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A SIMPLIFIED METHOD OF INELASTIC ANALYSIS FOR THE DESIGN OF SODIUM COOLED FAST REACTOR VESSELS

A. PELLISSIER-TANON

SENTA, Société d'Études Nucléaires et de Techniques Avancées,
(Groupe Creusot-Loire), F-92400 Courbevoie, France

SUMMARY

The analysis of the loadings supported by the structural elements of sodium cooled FNR primary circuit shows that progressive deformation is a major risk of failure.

To fill up the gap which appears between the numerical methods of inelastic analysis, which are accurate but costly, and simple code rules such as the rule 13 of the ASME code case 1331-5, which is very conservative, there is an urgent need for simplified methods of inelastic analysis, allowing to make sure that the accumulated deformations do not exceed some specified limit, although avoiding the explicit calculation to these deformations.

This paper presents the basis of such a method which has been set forth for the design analysis of the vessels of reactors of the integrated type such as PHENIX.

The calculation model is founded on the theory of thin shells, and assumes that the behaviour of a shell element is elastic-perfectly-plastic under the rather fast loading conditions associated with the start up and shut down transient periods of the reactor, and that visco-plastic creep may occur during the operating periods.

The analysis is performed in two stages: in the first one Melan's theorem is used to establish that shakedown occurs during the transient periods. In the second one, it must be checked that the point which represents the loading state of any shell element during the operating period, after shakedown, is located inside a boundary of limit loading for creep.

To obtain that the accumulated deformation at each cycle remains smaller than a specified limit $\delta\varepsilon$, the two conventional yield stresses of the model, σ_E^P for instantaneous plasticity and σ_E^f for the creep limit, are defined in the following way: in principle σ_E^P is the stress which corresponds to $\delta\varepsilon$ in a cyclic stress-strain relation appropriate to the transient loading conditions, and σ_E^f is the stress which corresponds to $\delta\varepsilon$ on an isochronous creep stress-strain relation which should take into account the previous cyclic hardening of the metal. In practice, lower values for σ_E^P and σ_E^f may be obtained from conventional test.

In addition, it appears that the method imposes bounds to the cyclic deformation which are low enough to prevent failure by fatigue.

The application of that method to the vessels of the reactor PHENIX has led to loading limits which are definitely higher than those which would have been imposed by the use of the rule 13 of the code case 1331-5.

1. INTRODUCTION

The purpose of simplified methods of inelastic analysis for the design of nuclear reactors operating at high temperature is clearly expressed in the ASME Code Case 1331-7 : "... Elastic and simplified methods of analysis can sometimes be justified and used to establish conservative bounds for deformation and to reduce the number of locations in a structure requiring detailed inelastic analysis".

For cylindrical structures of the LMFBR reactors, the major risk of distorsion comes from progressive deformation by thermal ratcheting, as the mechanical loads are low, and as high thermal stresses appear during the transient start up and shut down periods.

The only simplified rule now available to prevent progressive deformation is rule 13 of the Code Case 1331-7 : $(P_L + P_b)_{max} + Q_{range} < Sq$. In the case of the vessels a major part of the cyclic thermal stresses are membrane stresses, which the Code Case considers as primary stresses. These membrane stresses appear then as P_L in the first parenthesis of the left side of rule 13 ; by comparison with the results published by BREE (1) and MILLER (2) this term should be physically understood as expressing that part of the loading which is kept fixed during the cycle. Clearly then, the rules of elastic analysis of the Code Case 1331-7 do not apply to the particular loading conditions of the vessels.

This prompted us to look for a simplified method of inelastic analysis, taking into account the main characteristics of the structural behaviour of the vessels subjected to given loads.

2. THE PRINCIPLE OF THE METHOD

2.1 Description of the vessels and of the loading conditions

The description of the structure of the PHENIX reactor block has already been published (3).

For the analysis, each vessel may be looked at as a cylinder suspended at its upper end, and bearing a mechanical load representing the weight at its lower end (Fig. 1a).

