INELASTIC ANALYSIS IN LMFBR REACTOR VESSEL DESIGN

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SUMMARY

LMFBR reactor vessels are designed to withstand groups of severe thermal transients and operate at very high temperature for decades. The safe design of such nuclear components unavoidably requires knowledge of inelastic time-dependent and time-independent analysis. The criteria for inelastic design at elevated temperature are tentatively defined in ASME Code Cases 1331-5 through -7 of Section III. The limitations and definitions of different failure modes can also be found there. With the existing computational techniques it is demonstrated that a complete cyclic inelastic analysis can be obtained analytically for three-dimensional structures of a general nature. The results presented illustrate the up-to-date capabilities of inelastic analysis.

This paper is intended to narrow the gap between the theoretical discussions and the practical applications of inelastic analysis. For this purpose, it is attempted to answer the questions such as:

(1) how a reasonable inelastic analysis can be obtained by the finite element method,
(2) how the information needed for the practical designs is reduced and interpreted from the massive computational results,
(3) what difficulties and limitations are involved in the analysis and
(4) what precautions should be taken in advance.

It is known that the accuracy of inelastic analysis is ultimately limited by three main factors; these are (i) constitutive equations, (ii) material properties and (iii) loading history.

Unlike the elastic analysis, the factors affecting an inelastic result may become multiple-dimensional as the structure becomes complex. For the purpose of illustration, three complete analyses for the components of a sodium reactor vessel are presented. The examples include the determination of the ultimate strength of a physically complex supporting structure and two cyclic elastic-plastic-creep analyses with prescribed loading histories.

The materials presented in this paper may be helpful to designers and researchers at this state of the art to develop a reliable analytical method for inelastic analysis. It may be noted that the technology of inelastic analysis is still undergoing very rigorous development and refinement.

1. Introduction

Inelastic analysis involves nonlinearity both in mathematical formulation and material properties. Very often the time factor must also be considered. Lacking experimental verification, inelastic analysis is neither established nor well-defined at the present time. It is known that there are numerous factors involved in an inelastic design such as strain rate, temperature, stress relaxation, etc. The interaction of creep and plasticity introduces many complications which do not exist in low-temperature design. It must be remembered that the accuracy of an inelastic analysis is ultimately limited by three main factors. These are: 1) constitutive equations, 2) material properties, and 3) loading history. Stress analysts should be aware of these limitations prior to discussing the significance of any inelastic result obtained by analytical methods.

Liquid metal fast breeder (LMFBR) reactor vessels are designed to withstand groups of severe thermal transients and operate at very high temperatures for decades. The safe design of such nuclear components requires knowledge of inelastic time-dependent and time-independent analysis. The criteria for inelastic design at elevated temperature are tentatively defined in ASME Code Case 1331-5 through -7 of Section III [1]. The limitations and definitions of different failure modes can also be found there.

With the existing computational technique, it is demonstrated here that a complete cyclic inelastic analysis can be obtained analytically for three-dimensional structures of a general nature. Because of its generality, the finite element method has been evolved into a powerful tool for performing inelastic analysis of complex structures. MARC [2] and ANSYS [3] are the two well-known general purpose inelastic programs available in the United States. In order to illustrate the current analytical capabilities of inelastic analysis, some of the results obtained from the analyses of a sodium reactor vessel are presented in this paper.

Applications of inelastic analysis on the designs of LMFBR nuclear components in recent years have sparked various activities which are aimed at establishing methods and criteria for inelastic design. Oak Ridge National Laboratory's (ORNL) reports and publications [4] represent the latest effort to integrate the methods of inelastic analysis. The research program includes verification of the constitutive equations and establishment of parameters used in the practical design. Recently, Pugh, et al, [5] presented a set of constitutive equations, flow law, and hardening rule for cyclic elastic-plastic-creep analysis of stainless steel. It may be noted that the technology of inelastic analysis is still undergoing vigorous development and refinement.

