THE INFLUENCE OF NON-CIRCULARITY ON THE CREEP OF SMOOTH CURVED PIPES

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SUMMARY

Smooth curved pipes are frequently the most important components in a pipework system. In high temperature pipework where creep is present, there is evidence that the pipe bend will be even more important than under linear elastic conditions. However, the bulk of the design information presently available is based on perfectly circular cross-sectional shapes. Little has been reported for non-circular cross-sections even for linear elasticity and virtually nothing is available for creep. Pipe bends can be produced to close dimensional tolerances by hot forging techniques but most of the common manufacturing methods give rise to non-circular bends of varying thickness.

The present paper outlines a theoretical creep analysis which is an extension of an analysis on circular bends and recent linear elastic work on elliptical cross-sections. Pipe bend behaviour is of most interest in a bending situation where its flexibility can contribute significantly to the overall system design. Consequently, in-plane bending loading is considered in this first analysis. The analysis is based on an energy method coupled with a stationary creep constitutive relationship in the form of a simple power law. Conventional strain rate, displacement relations for rotationally symmetric thin shells, are used slightly modified. The initial cross-sectional shape is assumed to be elliptical but of constant thickness. Thickness variations are shown to be of less importance than the cross-sectional shape. A sufficiently general series expansion is assumed for the deformation of the bend and the energy dissipation rate minimised to obtain solutions for any desired numbers of terms in the series. Thus it is possible to check convergence of the solution at all practical values of the pipe bend geometric parameters. The minimisation is performed using a recent computerised optimisation technique and it is easily verified that the results are lower bounds on deformation.

Deformation results are defined relative to the behaviour of a straight pipe but can also be given in terms of the powerful reference stress technique, the latter being attractive because of the consequent economy in material data requirements. Stationary stresses can be found as factors related to the linear elastic stress in a simple beam. The results have profound implications for elevated temperature pipework design but are simple enough to be of direct use without a detailed knowledge of the theoretical background.
1. Introduction

Design concepts for pipework or ducting systems operating in the creep regime are still in their infancy despite the fact that some present day systems are expected to operate at temperatures sufficiently elevated to cause creep. Recently some information has become available which allows account to be taken of creep in an approximate way for simple uniplanar pipework systems \[1\] * . The work outlined in \[1\] indicated that the curved portions of pipe were usually the most important portions of the system as far as controlling the overall behaviour was concerned. This simply reinforces what has been already known for linear elastic systems. However, reported work on bends \[2\] has so far only considered initially perfectly circular cross-sectional shapes with constant thickness. Indeed, even for linear elastic situations \[3\] , enough information is not yet available for all loading conditions.

Of course, pipe bends can be produced with either short or long radius to close dimensional tolerances by hot forging techniques using mandrels, but there is an obvious economic incentive to use cheaper manufacturing methods such as hot or cold bending with or without suitable dies or fillers. These latter methods give rise to bends having a non-circular cross-section together with some local thickening or thinning at the inside and outside of the bend. Provided the thickness variation is reasonably symmetrical, it can be shown to be only a minor effect and is neglected in all that follows. Examination of a large number of 'as manufactured' bends has shown that the non-circularity is not quite symmetrical and could be described as 'pear' shaped. Nevertheless, the shape is not far removed from an elliptical cross-section and it is considered that a first analysis for non-circularity may be conveniently based on an initially elliptical cross-section. Details of the bend geometry are given in Fig.1. It is taken to be of constant thickness \(2h\), radius of curvature \(R\), and elliptical in cross-section everywhere with semi-axes 'b' in the plane of the bend and 'a' in a plane at 90° to the plane of the bend respectively. Usually but not always, \(b/a\) is less than unity.

What follows presents a stationary creep analysis of the defined bend under in-plane bending loading. It is a logical extension of \[2\] and widens the information available to the designer. Preliminary results for both deformations and stresses are presented and discussed. No previous information on the topic is known to the author.

2. Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>(a)</td>
<td>semi-axis of ellipse mid-surface at 90° to the plane of bend</td>
</tr>
<tr>
<td>(B)</td>
<td>constant or function of time in the creep law</td>
</tr>
<tr>
<td>(b)</td>
<td>semi-axis of ellipse mid-surface in the plane of bend</td>
</tr>
<tr>
<td>(C)</td>
<td>series coefficient in deformation rate</td>
</tr>
<tr>
<td>(\varepsilon^2)</td>
<td>(1 - \frac{b^2}{a^2})</td>
</tr>
<tr>
<td>(h)</td>
<td>half of the pipe wall thickness</td>
</tr>
<tr>
<td>(J)</td>
<td>integral function of (\lambda, \frac{b}{a}) and (g)</td>
</tr>
</tbody>
</table>

* References are given in the Bibliography
3. Theoretical Analysis

A stationary creep law is assumed as

$$\dot{\varepsilon} = B \sigma^n$$  \hspace{1cm} (1)

where n and B are material characteristics. In generalised non-dimensional form it can be written as

$$\left( \frac{\varepsilon}{\varepsilon_*} \right) = \left( \frac{\sigma}{\sigma_*} \right)^n$$  \hspace{1cm} (2)

where $\varepsilon_*$ and $\sigma_*$ are any values satisfying eqn 1.

