

## EXPERIMENTAL AND THEORETICAL ANALYSIS IN ELASTIC-PLASTIC FRACTURE MECHANICS

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### SUMMARY

The brittle fracture safety of the housing of a main coolant pump in a reactor primary circuit (PWR) will be examined. The housing is almost spherical and constructed from three cast steel components welded together.

The inner surface is clad with Austenitic Stainless Steel. The main dimensions are (1) maximum diameter 2750 mm, (2) wall thickness up to 550 mm, (3) weight 32 metric tons. Although a stress relief heat treatment was applied, residual stress up to  $10 \text{ kp/mm}^2$  has been taken into consideration.

Except for the exit nozzle, the housing can be regarded as rotationally symmetric and this assumption was made to simplify the stress analysis. The Finite Element method was used to examine the three most severe loading conditions in the working cycle (1) start up, (2) running, (3) shut down, together with the proof test loading. Also, the static and transient temperature fields have been determined by analytical procedures. The results of these analyses were used to specify regions of maximum stress level.

A brittle fracture safety evaluation is particularly important in this case because a ferritic steel of the type GS 20 Mo 4 (CO·20, Mo 0·4), instead of the more usual austenitic, was used for the casting. Also, experience shows that cast material is more susceptible to defects than the same plate material.

A previously published "systems approach" used to provide a logical procedure for brittle fracture assessment indicates that impact testing should be considered first. However, quantitative data were necessary in this case, so that neither impact testing nor the "next step", drop weight testing were completely adequate although they provided useful quality control information.

Consequently, 4 inch compact tension specimens were tested in the probable "proof test" temperature range. Fractures did not occur within the specified limits for a linear elastic treatment and some critical defect sizes were defined on the basis of the non-valid  $K_Q$  concept. Crack opening displacements (COD) were also measured employing a newly developed technique on a modified CT specimen geometry. The critical defect sizes derived from the COD at fracture were found to be considerably larger than those calculated from  $K_Q$ .

The COD, together with Rice's contour integral  $J$ , were also determined from elastic plastic finite element calculations. Thus, experimentally and theoretically measured values are compared and discussed for the first time.

Experimental and Theoretical Analysis in Elastic-Plastic Fracture Mechanics.

1. Introduction

Safety against brittle fracture is assured if tensile stress concentrations exceeding the locally encountered yield point are sufficiently relaxed by plastic deformation. In the presence of sharp ended defects, it is necessary that critical stress intensity factors,  $K_{Ic}$ , or crack opening displacements,  $\delta_c$ , should not be exceeded. However, it is not always necessary to determine  $K_{Ic}$  or  $\delta_c$  and simpler tests may provide sufficient information. A logical procedure to determine the necessary amount of information has been developed by one of the authors, [Varga 1,2].

When quantitative defect evaluation is required, linear elastic fracture mechanics is the current method employed [3]. Linear elastic fracture mechanics techniques allow a realistic description when brittle fracture is predominant, but become more and more conservative as the amount of plastic straining increases. In the cases of pressure vessel steels and welds, considerable plasticity can exist at the lowest service temperatures so that linear elastic fracture mechanics imposes defect tolerances and controls which are too stringent. This may become critical, if not only membrane stresses but also stress concentrations (nozzles) and residual stresses are taken into consideration. So parallel to the theoretical and experimental treatment, improved defect size evaluation becomes necessary.

Elastic-plastic fracture mechanics, taking account of additional plasticity, should give the more precise information required for quantitative fracture safety evaluations. Practical significance may be attributed to this approach beyond the validity limits of linear elastic fracture mechanics, i.e. at the lower end of the temperature induced brittle-ductile transition range. An attempt to demonstrate this will be presented in the following.

For this purpose, the brittle fracture safety of the housing of a main coolant pump for a nuclear reactor is considered. The housing, illustrated in Fig. 1, is almost spherical and is constructed from welded cast steel components. Apart from the outlet section, the casing as well as the upper structure may be regarded as axisymmetric.

The inside of the casing and the upper part (forged ferritic steel) are clad with austenitic steel. The casing weighs about 32 tonnes, has a maximum diameter of 2750 mm and wall thickness ranging from 100 to 550 mm.

2. Stress Analysis

The pump housing is subjected to the following sources of load:

- 1) Bolting forces between the housing and upper structure
- 2) Internal pressure
- 3) Non-uniform temperature field.

The finite element method (assuming axial symmetry) was used to determine the stress fields for the proof test loading together with the three service conditions, start, running and stop. As would be anticipated, the most severe stresses are generated during the proof test and these are represented in Fig. 2.