2. Inelastic Analysis and Design

Inelastic analysis may be done by (1) numerical computation, (2) theoretical approach, and (3) experimental method. However, only a few problems can be solved theoretically and the experimental method may not be practical in design. For these reasons, the computational methods become more important and in increased demand. Many computer programs are available. Among them, MARC and ANSYS are the two most generalized computer codes.

The scope and theoretical background of MARC computer code were discussed by Marcal [6, 7, 8] and MARC CDC manuals [2]. The computer program is developed in incremental fashion. It essentially reduces the inelastic analysis to solutions of successive elastic problems. Ayres [9] reported the successful use of MARC in analyzing some practical problems. Our own experience in using MARC to analyze structures such as nozzles, supporting structures, plate and shell interactions, vessel shells, vessel flanges, etc., has shown that reasonable and consistent inelastic results can be obtained for three-dimensional structures of a general nature.

A reasonable inelastic analysis by the finite element method usually requires:

1. Stability of results by changing grid size, connectivity, loading increment, etc.
2. Stability of numerical computation (convergence).
3. Reasonable representation of the overall structure response.
4. Reasonable predictions of local effects.

In the inelastic analysis, superposition and extrapolation are, in general, not applicable. The finite element model used in a nonlinear structure analysis should be (1) used with well-defined boundary conditions; (2) consolidated and acting as a total structure, and (3) modeled with a grid size as small as possible.

There are numerous factors that need to be considered in inelastic design. However, ASME Code Case 1331-5 though -7 reduces them to include only the calculations of the following:

1. Strain range (fatigue).
2. Accumulated inelastic strain (strain limitation).
3. Residual stresses or stresses existing at the long-term operational steady state (creep rupture).
4. Functional limitation (if any).
5. Critical time for creep buckling.

The other parameters are simply included in the material property curves.

To determine the strain range, the accumulated inelastic strain and residual stresses require cyclic analysis. Usually the required cycles in an inelastic analysis are:

1. At least one cycle to estimate the strain range.
2. At least one and one-half cycles to estimate the accumulated strain.
3. At least two and one-half cycles to estimate the ratchetting effect.

One cycle means a complete cycle of operation. A typical cycle usually includes normal heatup, operational steady state, and cooldown accompanying a scram transient. Sometimes, shakedown of a structure can be shown within two cycles of inelastic analysis.

The sequence of loading (Dowling [10]) and the cyclic plastic strain are important in the determination of the vessel life. In the real world, it is difficult to predict the sequence of the occurrences of these loadings. It may be suggested that the most severe loading condition be considered first, followed by several less severe loading conditions.

It may be noted that:

1. The result of inelastic analysis is no better than the input data.
2. Finite element program is no better than the postulated constitutive equations.
3. Extrapolation and superposition should be avoided in the inelastic analysis.
4. Loading path should be followed closely.
5. The criteria of the practical design are extracted from the experimental data of uniaxial or, at best, biaxial test and extrapolated to general stress states.

3. Example Problems

In order to illustrate some of the characteristics and current capabilities of inelastic analysis, three examples obtained from the analyses of a sodium reactor vessel are presented. These results are obtained using MARC computer code.

3.1. Ultimate strength analysis of a supporting structure

The design of LMFBR reactor vessel and the vessel supporting structure is required to withstand a hypothetical core disruptive accident (HCDA) without rupture. The supporting structure shown in Fig. 1 consists of three parts: (1) the axisymmetric supporting ring, (2) the fingers, and (3) part of the reactor vessel. The reactor vessel and the supporting ring are geometrically axisymmetric while the fingers are not circumferentially continuous. In the analysis, it is assumed that the reactor vessel and the supporting ring behave axisymmetrically and the fingers act as a bar. These three parts are connected at the boundary by specifying the displacement restriction in the finite element solution. The surface interaction between the supporting ring and finger is very complicated and may vary as the applied load is increased. The stiffness of the finger is retained by considering the total energy absorption of the finger. Furthermore, the supporting system consists of three different materials. The HCDA vertically downward load is applied monotonically in increments. When the plastic strain becomes large, the load increment should be small enough to assure the stability of the computation. The stress versus strain curves were approximated in five steps.