3.1 Deformation Characteristics

The analysis follows Odquist's type 1 energy theorem in the manner described in [5] for/
for circular bends. In the course of a linear elastic analysis [3] the strain displacement relations for the main components of strain in an elliptical bend were developed. In the usual thin shell formulation and differentiated with respect to time, they become

\[
\ddot{\epsilon}_\theta = \left[ \frac{3\dot{w}}{\dot{\theta}} + \dot{v} \cos \phi + \dot{\omega} \sin \phi \right] / R
\]

\[
\ddot{K}_\phi = \frac{1}{K} \frac{3}{3 \phi^2} \left( \frac{3 \dot{w}}{\dot{\phi}} - \dot{v} \right) - \frac{1}{K^2} \left( \frac{3 \dot{v}}{3 \phi} - \frac{3 \dot{\omega}}{3 \phi} \right)
\]

where \( R \) is the radius of curvature of the cross-section. In terms of the angle of the normal \( \phi \) it is

\[
R = a (b/a)^2 \left[ 1 - e^{2 \sin^2 \phi} \right]^{-3/2}
\]

The development of the equations assumed long radius bends, namely, that \( R >> r \), since these are the types most likely to be produced by the manufacturing processes which give rise to non-circular bends. The circumferential curvature rate term \( \dot{K}_\phi \) can be demonstrated to be small. It is further assumed, after Kármán [6], that the mid-surface meridional strain rate

\[
\ddot{\epsilon}_\phi = \frac{1}{K} \left( \frac{3 \ddot{v}}{3 \phi} + \dot{\omega} \right)
\]

is zero which gives a convenient relationship between \( \dot{v} \) and \( \dot{\omega} \).

A reasonable postulation for the deformation behavior of the bend when loaded is that the initial angle \( \alpha \) in Fig. 1 increases after some time to \( \alpha + \gamma \). This is taken to occur in conjunction with a change in cross-sectional shape into another more oval configuration. It is not another elliptical shape since critical examination of Marbec's [7] work shows that type of assumption to be too restrictive. The change in shape is the same everywhere along the bend and is approximated by the series

\[
\gamma = \sum_{p=1, 2, 3} \hat{C}_\phi \cos 2p \phi
\]

An expression for \( \dot{u} \) may simply be found from

\[
\dot{u} = \frac{\dot{v} \theta}{\alpha} b \sin \psi
\]

For the relation between \( \psi \) and \( \phi \), see Fig. 1 and reference [3]. Substitution of eqns 5 and 6 into 3 gives

\[
\ddot{\epsilon}_\theta = \frac{1}{R} \left[ \frac{b (b/a) \dot{v} \sin \phi}{\alpha \left( 1 - e^{2 \sin^2 \phi} \right)^{3/2}} + \sum \hat{C}_\phi \left( \sin \phi \cos 2p \phi - \frac{1}{2p} \cos \phi \sin 2p \phi \right) \right]
\]

\[
\ddot{K}_\phi = \frac{\left( 1 - e^{2 \sin^2 \phi} \right)^3}{\alpha^2 (b/a)^4} \left[ \sum \hat{C}_\phi \left( 4b^2 - 1 \right) \cos 2p \phi \right.
\]

\[
\left. - \frac{(3/2) e^{2 \sin^2 \phi}}{1 - e^{2 \sin^2 \phi}} \sum \hat{C}_\phi \left( 4b^2 - 1 \right) \sin 2p \phi \right]
\]

(7)
3.2 Energy

The approximate potential energy rate given in [5] was

\[ V = \frac{n}{n+1} \left( \frac{4}{3} \right)^{((n+1)/(2n))} \frac{2h \sigma_0}{\dot{E}_o^{1/n}} \int_{V_{ol}} \left[ \dot{\varepsilon}_T^2 + N \hbar \kappa_{\phi}^2 \right] dV_{ol} - M \dot{Y} \]

where \( N = \left[ \frac{n}{(2n+1)} \right]^{2n/(n+1)} \)