The distribution of temperature in the vessels is assumed to be axisymetric. The mean temperature in the wall during normal operating conditions and at a characteristic moment during the start up and shut down transient periods, is shown in Fig. 1b. The corresponding resulting forces and moments are shown in Fig. 1c.

Typical variations of the mean temperature and of the resulting forces and moments in an element of the vessel during one operating cycle are presented in Fig. 2.

2.2 The behaviour of a tensile specimen under loading conditions similar to those of an element of the vessel

The cyclic tensile test under fixed displacement conditions seems to give a good representation of the behaviour of an element of the vessel subjected to the loading conditions represented in figures 1 and 2.

Some experiments which have been carried out at a constant temperature on the A.I.S.I. 316 L stainless steel used for the vessels reveal the process of stabilisation of the stress-strain loop depicted in Fig. 3 : the stabilisation occurs practically after a few cycles and is maintained during a large number of cycles, until cracking begins. The results concerning stabilized loops obtained for different values of the imposed deformation can be brought together in the cyclic stress-strain curve represented in Fig. 4. This behaviour is described by MANSON (4), who proposed using the cyclic stress-strain curve in conjunction with a kinematical hardening model to calculate the deformation of structures under repeated loading.

The behaviour corresponding to a complete operating cycle may be represented to a certain extent by a test in which each cycle of varying deformation at a given temperature T_1 is interrupted by a period of fixed total deformation at zero stress and at a temperature T_2. This period causes a softening of the material which can be evaluated by comparing the cyclic stress-strain curve obtained with the loading pattern of Fig. 5b with the one obtained with the loading pattern of Fig. 5a. The actual working cycle is represented in Fig. 5c ; a relaxation of the stresses by creep during the operating period should also be taken into account. As a result of these effects, a stabilisation of the stress-strain loop may be assumed, as represented in the figure, but the loop width should be larger than the one of Fig. 5a, for the same value of the strain imposed.

The relaxation should theoretically be measured by means of tests performed according to the principle of Fig. 5c. However, upper bound values for the creep deformation under specified conditions of temperature and for a given length of the operating period can be obtained by replacing the period at fixed strain of Fig. 5c by a period at an appropriate fixed stress. Moreover there are experimental indications that the cyclic strain hardening should not increase the creep. So upper bound values could be obtained from conventional creep tests at constant stress.

The part that creep deformation might play with respect to plastic deformation in the progressive deformation process can be evaluated by comparing a tensile test to an isochronous stress-strain creep curve corresponding to the duration of the operating period. Fig. 6 presents the curves obtained with the A.I.S.I. 316 L steel used in PHENIX at 500ºC and 560ºC for 5.10^3 hours, which are the temperature upper limits in the different vessels. It appears that the creep should have no significant effects during the transient periods which are shorter and at a lower temperature, and could eventually have a minor effect during the operating periods.

The experiments from which the curves of Fig. 6 were obtained will be described elsewhere (5).

2.3 The simplified method of inelastic analysis

The behaviour of the structure under the conditions described in the preceding paragraphs will be, generally speaking, the following :

Under the cyclic loading associated with the transient periods one may expect a build-up of residual stresses in the vessels, leading to shakedown under favorable circumstances.

A more or less significant creep, which may occur only during the operating period, can modify these residual stresses. This would involve cyclic deformation due to the interaction of plastic strain and creep, as illustrated by Fig. 5c.

Now, if under the given loading condition shakedown occurs, and if at the same time there is no significant creep, the structure will be safe with respect to progressive deformation and low cycle fatigue.

The simplified method proposed consists of checking separately that each of these two conditions is satisfied. So the structure will be considered as safe if it can be demonstrated that :

1 - Shakedown occurs for the loading conditions of the transient periods alone. (The start up and the shut down).

2 - No significant creep occurs during the operating period.

The shakedown analysis has to be carried out by using MELAN's theorem which can generally be applied in two different ways. On the one hand the structure is considered as tridimensional, and the yield limit of the material must not be exceeded at any point after shakedown. On the other hand the shell element can be considered as the basic structural element. We have adopted the second point of view as presented at the beginning of the next paragraph.

