LIMIT IN-PLANE COUPLES OF NOZZLES IN CYLINDRICAL VESSELS AND BRANCH CONNECTIONS IN PIPING SYSTEMS

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SUMMARY

Nozzles in pressure vessels or branch pipes in piping systems are generally loaded by external forces as well as internal pressure. However, the magnitude of the external loadings cannot normally be determined. To evaluate the critical case, it is necessary to determine the limit loads due to each of the external loads, and their combination with the internal pressure.

This paper considers the case of a branch pipe or nozzle subject to an in-plane couple. The magnitude of this couple which will cause "collapse" as defined by the limit analysis will be sought. This limit will be approached from below by using the lower bound technique. Using the limit couple as an objective function, a lower bound is found by maximizing the couple over a "statically consistent stress field" subject to inequality constraints imposed by the material yield condition. The results are valid for all diameter ratios, applicable to both nozzles in cylindrical vessels and branch pipes in piping systems.

To the best of the author's knowledge, no safe (lower bound) solution for the title problem exists at the moment. It is believed that the solution presented here is a pre-requisite in an over-all evaluation of the critical cases due to external loadings.

The section III of the ASME Boiler and Pressure Vessel Code for Nuclear Vessels specifies that the limits on local membrane stress intensity and primary membrane plus primary bending stress intensity need not be satisfied, if it can be shown that the specified loadings do not exceed 2/3 of the lower bound limit load. In accord with the preceding paragraph, the need for a lower bound analysis becomes not only desirable but in a practical sense necessary.

1. INTRODUCTION

This paper is concerned with finding the limit in-plane couple applied to the branch (or nozzle) of a branch-pipe tee connection (or nozzle-vessel attachment). This limit will be approached from below by using the lower bound technique which has been employed in a number of previous papers [1,2,3,4].

The method of solution, except for some details, is similar to that of refs [3,4], and the reader should consult these papers for supplementary information. The common relations will not be repeated in this paper, and only the differences will be emphasized. However, the paper is self-contained in the sense that those interested in the outline of method of solution and results should not encounter any difficulties by reading only this paper.

2. GENERAL RELATIONS

The geometry of the normal intersection of two circular cylinders, the differential equations of equilibrium, the continuity of tractions and couples at the intersection are all independent of the applied loads. They are given in [3,4], and need not be repeated here. The boundary and symmetry conditions however, are functions of the applied loads, and will be elaborated herein.

2.1 Boundary Conditions

Figure 1 shows an in-plane couple applied to the branch (or nozzle) extremity. The boundary conditions in the run pipe (or vessel) will be taken in analogy with a fixed end beam subject to an in-plane bending moment (fig. 1). The applied couple at the end of the nozzle is balanced by in-plane couples and shear reactions at the run pipe (or vessel) extremities. It will be assumed that the couples are equivalent to a sinusoidally distributed membrane force, i.e., for the nozzle:

$$C_1 = \int_0^{2\pi} N \cos\theta \; . \; r \cos\theta \; . \; r \, d\theta, \tag{1}$$

or

$$N_z = \frac{C_1}{\pi r^2} \cos\theta,$$

and in non-dimensional form:

$$\text{for } z = 1, \quad n_z = c_1 \cos\theta, \tag{2}$$

where $c_1 = C_1/\pi r^2 N_{on}$, and N_{on} is the maximum plastic membrane strength of the branch pipe or nozzle cross section. All the other stress components in the branch extremity are equal to zero. The positive direction of stress resultants and stress couples acting on shell elements of branch pipe (or nozzle) and run pipe (or vessel) are shown in fig. 2.

Similarly, for the run pipe (or vessel), the boundary conditions are:

$$\text{for } y = \pm 1,\ n_y = -\frac{c_1}{4}\frac{\rho^3}{\eta\,\delta}\cos\phi,\ n_{y\phi} = \frac{3c_1}{4}\frac{\alpha\,\rho^3}{\eta\,\delta}\sin\phi, \qquad (3)$$

where $\rho = d/D$ is diameter ratio, $\eta = \sigma_{ov}/\sigma_{on}$ is yield stress ratio, $\delta = \rho T/t$ is a representative geometric parameter, and $\alpha = D/2L_1$, is the diameter-to-length ratio of run pipe (vessel). The other stress resultants are equal to zero. The last of eqs (3) is due to the in-plane shear distribution equivalent to the end reaction (fig. 1).