Eqns 7 inserted in 8 give after integration

\[ V = \frac{n}{n+1} \left( \frac{4}{3} \right)^{((n+1)/(2n))} \frac{2h \sigma_0 ab(b/a)^2}{\dot{E}_o^{1/n}} \left( \frac{b}{R} \right)^{1/n} \int_0^{2\pi} \left( \frac{1 + \beta_p(\sin \phi \cos 2\beta \phi - \frac{1}{2} \cos \phi \cos 2\beta \phi)}{1 - e^{2\sin \phi}} \right)^{1/2} d\phi \]

\[ + N \frac{\lambda}{4(b/a)^2} \left[ \frac{\lambda}{4} \right] \left( \frac{4b^2 - 1}{2b} \right) \left( 1 - e^{2\sin \phi} \right) (\sin 2\beta \phi) \]

where \( \lambda = \frac{2 \hbar R/a^2} \), \( \beta_p = \frac{c_b \lambda}{b} \)

Eqn 9 may be minimised with respect to \( \beta \) and \( \dot{Y} \). It is easily shown that

\[ \dot{Y} = \left( \frac{3}{4} \right)^{((n+1)/2)} \frac{R \dot{e}}{b} \left[ \frac{M \dot{E}_o^{1/n}}{2h \sigma_0 ab(b/a)^2 \dot{e}_o} \right] - \frac{1}{3} \frac{1}{n} \]

where \( \Upsilon \) is the minimum value of \( J \) for any values of \( \beta \). Minimisation of \( J \) was performed numerically and since the \( \beta \) coefficients are interdependent, was iterative and slow even on a high speed digital computer. By varying the number of terms \( p \) in eqn 5, convergence of the solution could be examined.

3.3 The Straight Elliptical Pipe

A straight pipe with the same cross-sectional properties as the bend may be easily analysed. From compatibility the longitudinal strain rate \( \dot{\varepsilon} \) in a length \( R \alpha \) is

\[ \dot{\varepsilon} = \frac{\dot{\varepsilon}_e b (b/a) \sin \phi}{R \alpha \left( 1 - e^{2\sin \phi} \right)^{1/2}} \]

where \( \dot{\varepsilon}_e \) is the end rotation rate for the straight pipe (or the curvature rate for unit length). Substitution of eqn 11 into the constitutive relation of eqn 2 gives

\[ \sigma = \frac{\sigma_0}{\dot{E}_o^{1/n}} \left[ \frac{\dot{\varepsilon}_e b (b/a) \sin \phi}{R \alpha \left( 1 - e^{2\sin \phi} \right)^{1/2}} \right]^{1/n} \]

Equilibrium/
Equilibrium of the external moment with the stress dictates

$$M = \int_0^{2\pi} 2h\sigma b\gamma_\Phi S\psi d\Phi$$

which rearranged leads to

$$\frac{\dot{\gamma}_e}{K} = \frac{R\dot{\gamma}}{b} \left[ \frac{M \dot{\gamma}_e \rho^n}{2h a b (b/a)^3 \sigma_o} \right]^{n/3} \int_o^\infty$$

where

$$\int_o^\infty \left[ \frac{(b/a) S\psi \Phi}{(1 - c^2 S\psi^2 \Phi)^n} \right]^{(n+1)/n} \left(1 - c^2 S\psi^2 \Phi \right)^{-3n} d\Phi$$

4. Preliminary Results

4.1 Deformation

Rather than evaluate eqn 10 directly, it is more convenient to present non dimensionless deformation factors in the form of flexibility factors $K$ defined as

$$K = \frac{\dot{\gamma}_e}{\dot{\gamma}} = \frac{\text{The end rotation rate of an elliptical bend in creep}}{\text{The end rotation rate of an elliptical straight in creep}}$$

Division of eqn 10 by eqn 12 yields

$$K = (\frac{3}{4})^{(n+1)/2} \int_o^\infty / \int_o^n$$

Factors have been evaluated from eqn 13 for specific $n$, $\lambda$ and $b/a$. The optimisation routine is expensive in terms of computer time and only selected results can be presented here.

Examples of the convergence of the solution together with some fully converged results are given in Fig.2 for $n = 1.5$, $b/a = 0.6$ and $n = 3$, $b/a = 1.2$, both covering the full range of practical $\lambda$. Values of $b/a$ less than unity exhibited different convergence characteristics from $b/a$ greater than unity and this is illustrated. As can be seen, 5 terms in the series were usually sufficient in most cases. The analysis is valid for all $b/a$ and obviously a wider spectrum of $n$ and $b/a$ require to be covered. It is also possible to redefine the factors in the manner outlined in [3] to give results based on an equivalent circular cross-sectional straight pipe if required. Furthermore, the deformation results may be alternatively presented in terms of the so called 'reference stress' technique. All of these aspects will be amplified in other publications.

4.2 Stresses

Having completed the deformation part of the analysis, it is possible to go back to the stress/strain rate equations with the necessary coefficients and evaluate the stresses.