The ultimate strength of a structure can usually be determined by plotting the displacement versus loading or effective stress versus loading curve. Figure 2 shows the plots of δ versus p and σ_e versus p curves. The extrapolation should be kept to a minimum. Here are some of the general rules to justify the accuracy of an analytical result.

1. Compare the calculated strain increment with the predicted strain increment.
2. Compare the calculated results with the input data e.g., reconstructing the σ-ε curve as shown in Fig. 3. Good agreement between the calculated

value and the input data can be seen in element A. However, element B responds somewhat stiffer than it should.

3. Check the equilibrium conditions.
4. Examine the shift of yielding surface.
5. Check the condition of $\varepsilon_{ii}{}^p = 0$.
6. Use engineering common sense.

3.2. Inelastic analysis of vessel shell and flange

This example is given to illustrate some of the characteristics of cyclic inelastic analysis. The structure analyzed is shown in Fig. 4. The finite element model consists of axisymmetric isoparametric ring element and axisymmetric shell element. The isoparametric element is used in the region of steep axial temperature gradients with a large shearing effect. The loading history for cyclic analysis includes a typical cycle of normal heatup, creep (at operational steady state), and cooldown accompanying a severe scram transient.

Figure 4A shows a plastic zone developed in the vessel wall at the vicinity of sodium level where large thermal gradients exist. This plastic zone gradually grew during heatup and cooldown and did not grow any further after the first cycle of loading. The plot of the plastic zones may assist designers in locating the critical areas of structures. A series of curves showing the distribution of stresses at different times during the transient are presented in Figs. 5 through 10. Analysts may be aware of these very typical characteristics of inelastic analysis resulting from the work-hardening of materials. The structure has been shown to shakedown eventually, to behaving elastically. However, the stress distribution is nowhere close to that obtained by direct elastic analysis. The shakedown of a structure may be shown by:

1. Reconstructing the cyclic σ-ε curve (Fig. 11).
2. Plotting the stress distributions at successive cycles such as the one shown in Figs. 12A and 12B.

Usually creep effect is only considered at operational steady state when hold time is relatively long and stress relaxation becomes important.

3.3. Calculations in a cyclic inelastic analysis

A complete inelastic analysis requires the evaluations of the cyclic strain range, residual stress, accumulated inelastic strain, and vessel life. The following example is given to illustrate how to calculate these design values.

The structure shown in Fig. 13 is designed to prevent core effluent from breaking the surface of the sodium pool. Its design data are given below.

1. Loading conditions -- including 120 cycles of normal shutdown, 705 cycles of x transient and 20 cycles of y transient: The temperature gradient through the thickness of the plate is very small during normal shutdown and the strain range resulting from this case is negligible. However, there will be residual stresses existing in the plate at all times during steady state. Thus, creep analysis must be considered for all cycles.

2. Sequence of loading -- it is assumed that the most severe transient y occurs first, followed by some less severe transient x.
3. Design life -- 20 calendar years operating at 75% availability; that is 1.315×10^5 hours.
4. Operational temperature: 1090 F.

Four cycles of inelastic analysis including two cycles of x transient and two cycles of y transient were performed. The reconstructed σ-ε curve of these two transients is shown in Fig. 11. Shakedown of the structure can be seen.

Currently the vessel life is estimated by a linear fraction rule of fatigue and creep damage. That is,

$$\Sigma \frac{n}{N_d} + \int \frac{dt}{Tr} \leq D \tag{1}$$

where: $0.6 \leq D \leq 1.0$.

Detailed discussion of the interaction effect of creep and fatigue may be found in Esztargar [11].

The $\Sigma \frac{n}{N_d}$ term may be determined by knowing the cyclic ranges, ε_T, experienced by the structure of all cycles. The strain ranges as well as the σ-ε curve may be determined by using the following equations.