2.2 *Symmetry Conditions*

The longitudinal and transversal planes are geometric planes of symmetry. However, when considering the distribution of the prescribed tractions, the symmetry is maintained with respect to the longitudinal plane only. As a consequence of symmetry, the following conditions must be imposed on the shear forces and twisting couples:

On the branch pipe (or nozzle) for

$$\theta = 0 \text{ (and} \pi), \qquad m_{z\theta} = n_{z\theta} = q_\theta = 0. \qquad (4)$$

On the run pipe (or vessel) for

$$\phi = 0 \text{ (and} \pi), \qquad m_{y\phi} = n_{y\phi} = q_\phi = 0. \qquad (5)$$

3. STATICALLY CONSISTENT STRESS FIELD

Two sets of statically consistent fields will be constructed, one for the branch pipe (nozzle) and the other for the run pipe (vessel). The requirements for such fields are that they must satisfy the conditions of internal and external equilibrium of forces and couples, the symmetry conditions, the continuity of tractions and couples at the intersection, and any other continuity imposed by the equilibrium equations.

Let the two sets of stress fields satisfying all the prescribed conditions be chosen in the following manner:

For the run pipe (or vessel)

$$n_y = \frac{1}{5\alpha} k_1^5 X_2 \cos\phi - \lambda X_1 \cos\phi - \frac{3}{2\pi}\lambda X_1 \cos\phi \sin\omega + f_1$$

$$n_\phi = -y k_1 X_3 \cos\phi + \alpha k_1 (k_1^2 - 3y^2) X_2 \cos\phi - 6\pi\alpha^2\lambda X_1 \cos\phi \sin\omega$$

$$n_{y\phi} = y k_1^3 X_2 \sin\phi + 3\alpha\lambda X_1 \sin\phi \cos\omega$$

$$m_y = \frac{1}{5\alpha} k_1^5 X_4 \cos\phi + \frac{1}{h_1\alpha} y f_4 \qquad (6)$$

$$m_\phi = -\alpha k_1 (k_1^2 - 3y^2)(X_4 - \frac{1}{h_1} X_2)\cos\phi - \frac{6\pi}{h_1}\alpha^2\lambda X_1 \cos\phi \sin\omega + \frac{1}{h_1} y k_1 X_3 \cos\phi$$

$$m_{y\phi} = - y\, k_1^3\, X_4 \sin\phi + \frac{1}{h_1}\,(y\, k_1^3\, X_2 \sin\phi + 3\alpha\,\lambda\, X_1 \sin\phi\cos\omega - 3\alpha\,\lambda\, X_1 \sin\phi)$$

$$q_y = -(y\, k_1^3\, X_2 + 3\alpha\,\lambda\, X_1 \cos\omega - 3\alpha\,\lambda\, X_1)\cos\phi + f_4$$

$$q_\phi = y\, k_1\, X_3 \sin\phi$$

where $k_1 = (1 - y^2)^{\frac{1}{2}}$, $\lambda = \rho^3/4\delta\eta$, $\omega = 2\pi y$, and the expressions for f_i's $(i = 1,4)$ are given in Appendix 1. The above expressions for the stress field in run pipe contain arbitrary parameters X_i as yet undetermined.

For the branch pipe (or nozzle)

$$n_z = -\frac{1}{3\beta}\, k_2^3\, X_5 \cos\theta + X_1 \cos\theta$$

$$n_\theta = (2\beta\, X_5 - X_6)\, k_2 \cos\theta$$

$$n_{z\theta} = k_2^2\, X_5 \sin\theta$$

$$m_z = -\frac{1}{3\beta}\,(X_7 + \frac{1}{h_2}\, X_5)\, k_2^3 \cos\theta + \frac{k_2}{\beta\, h_2}\, f_3 + f_2 \qquad (7)$$

$$m_\theta = (\frac{1}{h_2}\, X_6 - 2\beta\, X_7)\, k_2 \cos\theta$$

$$m_{z\theta} = - k_2^2\, X_7 \sin\theta$$

$$q_z = - k_2^2\, X_5 \cos\theta + f_3$$

$$q_\theta = k_2\, X_6 \sin\theta$$

where $k_2 = z - 1$, $\beta = r/(L_o+R)$, a characteristic length ratio of branch pipe or nozzle, and the appropriate expressions of f_i's $(i = 2,3)$ are given in Appendix 1.