An additional  $10 \text{ Kp/mm}^2$  was taken into account as a conservative estimate of residual stresses. The combined result served as a basis for a fracture mechanics evaluation. To simplify the calculation, the housing was divided into regions of maximum tensile stress (5,10,15,.....  $\text{Kp/mm}^2$ ).

### 3. Fracture Tests

The 'systems approach' for brittle fracture safety evaluation [1,2] indicates that impact testing (Charpy V) should be considered as a first step in the test procedure. However, due to wall thicknesses above 100mm and lack of similar operational experience with this particular cast steel, the next step in the test sequence was checked. These were limit value tests (e.g. Pellini drop weight testing) but were still not capable of providing sufficiently precise information on tolerable defect sizes. In consequence, quantitative procedures ( $K_{Ic}$  or COD testing) were necessary to determine the fracture resistance of the material.

Several 4T-CT specimens were tested over the temperature range  $-50^\circ\text{C}$  to  $80^\circ\text{C}$ . In addition, Charpy V and drop weight tests were made for quality control purposes.

### 4. Quantitative Evaluation Procedures

An interpretation of the fracture behaviour of sharply notched configurations in terms of linear elastic fracture mechanics is limited to loads under about 80 % of the plastic collapse load, with the single parameter  $K_{Ic}$  encompassing the important variables. This range has been extended somewhat arbitrarily with the  $K_Q$  concept but is usually conservative for the more commonly used materials. Beyond this limit it seems more realistic at present to use either Well's COD [5] or Rices J. integral [6] for a better representation. The non-linear behaviour associated with this range has made theoretical calibrations costly and inference of results from the usually measured parameters difficult.

#### 4.1 Fracture Mechanics Considerations

The Experimental program described herein on 4T-CT specimens

included displacement measurements at several locations along the crack flank. The gauges used were of the clip-in type [7] and their layout is shown in Fig. 3. With some uncertainty, the resulting crack profiles could be extrapolated towards the crack tip, the intercept there being a measure of COD [7]. This method probably gives a slight overestimate, especially when plastic straining is not extensive. The same configuration was examined theoretically using an elastic plastic finite element computer program, developed by one of the authors, [8]. The aim of this comparison was to establish calibrations between variables, easily measured experimentally, (clip gauge displacement  $V_g$  and external work  $U$ ) and the desired fracture parameters (COD,  $J$ ). The burden of elaborate instrumentation in future fracture test programs is then either considerably reduced or the interpretation procedures more satisfactorily defined. Also, the usefulness of the now highly developed elastic plastic stress analysis can be directly assessed.

As shown previously by Wells [9] and Boyle [8], the  $V_g$ -COD relationship is first parabolic followed by a linear response at higher COD's when hinging action dominates the overall deformation of the body. Nevertheless, it has been common practice [10] to relate  $V_g$  to COD [6] through a relationship of the form

$$\frac{\delta}{V_g} = \frac{r(w-a)}{r(w-a) + a + z} \quad (1)$$

Where 'r' is taken to be the rotation constant at plastic collapse and is approximately 0.45 for bend specimens, other quantities in equation (1) refer to the geometric dimensions of the test piece [4]. The merits of equation (1) will be examined in the next section and its serious limitations pointed out especially in the region within and somewhat beyond  $K_{Ic}$  testing.

A correlation between  $J$  or COD and external work  $U$  indicates that an even simpler interpretation technique may be possible. The connection between these parameters is apparently linear through all stages of deformation. Similar results were reported by Wells [11] and Boyle [8] for different configurations. However, further investigation will be necessary to establish this as a general rule.

#### 4.2 Comparison between Theoretical and Experimental Results from the Compact Tension Specimen.

The calculation capabilities of the elastic plastic finite element computer program used here are described elsewhere [8]. The element mesh (1052 elements) for the present computations is illustrated in Fig. 4 together with the 3 separate paths over which the  $J$  integral was evaluated. It may be noted that the percentage standard

deviation in J over these paths was always less than 1.40 %.

A comparison between the computed and experimental load (P) , displacement (Vg) responses is shown in Fig. 5. In the elastic range the agreement is good although the finite element result, as would be expected, shows less compliance. Specimen A5 (20°C) fractured with limited ductility, while A7 (50°C) was capable of absorbing extensive prefracture plastic straining, see Fig. 6. The fracture load for A7 was within 7 % of that predicted theoretically for the same Vg.

The agreement between the experimentally established and theoretically predicted crack profiles (at equal loads) is illustrated in Fig. 8.

