COLLAPSE OF DUCTILE HEAT EXCHANGER TUBES WITH OVALITY UNDER EXTERNAL PRESSURE

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SUMMARY

Conventional rules in boiler and pressure vessel code documents for computation of collapse pressures of long circular tubes under external pressure do not directly give consideration to the reduction of external pressure capability due to tube cross section ovality. Allowable stress is based on computation of collapse pressure of perfect circular cylinders utilizing tangent modulus material behavior concept or impirical formulae based on collapse tests of actual tubes. Design rule safety factors have accounted for maximum ovality expected in actual tubing.

In computation of external pressure of nuclear system heat exchange tubing, it is prudent to determine collapse pressure considering effects of ovality. Ovality can result from bending processes utilized in forming U-tube heat exchanger tubing and an assessment should be made of the external pressure capability reduction in contrast to the increase due to the torodial shell form consequent from bending.

An analytic technique has been devised for application to collapse pressure determination for long cylindrical tubes with uniform cross-sectional ovality using plastic limit analysis theory in computation of upper and lower bound limit pressure. The technique applies limit analysis theory to a geometric idealization of an oval tube where plastic material behavior governs the collapse mechanism rather than elastic instability. Theory includes the effect of elastic deformation changes on initial ovality prior to collapse. Yield criterion assumed is a bilinear representation of the parabolic yield surface consequent from the combination of bending and normal forces.

In corroboration of the application of plastic limit analysis theory to collapse pressure determination of long, oval cylindrical tubes, collapse tests of actual heat exchanger tubing from several sources have been performed. Careful experiments have been performed for yield strength determination of sample tube specimens. Results indicate reasonable agreement with theoretical determination of collapse pressure using plastic limit analysis theory for tubes of diameter to thickness proportions conducive to plastic hinge formation collapse mechanisms. Collapse experiments with U-bend tubing indicate the increase in external pressure capability of sharply bent tubing over that of long straight cylindrical tubes with similar ovality.

The agreement of experimental results with plastic limit analysis theory suggests praticality of consideration of tube ovality in formulating design rules for nuclear heat exchanger tubing under external pressure.

1. INTRODUCTION

Perhaps the most important consideration in the design of modern heat exchangers is that of tubing. The selection of appropriate tube wall thickness reflects heavily on the economics of the design since increased wall thickness effects both heat transfer rates and vessel size. Likewise, the integrity of the vessel hinges strongly on the tube thickness, for tubes form the barrier to mixing of incompatible fluids. In some nuclear applications, the barrier prevents penetration of primary loop fluid into the secondary fluid system. Tubing material for such application is costly and the design of such tubing must consequently consider both economics and integrity.

Inherent in design of many heat exchanger applications is consideration of ability of the tubes to sustain external pressure consequent from either steady state or accident conditions resulting in loss of tube internal pressure while maintaining external pressure. The literature reflects an abundance of studies directed toward the strength of tubes under external pressure since the mid-1800's. In historical perspective, papers by Fairbairn [1] Carmen and Carr [2] [3], Jasper and Sullivan [4], Sturm [5], Timoshenko [6], and others indicate development of a wide range of analytic and experimental considerations which have formed the basis for design rules over the years.

For the most part, design rules do not reflect consideration of tube ovality in computational procedures. The ASME Boiler and Pressure Vessel Code Sections VIII [7] and III [8] provide for rules based on either test results from a series of collapse experiments for which tube ovality was not carefully related or theoretical results based on the perfectly circular geometry. Although tube ovality sharply affects external pressure capability, this effect was considered only in establishing design safety factor. Consequently, the rules provide little ability to reduce required tube wall thickness where ovality is reduced.

For thin tubes, the collapse mechanism at failure is that of elastic instability, whereas for oval tubes of intermediate thickness the mechanism may be indistinguishable from that of plastic structural collapse, where limit analysis bounding techniques appear applicable. For such cases, this study attempts to indicate the usefulness of upper and lower bounds based on limit analysis for determining collapse pressures for straight cylindrical tubes of oval cross section. This is considered as an alternate to determing such collapse through utilization of large-deformation capability computer solutions. The degree of success with this technique has been determined through collapse testing of tubing with varying degrees of ovality.

Commercial heat exchanger tubing is generally manufactured with small ovality (less than 1%) in straight lengths. For U-tube heat exchangers, however, the bending process can give rise to higher degrees of ovality. The resulting toroidal shape of bent tubes gives rise to two effects on external pressure carrying capability. Cross section ovalization causes collapse pressure reduction, while the doubly curved toroidal surface increases the tube collapse pressure over that for the straight tube.