These stress fields were found by assuming stress distributions in terms of spatial coordinates and arbitrary parameters for three resultant stress $n_{y\phi(\text{or } z\theta)}$, $m_{y\phi(\text{or } z\theta)}$, and $q_{\phi(\text{or } \theta)}$. The other stress resultants were then obtained through the solution of differential equations of equilibrium, and satisfaction of the appropriate boundary conditions.

If the f_i's are not included in the stress fields (6) and (7), and the stresses (without f_i's) are substituted in the stress continuity conditions of the intersection, then these conditions will not be identically satisfied. Therefore, f_i's can be envisaged as remaining residual values, which are required to satisfy identically the stress continuity conditions at the intersection. It should be emphasized that the expressions for the f_i's, given in Appendix 1, are evaluated at the intersection. However, for all other regions removed from the intersection, these values remain unchanged.

Suppose that the values of f_i's are not small when compared to the stress resultants n_y, $m_{y(\text{or } z)}$, $q_{y(\text{or } z)}$. Then one would risk violating the corresponding boundary conditions for these stresses, see eqs (2) and (3). However, the magnitudes of the residual values can be obtained only after the numerical procedure, described in the next section, has been implemented. The numerical results so-obtained indicate that for the wide range of geometric parameters considered, the magnitudes of f_i's were always less than 10^{-2}. Consequently, for all practical purposes, the boundary conditions can be considered as being satisfied. Nevertheless, if in certain cases it is assumed that these boundary conditions should be satisfied more accurately, a statically admissible stress field could be easily constructed near boundaries which satisfies static requirements and everywhere is below yield, i.e., a rigid region. An example may be found in ref. [5]. The inclusion of latter stress would not affect the limit couple reported hereinafter, since it is the region near the intersection which will govern.

4. LOWER BOUND APPROACH

The lower bound theorem of limit analysis states that any load associated with a statically admissible stress field is a lower bound to the limit load, and the maximum among these loads is the limit load itself. In order that the statically consistent fields (defined earlier) be admissible, they should nowhere violate a prescribed yield condition. Thus, with this approach, the lower bound to limit in-plane couple may be formulated as:

$$\text{Maximize } X_1 \quad (X_1 \equiv c_1) \tag{8}$$

subject to

$$R_j \, (X_i) \geq 0 \qquad j = 1, 2$$
$$i = 1, \ldots, 7$$

where

$$R_1 \, (X_i) = f_R \geq 0 \qquad \text{for run pipe (or vessel)}$$

$$R_2 \, (X_i) = f_R \geq 0 \qquad \text{for branch pipe (or nozzle)}$$

and f_R is a specified yield condition in terms of stress resultants. $R_1 \, (X_i)$ and $R_2 \, (X_i)$ are obtained by substituting stress fields (6) and (7) in the specified yield surface, respectively. For the sake of consistency and comparison, the same yield surface as described in refs [3,4] will be considered herein. The relation between this surface and a number of other yield surfaces is discussed in refs [2,3]. Note that the formulation is such that any other yield surface suitable for a designer's special purpose, could be taken without any change in the formulation presented herein.

A further restriction may be introduced, stating that the limit couple of a tee connection (or nozzle-cylindrical vessel attachment), should not exceed that of the branch pipe (or nozzle), i.e.,

$$R_3 \, (X_i) = 1.3 - X_1 \geq 0 \tag{9}$$

This inequality constraint is optional and by no means could alter the final solution. It may however, be useful in numerical studies where a possible convergence problem may arise.

The problem thus posed is a non-linear programming problem and the numerical method adopted for the solution is a modified version of SUMT (Sequential Unconstrained Minimization Technique). This method is by now well understood, and has been explained in refs [1,3,4] among others. However, for the purpose of implementing SUMT, it is necessary to replace the yield functions $R_1\ (X_i)$ and $R_2\ (X_i)$ with a finite number of discrete inequalities,

$$R_j\ (X_i) \equiv R\ (X_i,\ \xi_k) \geq 0,\ k = 1,\dots,\ n \qquad (10)$$

where ξ denotes, symbolically, a generic point on the branch or run pipe.