The total strain in the plastic-creep range may be written as:

$$\varepsilon_{ij}^{T} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} + \varepsilon_{ij}^{c} \tag{2}$$

or,

$$\varepsilon_{ij,n}^{T} = \frac{1}{E}[(1+\nu)\sigma_{ij} - \nu\theta\,\delta_{ij}] + \underline{\alpha T\,\delta_{ij}}$$

$$+ \sum_{n=1}^{n} \varepsilon_{ij,n-1}^{p} + \frac{3}{2}\frac{1}{\sigma_e}(\sigma_{ij} - \frac{1}{3}\theta\delta_{ij})(\varepsilon_n^{p} - \varepsilon_{n-1}^{p})$$

$$+ \sum_{n=1}^{n} \varepsilon_{ij,n-1}^{c} + \frac{3}{2}\frac{1}{\sigma_e}(\sigma_{kj} - \frac{1}{3}\theta\delta_{ij})(\varepsilon_n^{c} - \varepsilon_{n-1}^{c}) \tag{3}$$

where: θ is the first stress invariant;

ε_{ij}^{T}, ε_{ij}^{p}, and ε_{ij}^{c} are the total, plastic, and creep strain components, respectively;

ε_n^{p} and ε_n^{c} are the plastic and creep strains at the nth increment; and

σ_e is the effective stress.

The stress components, effective stress, accumulated plastic and creep strains are usually given in the computer output. Thus, the total strain components, effective strain, and principal strains at each increment can be easily determined. In eq. (3), the αT term represents the free thermal expansion and should not be included to calculate the strain range. There is no definite method to determine the strain range. It is suggested that either eq. (4 or 5) be used to calculate the strain range.

$$\varepsilon_T = \frac{\sqrt{2}}{3} \left[(\varepsilon_{xx}-\varepsilon_{yy})^2 + (\varepsilon_{xx}-\varepsilon_{zz})^2 + (\varepsilon_{yy}-\varepsilon_{zz})^2 + 6\,\varepsilon_{xy}^{\;2} + 6\,\varepsilon_{xz}^{\;2} + 6\,\varepsilon_{yz}^{\;2} \right]^{1/2} \qquad (4)$$

where ε_{xx}, ε_{yy}, ε_{xy}, etc., are the strain components, and

$$\varepsilon_{xx} = (\varepsilon_{xx})_1 - (\varepsilon_{xx})_2,$$

$$\varepsilon_{yy} = (\varepsilon_{yy})_1 - (\varepsilon_{yy})_2, \text{ etc.}$$

Subscripts $_1$ and $_2$ indicate the two extremes in a loading cycle.

$$\varepsilon_T = (\varepsilon_e)_1 - (\varepsilon_e)_2 \qquad (5)$$

where ε_e is the effective strain.

Minami and Roberts [12] recommended the use of the range of maximum strain components. For conservatism, the largest strain range resulting from these three approaches may be used to calculate the fatigue damage factor.

Figure 11 shows the strain range obtained analytically for virgin material and subsequent cycles. Thus, the total usage factor contributed by fatigue can be determined as:

$$\Sigma \frac{n}{N_d} = \frac{1}{N_{\varepsilon_{T,\text{ virgin material}}}} + \frac{19}{N_{\varepsilon_{T,\text{ y transient}}}} + \frac{705}{N_{\varepsilon_{T,\text{ x transient}}}} + \frac{120}{\infty} \qquad (6)$$

at 1090 F.

The creep damage effect may be estimated by the integral, $\int \frac{dt}{T_d}$ or $\frac{\Delta t}{T_d}$. T_d is time to rupture at a corresponding stress σ_e. It can be determined from the curves given in Code Case 1331-5 through 7. ΔT is the duration time of creep for each loading cycle. Hence,

$$\Sigma \frac{\Delta t}{T_d} = \left(\frac{20/725}{(T_d)_{\sigma_{e,\text{ y transient}}}} + \frac{705/725}{(T_d)_{\sigma_{e,\text{ x transient}}}} \right) \times t_{\text{vessel design life}} \qquad (7)$$

where:

$t = 1.315 \times 10^5$ hours

σ = the average residual stress during creep divided by 0.9.

Only the locations with the worst combination of cyclic plastic strain and residual stress need fatigue and creep damage evaluation.