Hence, the studies have been directed toward the end of providing a quantitative assessment of tube collapse pressure for tubes with ovality in which failure is not elastic instability and a qualitative assessment of the effect of the bent tube shape on increasing tube collapse pressure.

2. ANALYTIC APPROACH

Methods of plastic limit analysis [9] were utilized to determine upper and lower bounds for the external collapse pressure of long, slightly oval straight tubes of intermediate thickness. The development of upper and lower bounds is described in detail in the Appendix.

The usual assumptions of plastic limit analysis were utilized, assuming a linear elastic - perfectly plastic material and a "fixed" cross sectional geometry idealized as two semicircles connected by short flat regions (Figure 1). The bilinear yield diagram (Figure 2) was used to allow interaction between circumferential bending moment and normal force. Yielding due to shear or axial forces was neglected.

Upper bound collapse pressures were obtained by inserting a sufficient and judicious choice of plastic hinges, equating the external work to the internal plastic work during an infinitesimal motion of the mechanism. The hinge plastic work is governed by the flow-rule associated with the yield diagram which specifies that the ratio between the rotation and extension or compression shall correspond to a normal to the yield curve.

Lower bounds were found from equilibrium satisfaction for the given choice of plastic hinges and observation that yield is exceeded only over the flat region. Since the equilibrium relations are linear, lower bounds were calculated by multiplying the upper bound collapse pressure by the appropriate "center-of-flat" moment ratio which reduces to the yield curve.

The effect of elastic deflection of the cross section prior to collapse was included, in an approximate way, by considering the total ovality of the elastically deflected shape just prior to collapse as the relevant parameter. Hence, for the same idealized oval tube geometry, two collapse pressures can be determined, one based on initial ovality and another final ovality due to elastic distortion prior to collapse.

The use of plastic limit analysis for what appears, on first observation, to be an instability problem is of particular concern. Plastic instability is difficult to define. From one viewpoint [10], it is a function of the loading history and therefore cannot be uniquely defined. Therefore, an explanation for the success of the present technique in correlating straight oval tube test data must await results of further study. Nevertheless, it appears that the present problem is actually of the load-deflection type, analogous to the eccentrically loaded column (imperfection stability concept), and that the tube deflections prior to collapse are small enough such that the crucial limit analysis assumption of "fixed" geometry is closely approximated.

For curved U-bend tubes, no similar analysis was attempted because of geometric difficulties and unique collapse modes are not apparent. However, tests of such tubes indicate qualitatively that curved tubes have external pressure capability over that of straight tubes to a degree inverse with tube bend radius.

3. EXPERIMENTAL RESULTS

In order to assess applicability of plastic limit analysis theory in determination of collapse pressure of heat exchanger tubing, a series of tests were performed on nominal 3/4 - .043 inch and 7/8 - .050 inch straight lengths of Inconel tubing with varying degrees of ovality. Furthermore, tests were performed on as-manufactured 7/8 - .050 inch tubing with varying bend radius and ovality.

Tests were performed in a cylindrical chamber which could accommodate either straight or U-bend tubing. The straight tubes were closed at one end and fixed to one head of the chamber at the other end in such a manner that the tube penetrated the head and the inside of the tube was open to atmosphere. The inside of the tube was filled with fluid at atmospheric pressure and tube collapse was determined by sudden issuance of fluid from within the tube due to the rapid contraction of internal tube volume on collapse.

Ovalization was induced in straight tube lengths in two ways. For the 3/4 inch and one set of 7/8 inch tubes a rolling process was utilized within a fixture which controlled the tube cross section shape. In a second set of 7/8 inch tubes, ovalization was induced by means of a press. In the first two sets of tubes, the residual stress system caused by ovalization was alleviated by stress relief. The third set of tubing was tested without stress relief.

Subsequent to the ovalization and stress relief process (if utilized), the material yield strength was determined. For the 3/4 inch tubes and a single set of 7/8 inch tubes, ring specimens were removed from the tube and ring loading tests performed on the specimens from which yield strength was determined from the load-deflection results of the test. For the 7/8 inch un-stress relieved test specimens, conventional tube tensile test data was utilized.