The program developed for this problem, automatically generates a mesh, and constraints (10) are enforced at mesh points. For the purpose of parametric study reported in the following section, the yield conditions were enforced at about 240 points.

5. RESULTS AND DISCUSSION

For a complete parametric study one has to vary six non-dimensional variables. Three parameters are associated with the diameter and thickness of the components: D/T (diameter-to-thickness ratio of the run pipe or vessel), $\rho = d/D$ (branch-to-run pipe diameter ratio), $\delta = \rho\ T/t$, a parameter frequently used by designers, and associated with the nominal hoop stress ratio in the case of pressure loading. For a fixed diameter ratio, the latter variable is proportional to thickness ratio of run (vessel) to branch pipe (nozzle). Two other parameters are related to the length of the components, i.e., $\alpha = D/2L_1$ = diameter-to-length ratio of run pipe, and $\mu = L_o/L_1$ = branch pipe-to-half of run pipe length ratio (fig. 1). Finally, the sixth variable represents the difference in material properties of branch and run pipe. This parametric is denoted by $\eta = \sigma_{ov}/\sigma_{on}$ = yield stress ratio of run to branch pipe.

The limit in-plane couple varies with each of these parameters. Therefore, for the graphical representation, four parameters are kept constant, and variation of limit in-plane couple versus diameter ratio is plotted for different values of a specific variable. For example, fig. 3 shows variation of limit couple against diameter ratio for three values of D/T, and fixed values of δ, η, α and μ. It could be noticed that the influence of D/T is quite pronounced in the region of small diameter ratio. The minimum strength for $\delta = 1$ is obtained when the diameter ratio is bounded by $0.20 < d/D < 0.35$.

Figure 4 indicates the effect of run to branch pipe thickness ratio for a given diameter ratio. It can be noted that the limit couple increases with the increase of branch pipe wall thickness, and the increase in the limit couple of tee connections is greater than that of its branch component.

The effect of diameter-to-length ratio of run pipe on the limit couple is shown in fig. 5. It is noted that the predicted in-plane limit couple increases slightly with the increase of this ratio. However, the influence of this parameter is not of the same order as those of D/T and T/t.

6. CONCLUSIONS

Lower bounds to the in-plane couple of branch-pipe tee connection (or nozzle-cylindrical vessel attachment) are found through formulating the problem as a non-linear programming one. No simplifying assumptions are made either regarding geometry of the intersection or any other geometric or material variables except those inherent in limit analysis and thin shell theory. However, the analysis presented treats an idealized intersection, thus ignoring the effect of fillets and reinforcements around the junction. Although the method could be easily extended to take into account the effect of local reinforcements, in the context of limit analysis the solution presented herein is a lower bound for any cylinder-cylinder intersection [4].

7. ACKNOWLEDGEMENTS

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8. APPENDIX 1

The expressions for f_i functions contained in eqs (6) and (7).

$$f_1 = -\frac{1}{5\alpha} X_2 \, (1 - \rho^2 \cos^2\theta)^{5/2} \cos\phi + \lambda X_1 \cos\phi + \frac{3}{2\pi} X_1 \cos\phi \sin(2\pi\rho \cos\theta) +$$

$$n_\phi(\phi) + A_1 n_{y\phi}(\phi) - A_2 n_z(\theta) + A_2 n_\theta(\theta) + A_3 n_{z\theta}(\theta)$$

$$f_2 = \frac{1}{3\beta}\left(\frac{1}{\rho}\cos\phi - 1\right)^3 \left(X_7 + \frac{X_5}{h_2}\right)\cos\theta - \frac{1}{\beta h_2}\left(\frac{1}{\rho}\cos\phi - 1\right) f_3$$

$$+ A_4 m_\theta(\theta) + A_5 m_{z\theta}(\theta) - A_6 m_y(\phi) + A_6 m_\phi(\phi) + A_7 m_{y\phi}(\phi)$$

$$f_3 = \left(\frac{1}{\rho}\cos\phi - 1\right)^2 X_5 \cos\theta - A_8 n_z(\theta) - A_9 n_\theta(\theta) - A_{10} n_{z\theta}(\theta)$$

$$+ A_{11} n_\phi(\phi) + A_{12} n_{y\phi}(\phi) - A_{13} q_\phi(\phi)$$

$$f_4 = \left[\rho X_2 \, (1 - \rho^2 \cos^2\theta)^{3/2} \cos\theta + 3\alpha\lambda X_1 \cos(2\pi\rho\cos\theta) - 3\lambda\alpha X_1\right]\cos\phi +$$

$$A_{14} n_z(\theta) + A_{15} n_\theta(\theta) + A_{16} n_{z\theta}(\theta) - A_{17} n_\phi(\phi) - A_{18} n_{y\phi}(\phi)$$

$$+ A_{19} q_\theta(\theta) - A_{20} q_\phi(\phi)$$

where the expressions for $A_1, \ldots, A_{20}$ are identical to those given in ref. [4].

