM 360/75 digital computer. To analyse the velocity influence it is necessary to keep into account at least the second -mode approximation i.e. a system with 4 equations.

In general, with n uncoupled frequencies, we have 2.n coupled frequencies, i.e. a polynomial equation of 2.n order, corresponding to n in-phase and n out-of-phase frequencies (fig. 3).

In fig. 4 we can read the natural frequency value of the coupled system as a function of the fluid velocity; the various curves concern the different mode of vibration, while the three groups of curves have been determined for three different values of fluid velocity in the internal tube.

It may be observed that the natural frequency is highly affected above a given critical fluid velocity in the annular region and that this critical velocity depends by the fluid velocity in the internal tube.

Appendix

V	velocity of fluid
EJ	flexural rigidity
y	transvers displacement
x	axial coordinate
M	mass of fluid per unit length
m	mass of tube per unit length
P	resultant force per unit length of fluid acting on structure
R	rod radius
L	length of tube
p	fluid pressure
ρ	fluid density
Ω	circular frequency
ϕ	modal functions
ω	natural frequencies

Index 1 and 2 refer to annular and natural region, respectively

Reference

|1| CHEN, S.S., "Free vibrations of a coupled fluid structural system", Journal Sound and Vibration 21, 1972

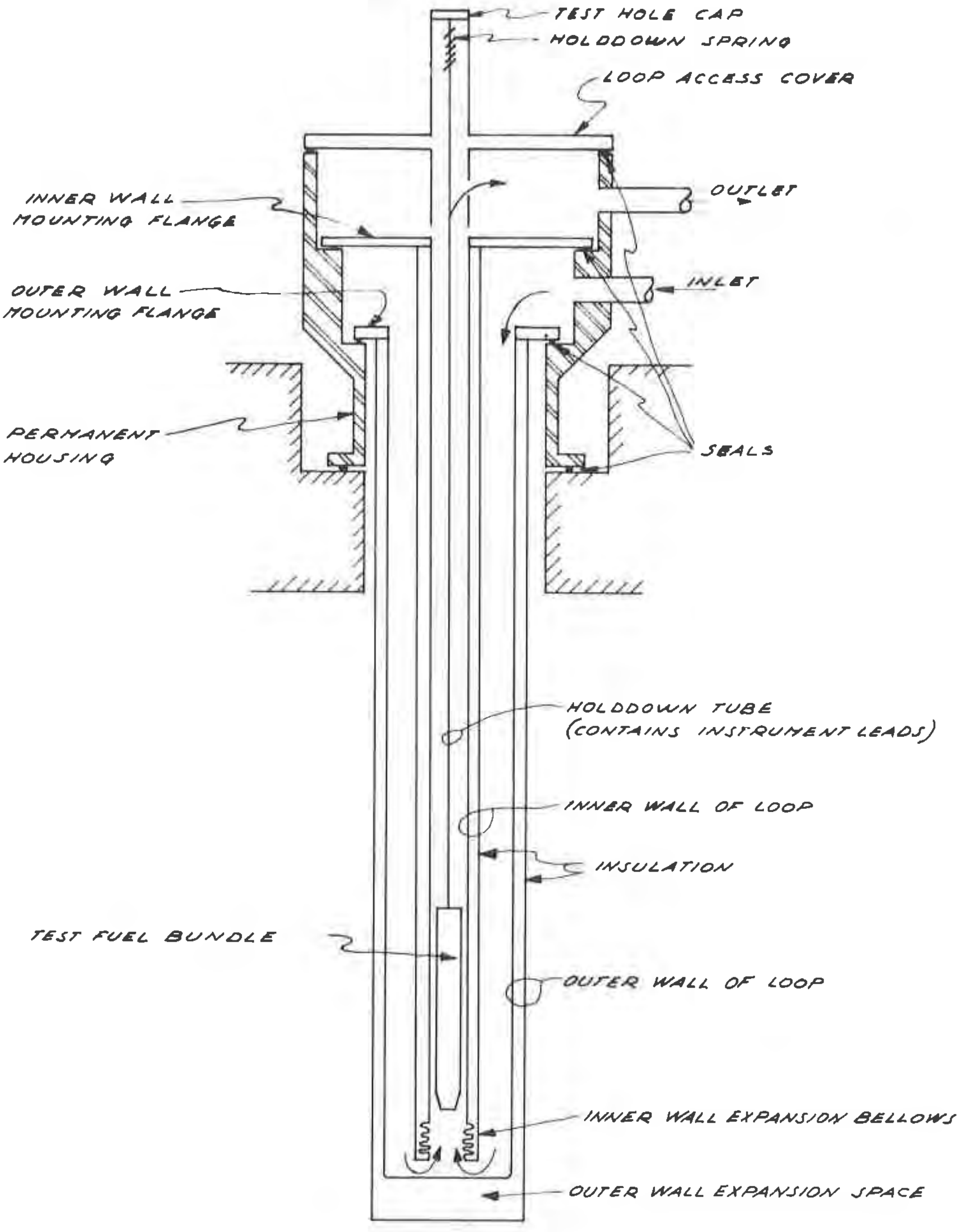

Fig. 1 - Standard re-entrant loop

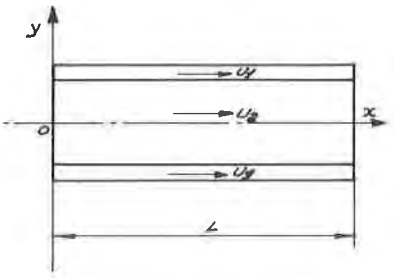

Fig. 2 - Loop model

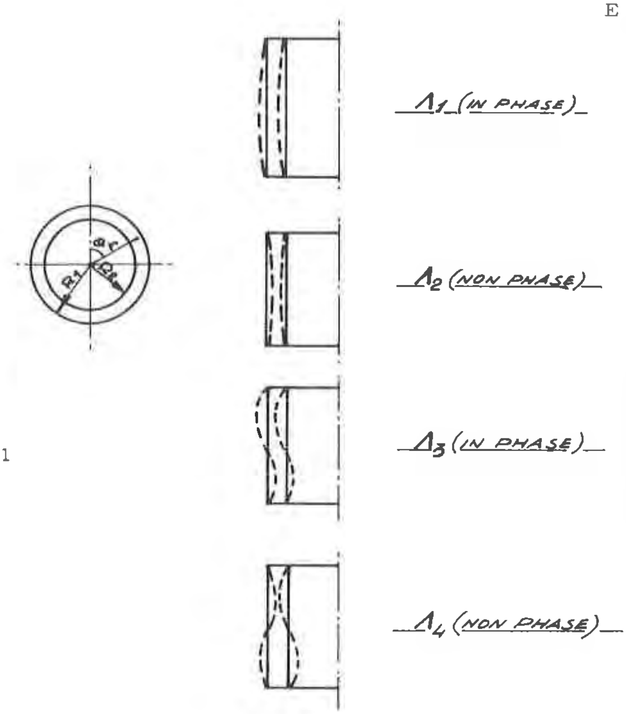

Fig. 3 - Vibrations mode

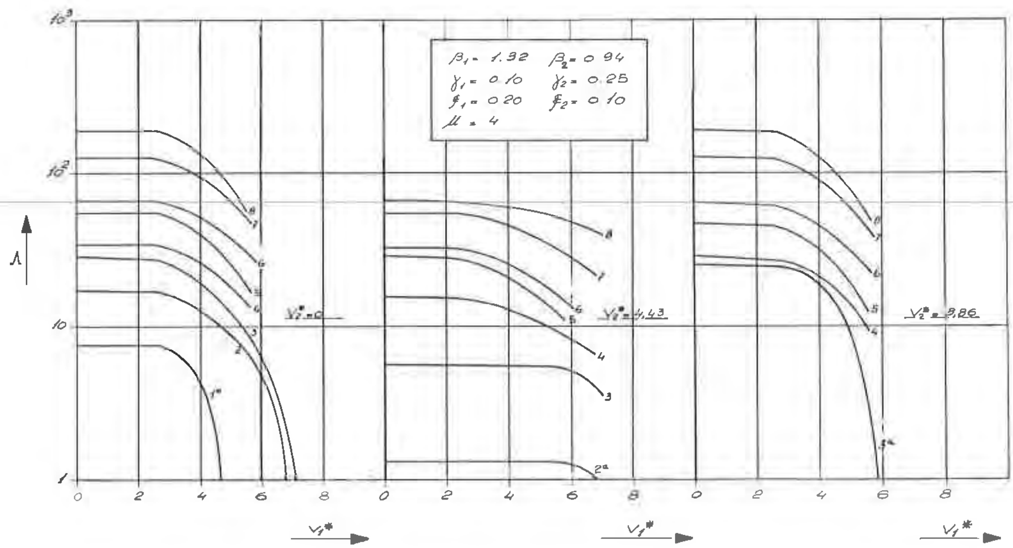

Fig. 4 - Natural frequency as a function of fluid velocity