In any case the method involves the use of material characteristics, which are obtained by uniaxial tests according to a specific program :

The yield stress is taken as equal to the proportional limit of the cyclic stress-strain curve obtained by the experiment of Fig. 5b, where the temperature T_1 is the mean of the temperatures corresponding to the two conditions of maximum loading during the transients, and the temperature T_2 is the operating temperature.

The creep analysis is carried out by checking that the state of stress (including the residual stresses) in any part of the vessel during the operating period is located inside a boundary based on the proportional limit of an isochronous creep curve, obtained in such a way as to give upper bound values for the creep deformation, as discussed in paragraph 2.2.

2.4 Shakedown analysis of the vessels according to MELAN's theorem

MELAN's theorem states that a structure will shakedown if any admissible distribution of residual stresses can be found which, when taken together with the stresses due to the actual loading cycle (assuming perfectly elastic behaviour), constitutes a system of stresses within the yield limit at any moment and anywhere within the structure.

To apply this theorem within the frame of the theory of shells, we have considered that a shell element has an elastic behaviour up to the yield limit.

In the elastic domain the KIRCHOF-LOVE hypothesis is assumed.

The flow condition for shells of perfectly plastic TRESCA material is adopted as described e. g. by HODGE (6) or SAVE and MASSONNET (7), and as represented in Fig. 7a. For the case of axial symmetry assumed here, the only resultant forces which need to be taken into account are N_x, N_θ, M_x (see : Fig. 1a).

The use of this criterion implies the neglect of the elastoplastic deformations of the shell element subjected to bending, which can eventually cause progressive deformation as illustrated by the work of BREE (1). But the choice of the proportional limit of the cyclic stress-strain curve as the yield limit for the material minimizes this danger, due to the benefit of the strain hardening characteristics of the cyclic stress-strain curve.

A complementary study of this point will be made later. It will discuss the practical interest of the safe but conservative criterion proposed by LECKIE and PENNY (8), who have in fact considered as the flow condition the limit of the proportional behaviour of shell elements of an elastic-plastic material ; this condition is represented in Fig. 7b.

The admissible residual stresses must obey the equations of equilibrium of a cylindric shell under axialy symmetric loading :

$$\left\{ \begin{array}{ll} dN_x = 0 & (1) \\ \dfrac{d^2 M_x}{d_x 2} = - \dfrac{N_\theta}{r} & (2) \end{array} \right.$$

The first equation leads to $N_x = 0$ for free end conditions. The second one can be used to find admissible stress fields from a parametric analytical representation of one of the functions $N_\theta(x)$ or $M_x\ (x)$, the other one then being deduced. The search for the best values of the parameters to insure shakedown may be carried out by trial and error methods, either by hand or by numerical programming (8).

3. APPLICATION OF THE METHOD TO PHENIX VESSELS (PRELIMINARY RESULTS)

3.1 The choice of values for σ_ε^p and σ_ε^f

The data from the conventional stress-strain curves and the isochronous curves illustrated by Fig. 6 were used, because the results of experiments under cyclic loading mentioned in paragraph 2.3 were not yet available. Besides, there are indications that the 316 L steel exhibits strain hardening under cyclic loading, and that this strain hardening lessens creep.

σ_ε^p and σ_ε^f have been chosen as the stresses corresponding to an inelastic deformations $\delta\varepsilon = 10^{-4}$, which is approximately the degree of precision that can be obtained by these measurements. Arguments can be given showing that if a variation of an appropriately defined equivalent strain of 10^{-4} occured at each cycle, the maximum value of the cumulated principal deformations after a specified number of cycles, say $N \sim 300$, would not be larger than 1 %. However, it seems useless to develop these considerations here because of the uncertainties of the stress-strain curves.