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9. NOTATION

C_i = in-plane couple applied to branch or nozzle (fig. 1)

$c_i = C_i/\pi r^2 N_{on}$ = non-dimensional in-plane couple

$\bar{c}_i$ = non-dimensional lower bound to limit in-plane couple

D,d = mid-surface diameter of run pipe (vessel) and branch pipe (nozzle), respectively

$h_1 = M_{ov}/RN_{ov}$, $h_2 = M_{on}/rN_{on}$

L_o = effective length of branch pipe or nozzle (fig. 1)

L_1 = half length of run pipe or vessel (fig. 1)

$M_{ov} = \frac{1}{4}\sigma_{ov}T^2$ = maximum plastic bending moment capacity of run pipe or vessel cross section

$M_{on} = \frac{1}{4}\sigma_{on}t^2$ = maximum plastic bending moment capacity of branch pipe or nozzle cross section

m_y, m_ϕ, $m_{y\phi} = M_Y/M_{ov}, \ldots$ = non-dimensional moments per unit length in run pipe or vessel

m_z, m_θ, $m_{z\theta} = M_Z/M_{on}, \ldots$ = non-dimensional moments per unit length in branch pipe or nozzle

$N_{ov} = \sigma_{ov}T$ = maximum membrane plastic strength of run pipe or vessel cross section

$N_{on} = \sigma_{on}t$ = maximum membrane plastic strength of branch pipe or nozzle cross section

n_y, n_ϕ, $n_{y\phi} = N_Y/N_{ov}, \ldots$ = non-dimensional membrane forces per unit length in run pipe or vessel

n_z, n_θ, $n_{z\theta} = N_Z/N_{on}, \ldots$ = non-dimensional membrane forces per unit length in branch pipe or nozzle

q_y, $q_\phi = Q_Y/N_{ov}, \ldots$ = non-dimensional transversal shear forces per unit length in run pipe or vessel

q_z, $q_\theta = Q_Z/N_{on}, \ldots$ = non-dimensional transversal shear forces per unit length in branch pipe or nozzle

R, r = mid-surface radius of run pipe (vessel) and branch pipe (nozzle), respectively

T, t = wall thickness of run pipe (vessel) and branch pipe (nozzle), respectively (fig. 1)

X, Y, Z = Cartesian coordinate system (fig. 2)

$y = Y/L_1$, $z = Z/L_o+R$, dimensionless coordinates

θ = cylindrical coordinate in branch pipe or nozzle (fig. 2)

σ_{on}, σ_{ov} = yield stress of branch pipe (nozzle) and run pipe (vessel) material, respectively

ϕ = cylindrical coordinate in run pipe or vessel (fig. 2)

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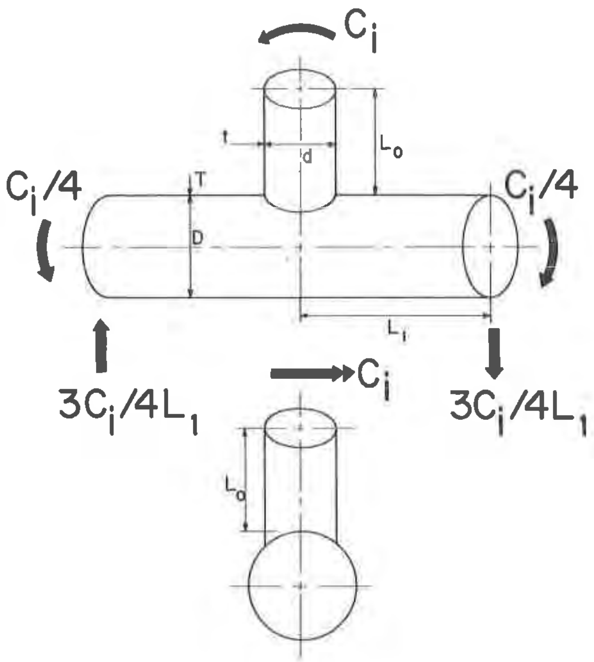

Fig. 1 A branch-pipe tee connection (or nozzle-cylindrical vessel attachment) subject to an in-plane couple, with the prescribed stress distribution at the extremities.