In solution, the primary creep may be repeatedly applied at the beginning of each cycle of loading. This will yield the most conservative accumulated creep strain and quite possibly a more pronounced ratchetting effect. The ratchetting effect may be determined from the plot of the strain increment $\Delta\varepsilon$ per cycle versus number of cycles of loading. It is our experience that unless the mechanical load or the stress relaxation and redistribution is very pronounced during creep, the ratchetting effect is often negligible. It may be noted that to perform a ratchetting analysis, the parameters such as temperature, loading increment, loading sequence, finite element used, etc., should be carefully studied in advance. Otherwise, it will be difficult to justify the accuracy of the analytical results at the end of the calculation. It may be desirable to use highly sophisticated elements in a ratchetting analysis.

4. Conclusions and Discussion

Although the inelastic analysis can be obtained for complex structures and arbitrary combination of loadings with existing computational techniques, inelastic analysis is still undergoing very vigorous development. Some questions regarding the postulations of inelastic theory need further verification,e.g., the assumption that creep has the same mechanism as that of plasticity whether it is subjected to mechanical loading or exposed to thermal shock, is arbitrary. Acquisition of material properties by multiaxial test is a necessity to clear the unverified postulates. Inelastic analysis is extremely complicated and the present proposed theory may not be applicable to materials other than stainless steel, e.g., yielding criteria may become extremely irregular for some materials and creep law is expected to be different from one material to another. In practical design, fatigue and creep damage introduce many other complications.

The present difficulties (if any) in inelastic analysis essentially result from:

1. Limitation of computer facility dealing with large system of finite element analysis.
2. Limited accessibility of computer program.
3. Large computer time involved.
4. Inadequate knowledge of material behaviors in the inelastic range.

Finite element methods will continue to be one of the most important and powerful tools in inelastic analysis for decades to come. The information presented in this paper may be helpful to designers and researchers in this state of the art in developing a reliable analytical method for inelastic analysis.

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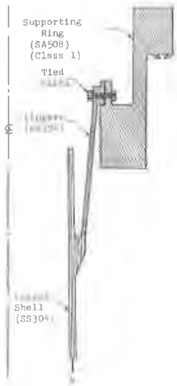