Collapse test results for ovalized straight tube specimens are shown plotted on Figure 5 showing relationship between collapse pressure circumferential membrane stress normalized to material yield strength and % ovality of specimens. Also indicated on Figure 5 are theoretical plastic analysis upper and lower bounds based on both initial ovality and total ovality (including elastic deformation effects prior to collapse). The theoretical results are those computed for the 7/8 - .050 inch tube dimensions.

Collapse tests were performed for 7/8 - .050 inch U-bend tube specimens of several different bend radii and results shown on Figure 6. Bent tube ovality varies around the bend periphery and the maximum measured ovality was utilized in plotting the Figure 6 collapse pressure results. Straight section specimens of the same tube were also tested and provided the datum for the effect of bend radius on collapse pressure. Results are shown related to actual pressure at collapse of tubes.

4. DISCUSSION OF RESULTS AND CONCLUSIONS

In evaluating test data shown in Figure 5 for ovalized straight tubes it is recognized that test data for each set of specimens must be corrected to provide for results compatible with theory at zero ovality. Results for the 7/8 - .050 inch tubes show remarkable agreement with theory for the upper and lower bounds based on initial ovality.

Since data for zero ovality tubes exceed the theoretical $pR/\sigma_y t$ value of 1.0, it can be seen that data correction will provide for reasonable agreement with the theoretical upper and lower bounds based on ovality corrected for elastic deformation at collapse. Data for 3/4 - .043 inch tubes can likewise be corrected downward for reasonable correlation with theoretical bounding curves.

Data for the un-stress relieved 7/8 - .050 inch tubes shows considerable variation from theoretical bounds. Significant in evaluating these results is that the tube yield strength determination and ovalization process differed from that of the foregoing data and the tubes were not stress relieved after ovalization.

Figure 6 clearly indicates the effect of tube curvature in increasing the collapse pressure capability of tubing. While the data for the bent specimens shows some variation, the trend of results is such that the collapse strength of tube is reduced as the specimens approach straight tube configuration. The collapse mode within the bend region is complex and analytic techniques are not provided for prediction of collapse pressures of U-bend tubes. Furthermore, the data is related to maximum ovality and uniform wall thickness, while measurements show a wide range of ovality around the tube bend as well as thickness variation around the tube circumference consequent from the bending process.

From the test results, it is apparent that reasonable predictions of collapse pressure of long straight oval tubes can be made utilizing plastic limit analysis. The precision of prediction is, however, a function of accurate determination of material yield strength within the tube and its relation to the assumed material behavior in the theoretical determination of limits.

Heat exchangers with U-bend tubes can benefit from the curvature effect of the torodial shape of the tubes, particularly for the small bend radii. Nevertheless, it appears design rules should be based, with confidence, on the relationships determined by limit analysis for straight tubes.

5. ACKNOWLEDGMENT

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APPENDIX

DERIVATION OF UPPER AND LOWER BOUNDS

1. NOMENCLATURE

t = Tube wall thickness

R = Mean nominal tube radius

D = $2R$

e = $(D_{max} - D_{min})/D$ = initial ovality

ε = Elastic ovality

e_t = Total ovality = $e + \varepsilon$

p = Collapse pressure

p_u = Upper bound collapse pressure

p_ℓ = Lower bound collapse pressure

Ψ = Yield hinge rotation

ε_i = Compression of Yield Hinge i

M = Circumferential bending moment

N = Compressive normal force

M_o = $\sigma_y t^2/4$ = Yield moment

N_o = $\sigma_y t$ = Normal force

σ_y = Yield Stress

W_i = Internal work at a yield hinge

W_e = External work for one quadrant

W_{it} = Total internal work for one quadrant

E = Young's Modulus of Elasticity

ν = Poisson's ratio

f = Lower bound factor ($p_\ell = f\, p_u$)

2. ANALYSIS FOR STRAIGHT TUBES OF SMALL OVALITY

Plastic limit analysis theorems [9] are applied to obtain upper and lower bounds for the collapse pressure of p of long, straight, slightly oval tubes of cross section shown in Figure 1, idealized as two semicircles connected by parallel "flats" (e=ovality). The bilinear interaction curve [9] of Figure 2 is utilized such that both bending moment and normal force yielding are considered. No yielding due to shear or axial forces is included.