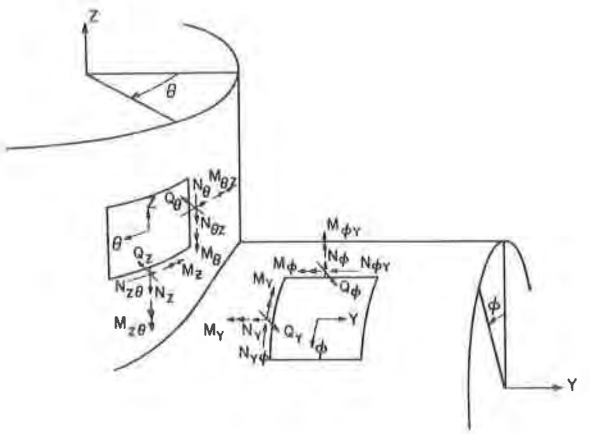

Fig. 2 Elements of cylindrical shells with notation and positive direction of forces acting on them.

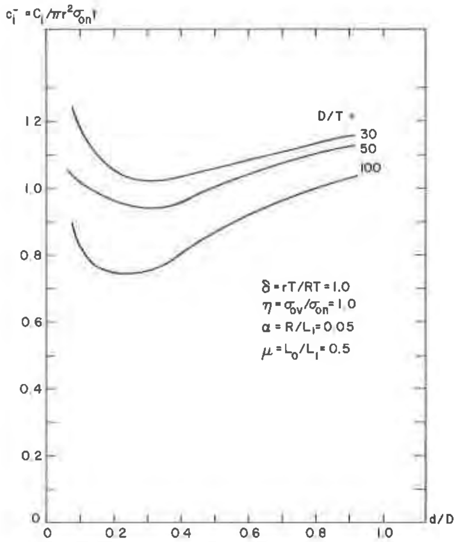

Fig. 3 Variation of lower bound to in-plane couple with diameter ratio for various thickness to diameter ratio of run pipe (or vessel).

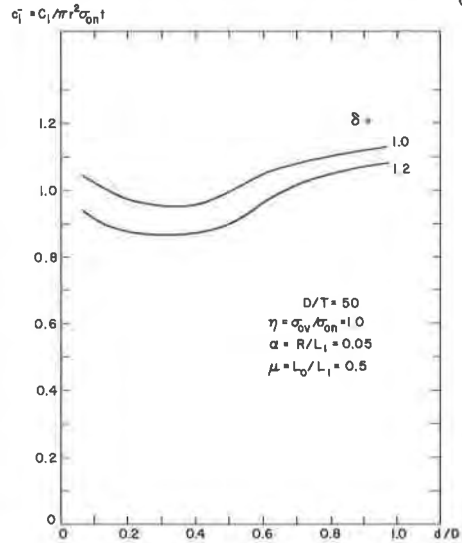

Fig. 4 The lower bound to limit couple versus diameter ratio for various thickness ratios.

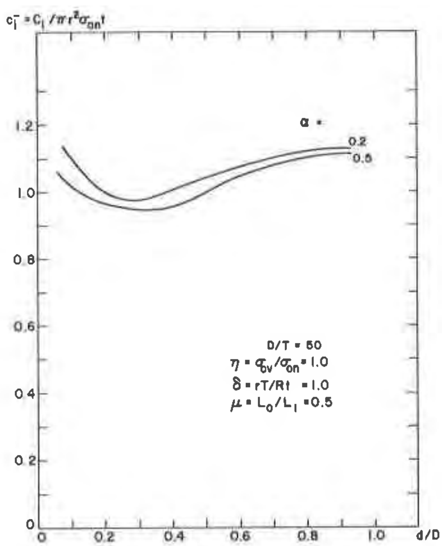

Fig. 5 Variation of lower bound to limit couple against diameter ratio for different diameter-to-length ratio of run pipe (or vessel).