3.2 Shakedown analysis

In the case of the PHENIX vessels, the values of N_x is nearly constant during one cycle ; at the same time we have the residual force $N_x = 0$. So the shakedown analysis involves only the N_θ and M_x components. This allows us to use as a yield criterion the contour which is the cross-section of the yield surface of Fig. 7a corresponding to the actual value of N_x in the element considered. Such cross sections have been represented on Fig. 7a and 7b.

The shakedown analysis is carried out as follows :

If by appropriate translations the loading paths for all elements can be inscribed in the yield contour, and if these local translations can be fitted into a self equilibrated system, shakedown occurs. If the loading path for any element is too large to be inscribed, shakedown is impossible.

Some typical situations are illustrated in Fig. 8a, which represents the loading paths and yield contours in several elements in the PHENIX vessels obtained by means of the plotter of the computer, and in Fig. 8b, which represents the loading paths inscribed in the yield contours after the corresponding translations. The dots on the loading paths represent the operating period.

The proof that the local translations form a self-equilibrated system is not given here. However, it is possible to show that such a system can be found using the parametric expressions of admissible stress fields mentioned in paragraph 2.4.

3.3 Checking for creep

The checking for creep has been carried out in $N_\theta - M_x$ coordinates by comparing the point representing the operating condition on the loading path after translation with a yield contour similar to the one used for shakedown, but based on the value of σ_E^f . As the real residual stress field is not known, the estimate of the residual stress state in each shell element can be given by placing the loading curve tangent to the plastic yield contour in the worst possible situation for creep, as represented in Fig. 8c.

4. CONCLUSIONS

This method of simplified inelastic analysis results from an attempt to find a simple but rational approach to the analysis of the vessels, involving both a model of the structural response of the vessel and a program of experiments to describe the material behaviour.

The program of experiments is at the moment under way, and its results should allow us to confirm the basis of the method and to apply it more rigourously.

It is felt that such a simplified method may be more reliable than a sophisticated finite-element elasto-visco-plastic analysis when the complex material characteristics that it requires to describe accurately the behaviour of the material during the first loading cycles are not available. However, the simple experimental program which we have described may also be considered as a first step towards such an accurate description of the material behaviour.

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6. REFERENCES

[1] BREE, J., "Elastic Plastic Behaviour of thin tubes subjected to internal pressure and intermitent high-heat fluxes with application to fast - nuclear - reactor fuel elements", *Journal Strain Analysis*, *2*, 3, 226-238, 1967.

[2] MILLER, D. R., "Thermal-Stress Ratchet Mechanism in Pressure Vessels", *Journal Basic Eng.*, 190-196, 1959.

[3] "The World's Reactor nº 53 PHENIX", *Nuclear Engineering International*, *16*, 182, 1971.

[4] MANSON, S.S., *Thermal Stress and Low-Cycle Fatigue*, Mc Graw Hill, 1966.

[5] BOULISSET, R., DOLLET, J., "Etude des déformations plastiques des aciers soudables à l'entrée dans le domaine du fluage", à paraître dans *Mécanique Matériaux*, 1973.

[6] HODGE, P. G., *Limit Analysis of Rotationally Symmetric Plates and Shells*, Prentice Hall, 1963.

[7] SAVE, M. A., MASSONNET, C.E., *Plastic Analysis and design of plates shells and disks*, North Holland, 1972.

[8] LECKIE, F. A., PENNY, R. K., "Shakedown loads for radial nozzles in spherical pressure vessels", *Int. J. Solids Structures*, 3, 743-755, 1967.