3. UPPER BOUND ANALYSIS

An upper bound p_u for the collapse pressure may be obtained by inserting sufficient hinges (A, B, C, etc. Figure 1) to convert the tube cross section to a mechanism and then equating external and internal plastic work. Note that a "double hinge" (two arbitrarily close hinges on each side of the horizontal center line) is used at A and D in Figure 1 in order to utilize their full hinge compression in each quadrant. Using the bilinear yield diagram of Figure 2(together with the fact that a hinge rotation Ψ and compression ε_i are components of an exterior normal such as OD or OE) it follows that on AB and BC, respectively:

$$|\varepsilon/\psi| = 3M/2N = 3t/8 \;;\; |N/N_o| > 1/2 \qquad (1)$$

$$|\varepsilon/\psi| = M_o/2N_o = t/8 \;;\; N/N_o| < 1/2 \qquad (2)$$

The internal plastic work W_i at a yield hinge is

$$W_i = N_\varepsilon + M\psi \qquad (3)$$

where ε and ψ are related by equations (1) or (2) and N, M must be on the yield curve

$$|N/N_o| + 2/3|M/M_o| = 1 \;;\; |N/N_o| > 1/2 \qquad (4)$$

$$1/2|N/N_o| + |M/M_o| = 1 \;;\; |N/N_o| < 1/2 \qquad (5)$$

Use of equations (4) and (5) in equation (3) gives the simplified forms

$$W_i = N_o|\varepsilon| \;;\; |N/N_o| > 1/2 \qquad (6) \qquad W_i = M_o|\psi| \;;\; |N/N_o| < 1/2 \qquad (7)$$

Since the normal force is large for small ovalities, it is reasonable to assume initially that $|N/N_o| > 1/2$ for both hinges 1 and 2 at A and B, respectively, Figures 1 and 3). Since ψ is the same for both hinges, it follows that $\varepsilon_1 = \varepsilon_2$, and the total internal work is

$$W_{it} = 2\, N_o|\varepsilon_1| \;;\; |N/N_o| > 1/2 \text{ per quadrant} \qquad (8)$$

Using the collapse mechanism of Figures 1 and 3, the external work W_e for one quadrant is

$$W_e = p_u eR(\varepsilon_1 + \psi R) + p_u R(\varepsilon_1 + \varepsilon_2) \qquad (9)$$

Substitution of equations (1) and use of $\varepsilon_1 = \varepsilon_2$ in equation (9) and equating $W_{it} = W_e$

$$p_u R/\ _y t = 2/(e(1+\frac{8}{3}\frac{R}{t})+2) \ ; \ |N/N_o| > \ 1/2 \qquad (10)$$

Since the minimum normal force $N = p_u R$ occurs at upper hinge 2 (Figure 3), the condition

$$|N/N_o| = p_u R/\sigma_y t = 1/2 \qquad (11)$$

shows that equation (1) applies for $0 \le e \le 2/(1+\frac{8R}{3t}) = e_a$ (12)

For the particular case of a 7/8 inch outer diameter tube with .05 inch wall thickness, where $R/t = 8.25$, the upper limit for use of equation (10) is $e_a = 2/23 = 0.087$. For initial ovalities above e_a, the hinges 2 and 1 move successively onto segments BC and DE of the yield curve of Figure 4. The upper bound analysis is analogous to that given previously

$$p_n R/\sigma_y t = 5/(3e(1+\frac{8}{3}\frac{R}{t})+4) \ ; \ e_a \le e \le 6/\ 3(1+\frac{8}{3}\frac{R}{t}) - 10 \ = e_b \qquad (13)$$

$$p_u R/\sigma_y t = 4/(2+e(1+\frac{8R}{t})) \ ; \ e_b \le e \qquad (14)$$

$$(0.087 \le e \le 6/59 = .1017 \text{ for } R/t = 8.25)$$

4. LOWER BOUND ANALYSIS

Lower bounds correspond to equilibrium solutions for moment and normal force within the yield curve (Figure 2). Utilizing upper bounds of equations (10, (13), or (14), equilibrium is shown preserved over the semicircular arcs except for flat regions CB and EF (Figure 1). Since maximum moment occurs at the center of the flat, a lower bound may be obtained by multiplying p_u by the appropriate center moment ratio that brings HJ (Figure 4) within the yield curve. This lower bound p_ℓ is less than the upper bound of equations (10) and (13) for $0 \le e \le 0.1017$ and $R/t = 8.25$. For example, the factor $f(p_\ell = f\ p_u)$ for the range $0 \le e \le 0.087$ of equation (10) is

$$f = \frac{1 + (8R/3t)}{1 + (8R/3t)(1+e)} \qquad (15)$$

Equations (10), (13), and (14) are shown plotted in Figure 5.