Being a stationary analysis the stresses so obtained are stationary ones and give no indication of the time taken in redistribution between the initial elastic and the stationary state. This is unlikely to be a significant limitation in the context of pipework design and in any case simple guide lines are available. It is convenient here to take the equation in an approximate form in terms of the energy function used in the earlier part of the analysis [2].

The component of stress which is usually of most interest is the meridional bending one which has its maximum value at $\Phi = 0$. Using the strain and curvature rates of eqn 7, after some manipulation,
manipulation, the bending stress can be shown to be

\[
\sigma_{\phi b} = \frac{M_r}{I b} = \pm \left( \frac{n}{2n+1} \right)^{\frac{n+1}{(n+2)}} \left( \frac{4}{3} \right)^{\frac{n+1}{2n}} \frac{B(\phi)}{(b/a)^2 \lambda_0} \sqrt{\nu n \left[ A_3 + \frac{N A_4}{4} \right]}^{(n-1)/(2n)} A_4 \lambda / 2 .
\]

where

\[
A_3 = \left[ \frac{(b/a) \sin \phi}{(1 - e^2 \sin^2 \phi)} \right] + \frac{1}{2p} \cos \phi \sin 2\phi - \frac{1}{2p} \cos \phi \sin 2\phi \sin \phi.
\]

\[
A_4 = \left( \frac{1 - e^2 \sin^2 \phi}{b/a} \right)^{\frac{3}{2}} \left[ \frac{\sin \phi}{(1 - e^2 \sin^2 \phi)} \right]^{(4b^2 - 1) / 2b} \sin \phi \sin 2\phi \sin \phi.
\]

\[
B(\phi) = \frac{(b/a)^2}{(b/a)^2} \int_0^{2\pi} \sin^2 \phi \sin^2 \phi \cos \phi \sin 2\phi \cos \phi \sin 2\phi \cos \phi \sin 2\phi \cos \phi \sin 2\phi \cos \phi \sin 2\phi .
\]

I = 2hab^2 B(\phi) is the second moment of area. Values of B(\phi) have been given in [3]. Inspection of eqn 14 reveals that despite its complexity all the variables \( n \), \( \lambda \), b/a, \( \beta \) and K, have either been specified or evaluated. It is thus a straightforward but tedious matter to compute the bending stress for selected values at any particular angle.

Fig. 3 presents results for the maximum meridional bending stress (\( \phi = 0 \)) for \( n = 1.5 \), a range of b/a and all \( \lambda \). Fig. 4 presents distributions of the bending stress around the cross-section for a specific \( \lambda \), (0.5), two fairly extreme b/a (0.6 and 1.4) and several values of the stress index \( n \) including \( n = 1 \) which by analogy gives the linear elastic result. Since designers dealing with the small departures from circularity will prefer to work from the original circular dimensions before manufacture, Fig. 3 has been referred to an initially circular pipe with a radius taken as (a + b)/2. Thus it should be noted that the ordinates in Figs. 3 and 4 are different.

5. Discussion

It will be apparent that the present work can be checked in a number of special cases where it touches upon previously published analyses. When b/a = 1 the results reduce to and check with those in [5]. If \( n = 1 \), then by analogy the results are applicable to linear elasticity and because of the method of presentation utilized here the factors presented become identical to the corresponding results in [3]. In the case where b/a = n = 1, the results agree with those in [6] and other subsequent similar works.

It is encouraging that for moderate ovality there seems to be nothing to prevent the designer logically allowing for the shape of the bend cross-section in his design. When compared with the circular case, the present results clearly indicate that for moderate ovality use of the nominal circular dimensions will be conservative. However, the difference in flexibility between even moderate ovality and the circular case is significant and could be of advantage in design. Of course, the stresses would also need to be considered. Although
the work reported here is for steady loading, because of the method of presentation the results will be applicable in at least an approximate manner to the real situation.

6. **Conclusions**

A theoretical analysis has been presented and preliminary results detailed which will help designers to have an appreciation of the effect of non circularity on the creep behaviour of pipe bends under in-plane bending loading. Although further results are required, those to date give confidence in the analysis because of the variety of other analyses which check as special cases.

7. **Bibliography**

[1] SPENCE, J.

"An analysis for pipework systems under creep conditions", 1st Int. Conf. on SMIRT, F3/5, Berlin (1971).

[2] SPENCE, J.


[5] SPENCE, J.


[7] Marbec, M.

Figure 1. Elliptical Bend Geometry and Deformation.

Figure 2. Convergence of Flexibility Factors.
FIG. 3. MAXIMUM MERIDIONAL BENDING S.C.F.

FIG. 4. STRESS DISTRIBUTION: MERIDIONAL BENDING.