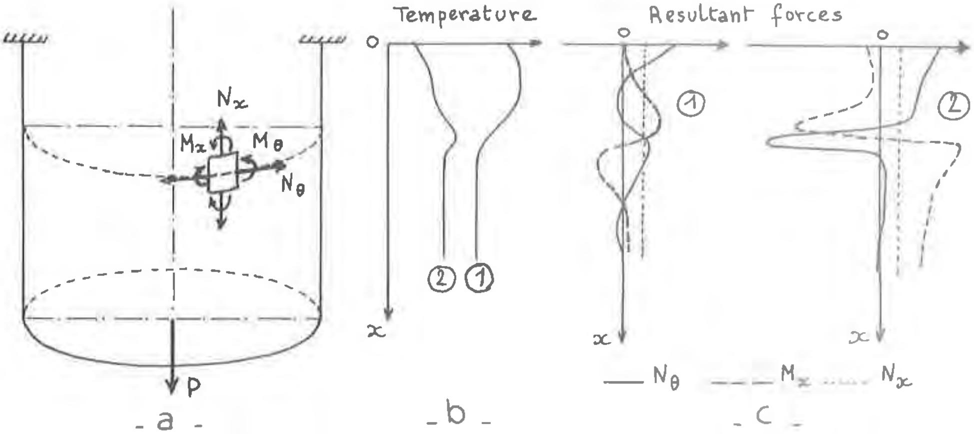

Fig. 1. The loading conditions of the vessels :

a – Model of a vessel.

b – Typical distribution of the mean wall temperature along the vessel, 1 at operating conditions, 2 at a characteristic moment during the start up period.

c – Typical distribution of the resultant forces and moments along the vessel for conditions 1 and 2 .

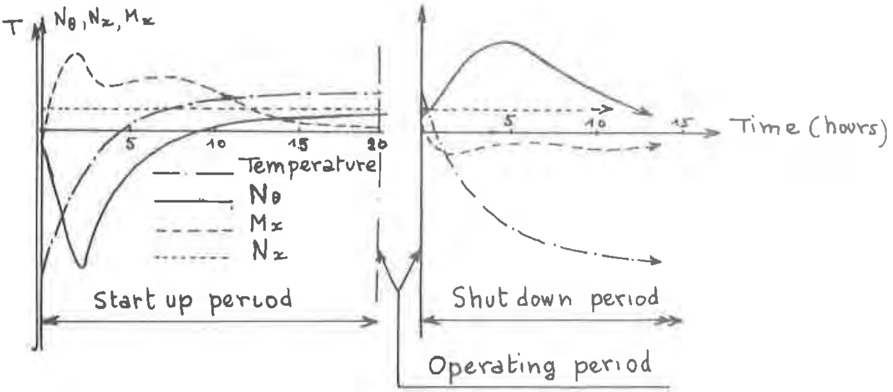

Fig. 2. An example of temperature variation and the resulting forces and moments in a vessel element during one complete operating cycle.

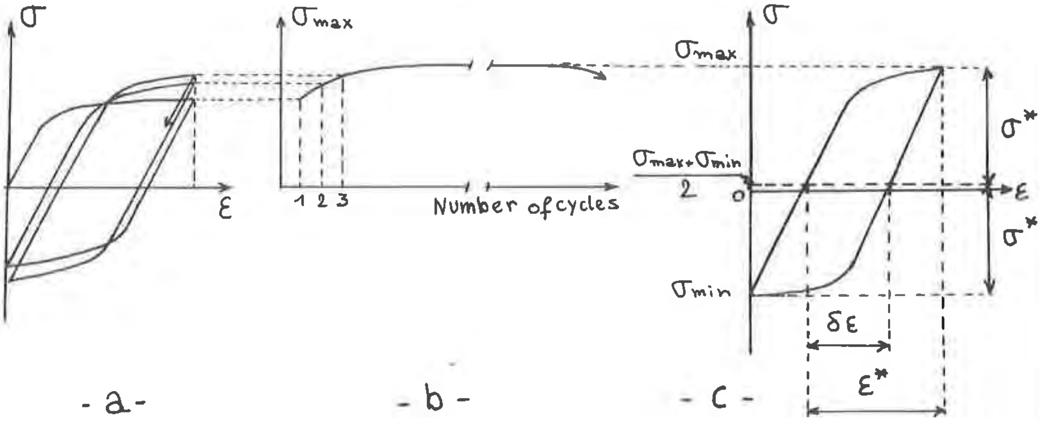

Fig. 3. The behaviour of a tensile specimen of AISI 316 L steel under cyclically variable imposed strain :

a - The evolution of the stress-strain loop during the first cycles.

b - The stabilisation of the stress-strain loop revealed by the stabilisation of the value of σmax.

c - The stabilized stress-strain loop (corresponding to the plateau of fig. b).