5. EFFECT OF ELASTIC DEFLECTIONS

The preceding upper and lower bound analyses are based on idealized initial geometry of Figure 1. For slightly oval tubes, the increase in ovality ε due to elastic deflection prior to collapse is significant. It would appear realistic to utilize the elastically deformed shape at collapse. An approximation to this elastic increase in ovality ε may be obtained by replacing initial ovality by the "total" ovality $e_t = e + \varepsilon$ is given by

$$\varepsilon = 8e\ pR^3/Et^3 \ ; \ \beta = 3(\pi-2)(1-\nu^2) = 0.39 \text{ for } \nu = .03 \qquad (16)$$

The value of ε, equation (16) is obtained from elastic solution of a slightly oval tube with cross section of Figure 1. The ovality at collapse is now the total value e_t. Furthermore, in the lower bound factor f of equation (15), e must also be replaced by e_t to determine upper bound with elastic ovality effect. The bounding collapse pressures, which include elastic effects, are shown on Figure 5, plotted as a function of the initial ovality e. Figure 4 shows how the same loci of stress resultants may be viewed as corresponding to different initial ovalities, since it is the total ovality at collapse that is the governing parameter. It follows that, when the elastic deformation is considered, the ranges of initial ovalities for which the collapse pressure equations (10), (13), and (14) apply, are altered. For example, when e is replaced by e_t in equations (10) and (11) the upper limit e_a is reduced from 0.087 to 0.072, for $R/t = 8.25$ and $\sigma_y/E = 1.64 \times 10^{-3}$.

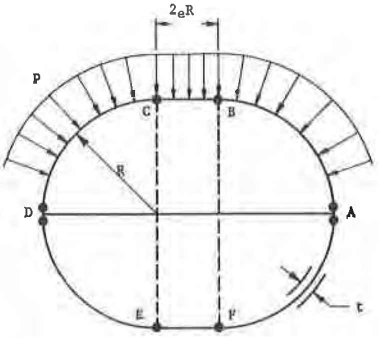

Figure 1 Idealized Tube Cross Section and Location of Yield Hinges

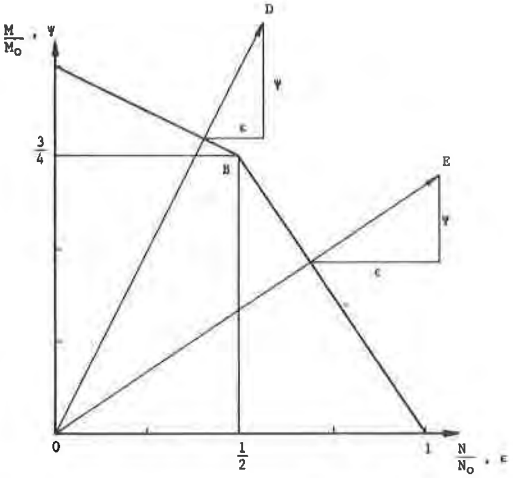

Figure 2 Bilinear Yield Diagram Showing External Normals and Relation Between Plastic Rotation and Extension

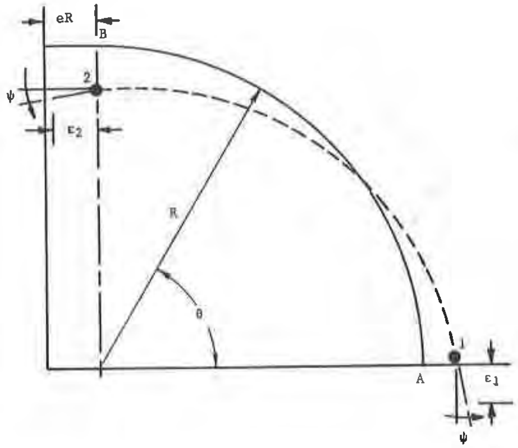

Figure 3 Collapse Mechanism for Quadrant of Tube Cross Section Showing Plastic Hinges at Locations A1 and B2