Of greater importance in fracture testing is the value of 'r' in equation (1), which in the present investigation could be measured both experimentally and theoretically. Fig. 9 illustrates the comparison and both sets of results are consistent through all levels of deformation. The limitation of equation (1) with  $r = 0.45$  is readily apparent as the maximum value never exceeded 0.35 although the curve tends asymptotically towards  $r = 0.45$ . However, this value is not reached theoretically until Vg is infinite. In the valid  $K_{IC}$  range the error in COD is of the order of 200 - 400 %, decreasing as deformation progresses. Even in the more ductile A7 test, the error in COD at the fracture point would be 20 %. This may account for the wide scatter of test results often apparent in critical COD measurement especially in the lower toughness range.

The Vg - COD relationship using the present procedures is shown in Fig. 10 and the agreement between experiment and theory hardly needs further comment. Also demonstrated in Fig. 10 is the reliability of the Wells [12] relationship, although the constant  $\gamma$  in this equation is not correct for the standard C.T. geometry (In this paper  $\gamma = 3.69$  was used while  $\gamma = 2.45$  is suggested by Wells). However, this can easily be calculated for particular geometries when the elastic compliance is known.

Fig. 11 illustrates the correlation between COD, J and external work U, where a linear relationship results through all stages of deformation. The theoretically predicted linear relationship is confirmed by experiment although there is a 14 % difference in the constant connecting COD with U.

From Theory:  $\sigma_Y \delta = 1.050 U/A$

From Experiment:  $\sigma_Y \delta = 1.222 U/A$

Where A is the ligament area of the specimen.

Finally, COD's were measured after fracture, as illustrated in Fig. 12, utilising a scanning electron microscope. As yet this

technique for determining  $\delta_c$  has not been fully developed at the authors laboratories, but results obtained so far are encouraging and will be presented in a future publication when more results are available.

5. Determination of Critical Defect Sizes from COD or J.

Improvements in the procedures for interpreting fracture test results were discussed in the foregoing section. However, before these parameters can be used for evaluating the applicability of a material in a particular service environment, further comment is necessary.

Under service conditions, yield stresses are normally not exceeded across complete cross sections as may be the case in corresponding fracture test specimens, Fig. 5. In these cases, linear elastic fracture mechanics analysis should be appropriate to real structures, although the question remains, is the critical COD or J measured or inferred under general yielding conditions relevant to essentially elastic conditions? Also, it may be realistic in some service conditions to consider loading beyond the linear range before critical COD's would be exceeded. Detailed consideration is not given to this aspect in this report, but work is already in progress to establish non-linear calibrations for various practical geometries.

A recent test program reported by Begley and Landes [13] suggested that the critical value of J at initiation (measured as a potential energy rate) under extensive yielding conditions was equal to that measured under valid elastic conditions. Since then, it has been verified numerically for an incremental plasticity flow rule (Boyle and Wells [14]) that the potential energy rate is equal to the J integral provided the method of crack extension is carefully defined.

For the cases when the linear elastic fracture mechanics relationships are valid:

$$K_{Ic}^2 = n\sigma_y E' \delta_c \tag{2}$$

For any configuration, K is related to crack length as:

$$K_I = C\sigma\sqrt{\pi a/Q} \tag{3}$$

where C and Q are well tabulated constants for the case of a surface crack in a thick walled vessel. Equations (2) and (3) may be combined so that:

$$a_c = \frac{nE'\sigma_y\delta_c Q}{C^2\sigma^2\pi} \tag{4}$$

where the subscript 'c' denoted critical values. The value of 'n' may be taken as 2 for plane strain and 1 for plane stress.

This does not mean that longer defects may be tolerated in plane strain as the value of  $\delta_c$  will be considerably smaller for this case. The stress applied normal to the crack direction is  $\sigma$  and for design purposes will be the maximum tensile stress in the vessel.

When linear elastic procedures become invalidated by excessive yielding then it is necessary to make an elastic plastic stress analysis for the particular structure with the appropriate loading condition and defect geometry. In this way the COD/load response can be obtained and a critical defect size determined, such that  $\delta_{service} < \delta_c$ .

Alternatively a factor of safety may be defined as  $\frac{\delta_c}{\delta_{service}}$  or  $\frac{P_c}{P_{service}}$ . With this procedure, it is hoped that design charts may be established for typical practical configurations such as nozzles.

6. Comparison between critical defect sizes from  $K_Q$  and COD.

Critical defect sizes calculated on the basis of the  $K_Q$  and COD concepts are illustrated in Fig. 13. It is readily apparent that  $K_Q$  is unable to take advantage of the extra ductility available within and beyond the transition temperature range. In the present application, these are the temperatures of interest and tolerable defect sizes defined on the basis of  $K_Q$  are much too stringent.