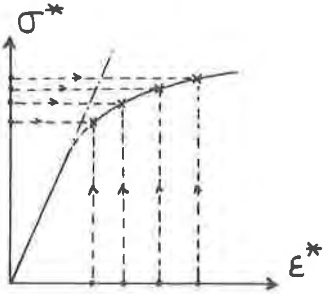

Fig. 4. The cyclic stress-strain relation obtained from a series of the experiments of fig. 3, carried out for different values of the imposed strain.

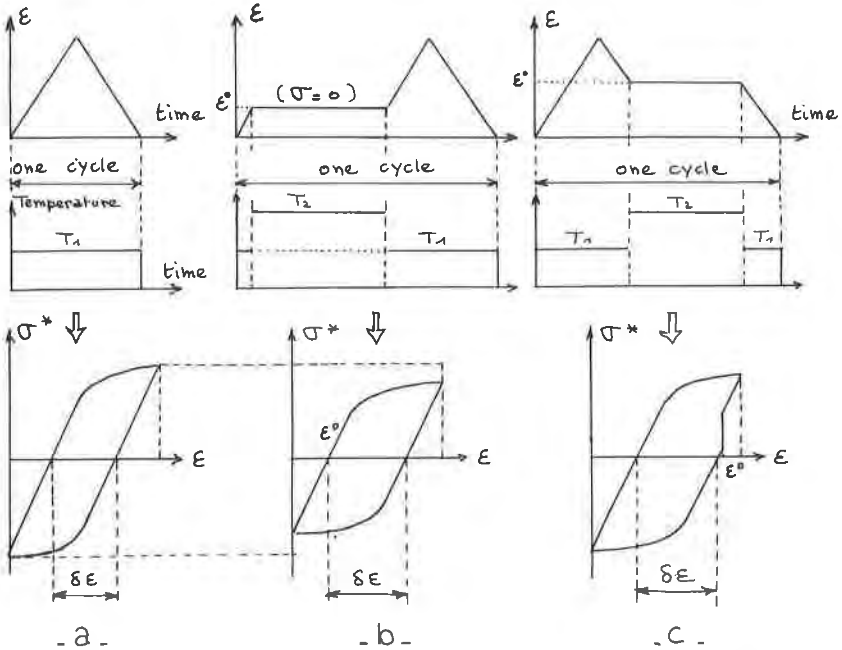

Fig. 5. The behaviour of a tensile specimen under a cyclically variable imposed deformation :

a - Behaviour during one cycle without hold period.

b - Behaviour during one cycle with a hold period at zero stress.

c - Behaviour during one cycle with a hold period at fixed deformation.

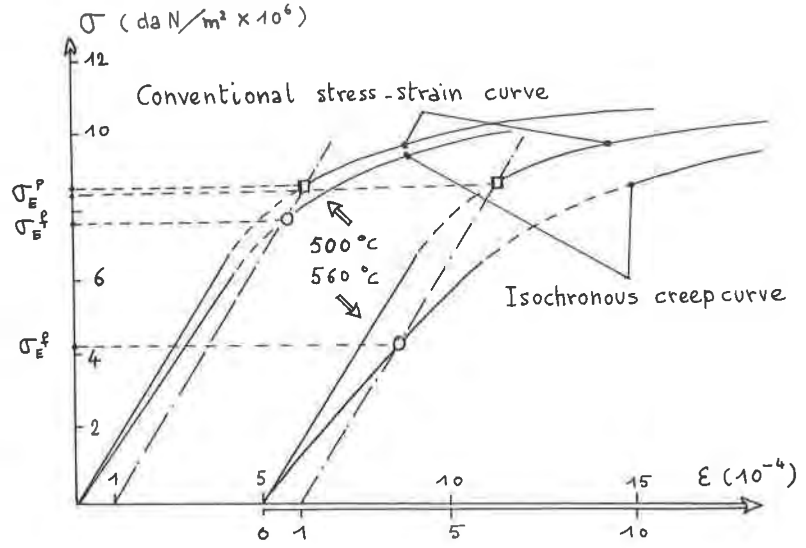

Fig. 6. Conventional and isochronous creep stress-strain curves for the AISI 316 L steel of PHENIX at 500 and 560ºC.