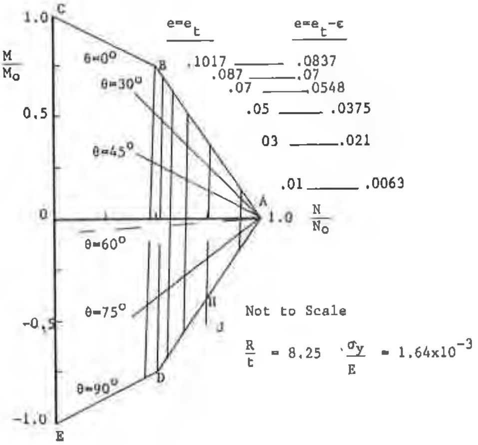

Figure 4 Loci of Stress Resultants for Upper Bound Pressure and Initial Ovalities

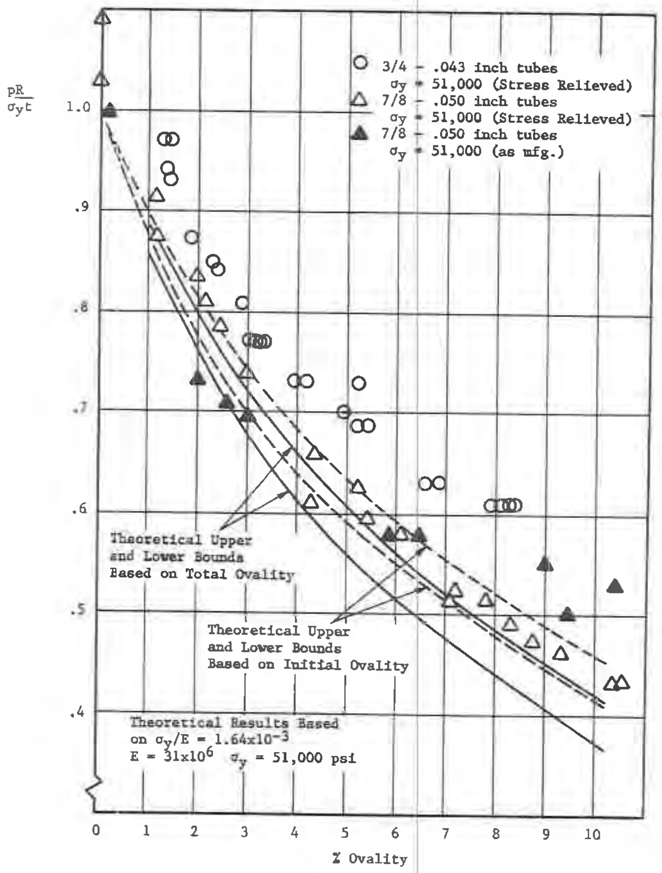

Figure 5 Comparison of Experimental Results for Collapse of Long-Oval Tubes under External Pressure with Theoretical Upper and Lower Bounds

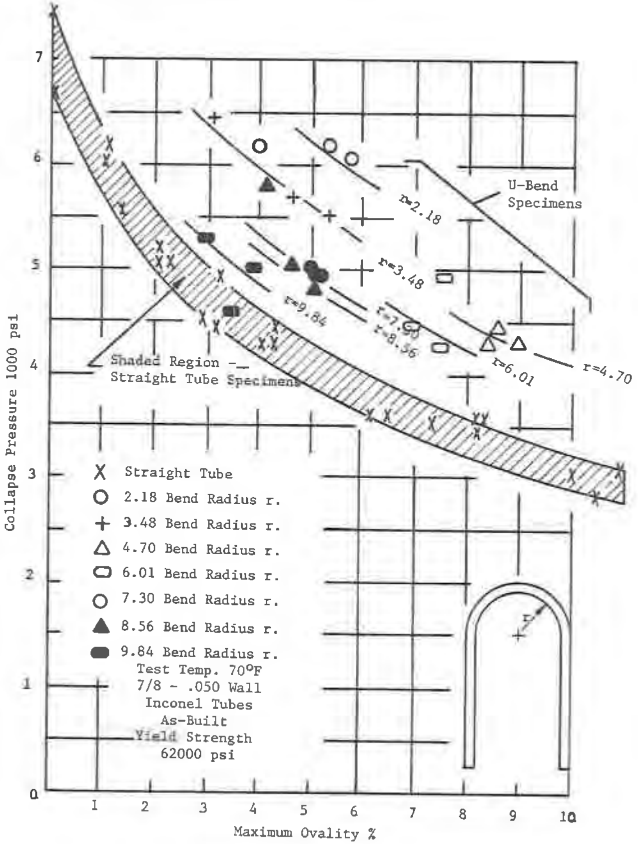

Figure 6 Experimental Results Showing Effect of Tube Bending Configuration on External Collapse Pressure