Fig. 1: The system of supporting structure

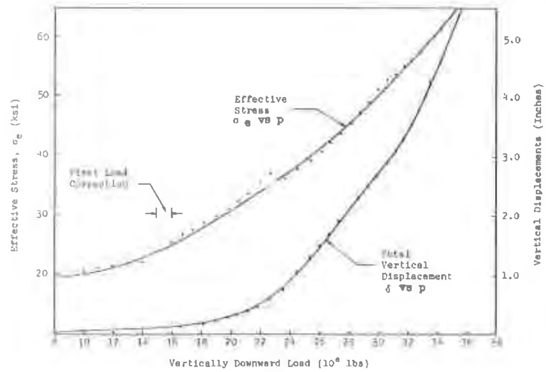

Fig. 2: Stress and displacement vs. load curves

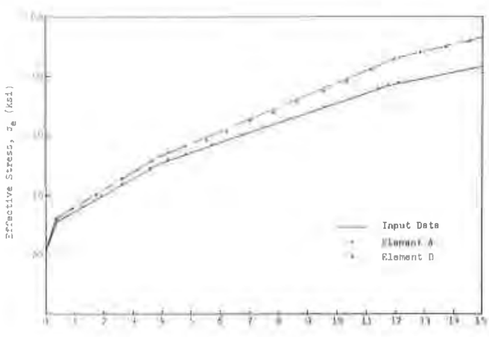

Fig. 3: Stress-strain curve

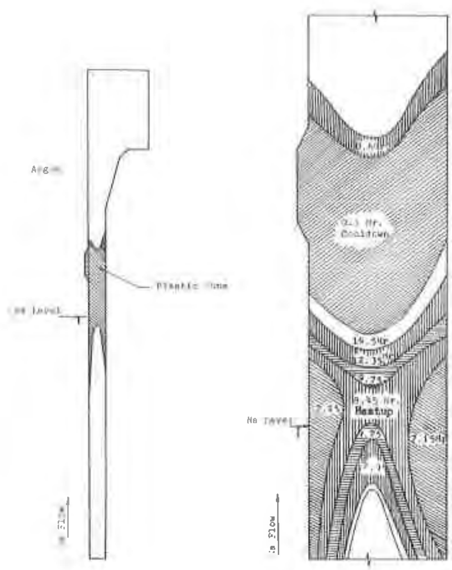

Fig. 4: Vessel wall
Fig. 4A: Vessel wall plastic zone close-up

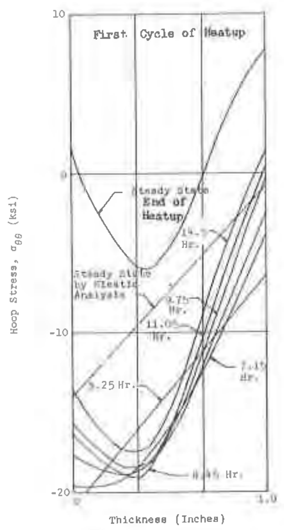

Fig. 5: Vessel wall hoop stress distribution

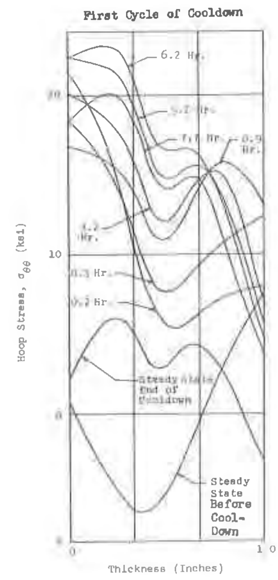

Fig. 6: Vessel wall hoop stress distribution

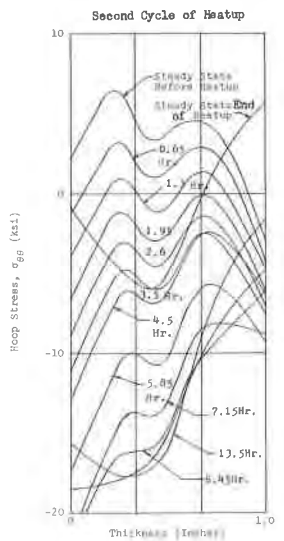

Fig. 7: Vessel wall hoop stress distribution

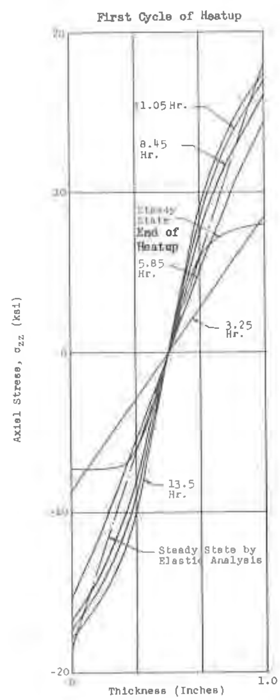

Fig. 8: Vessel wall axial stress distribution