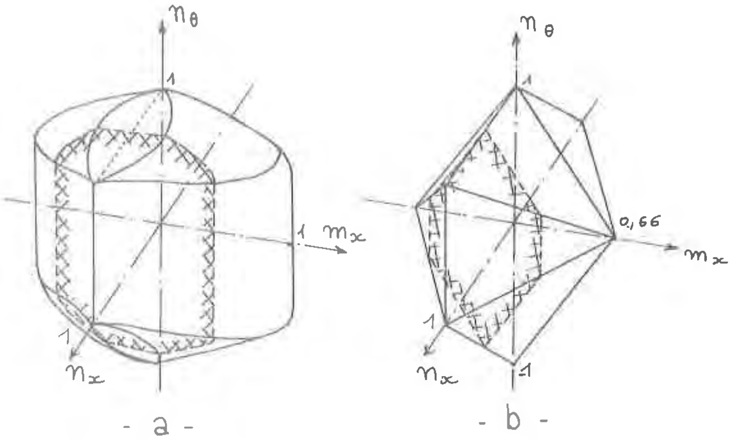

Fig. 7. The yield surfaces in N_θ, N_x, M_x coordinates (7) :

a - For the perfectly plastic flow of a shell element.

b - For the limit of proportional behaviour of a shell element

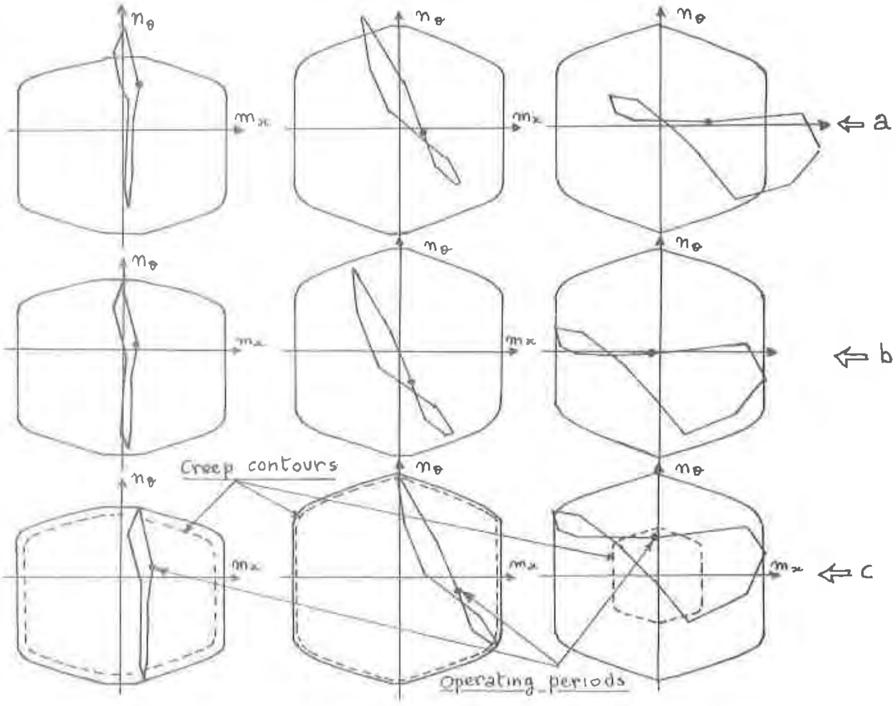

Fig. 8. Typical results of the application of the method to the PHENIX vessels:

a - The yield contour and the loading path as given by the computer.

b - The loading path inscribed in the yield contour.

c - Checking for creep.