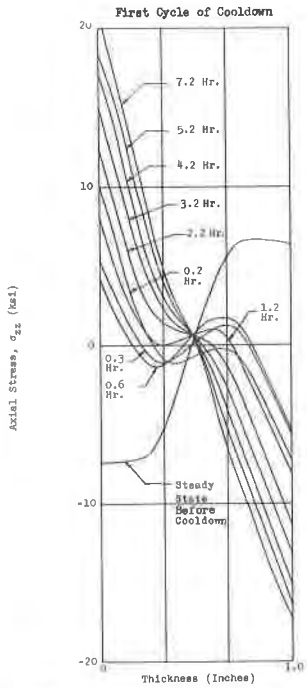

Fig. 9: Vessel wall axial stress distribution

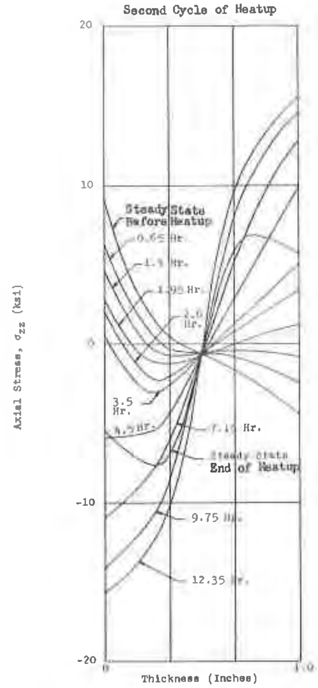

Fig. 10: Vessel wall axial stress distribution

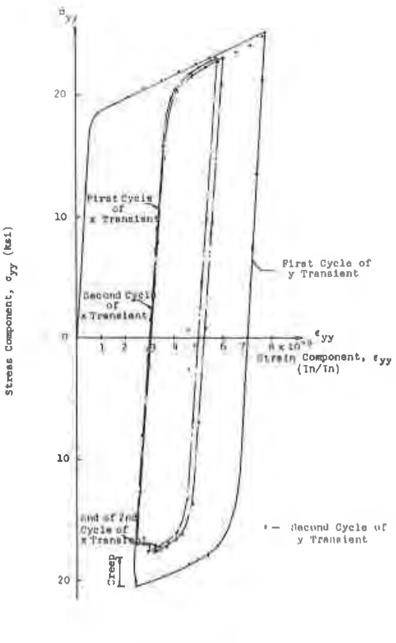

Fig. 11: Reconstructed stress vs. strain curve for cyclic analysis

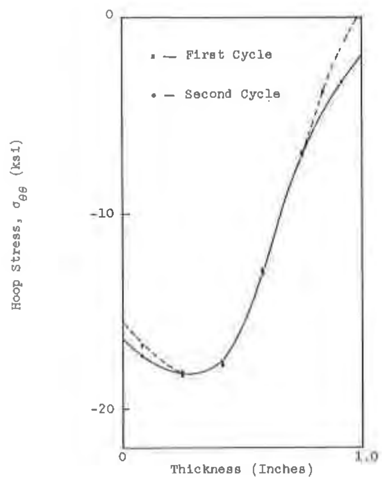

Fig. 12A: Hoop stress at the first and second cycles of heatup

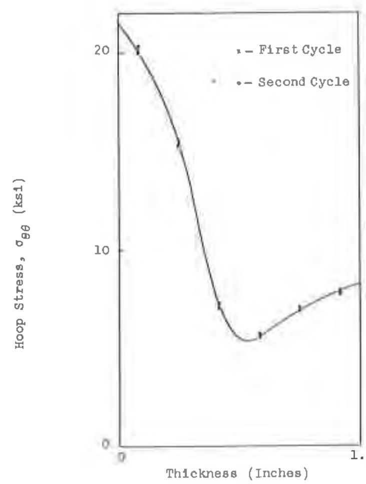

Fig. 12B: Hoop stress at the first and second cycles of cooldown

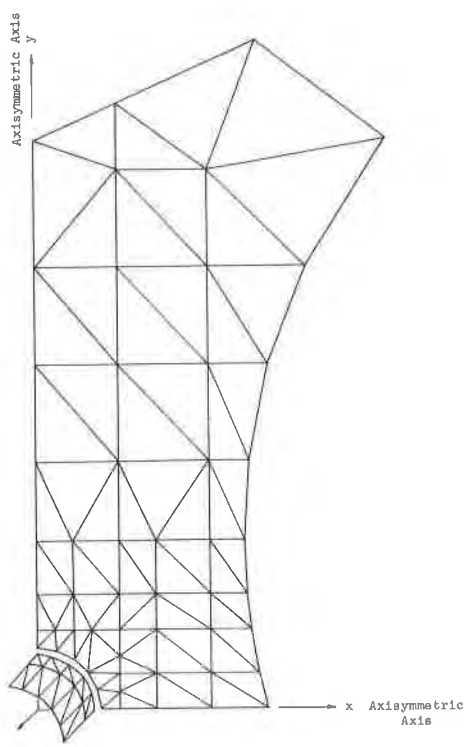

Fig. 13: Structrue designed to prevent core effluent from breaking the surface of the sodium pool