7. Conclusions

Based on the results presented, the following conclusions may be drawn:

- 1 The critical defect size predicted from  $K_Q$  is much more conservative than the corresponding prediction from COD.
- 2 The theoretically established relationships from elastic plastic finite element calculations are confirmed experimentally.
- 3 A linear relationship appears to exist between external work done and COD or J for the geometry and material examined.

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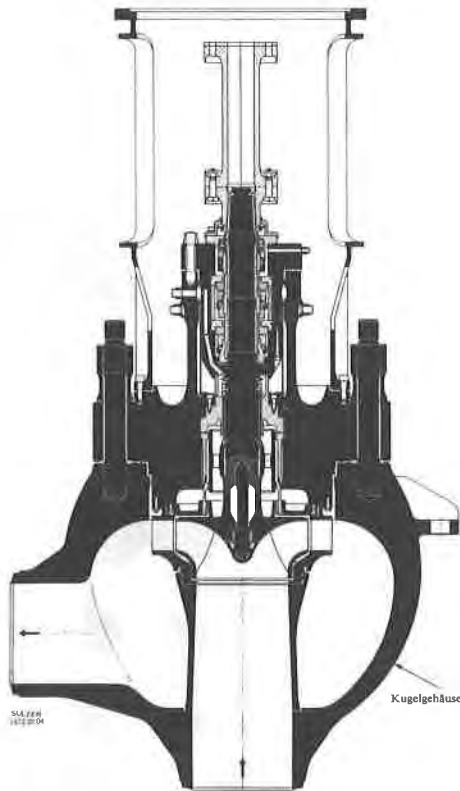


Fig. 1 Section of main reactor coolant pump. Fracture evaluation was made on the spherical casting (cast ferritic steel).

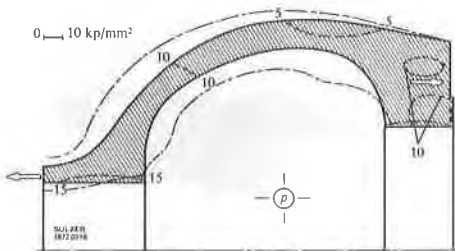


Fig. 2 Stress contours for the pump housing. - - - Lines of const. hoop stresses - - - Meridional stresses on surface

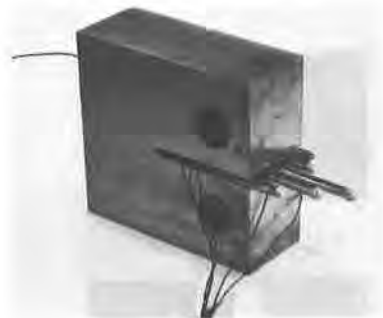


Fig. 3 Layout of crack profile measuring gauges.

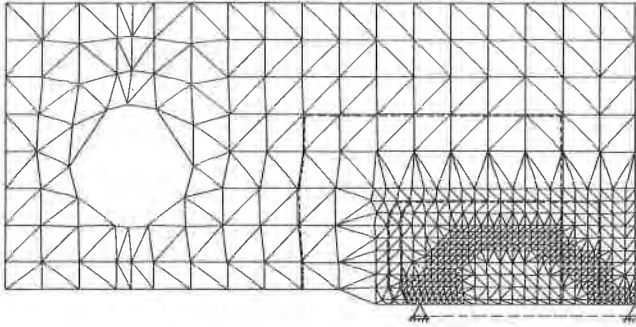


Fig. 4 Element Mesh (1052 elements) for the analysis of CT specimen.

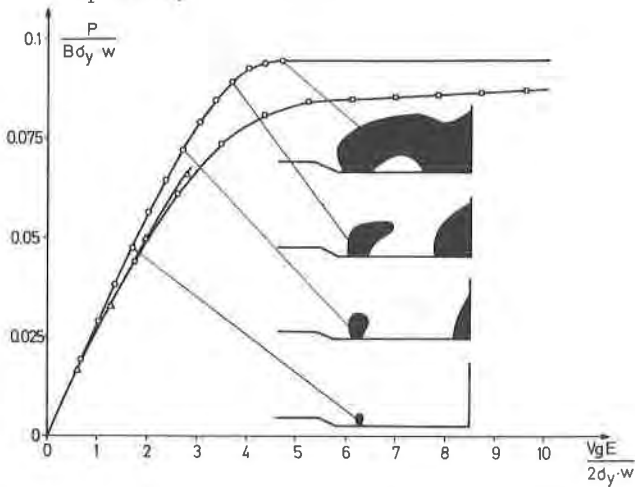


Fig. 5 Comparison between theoretical and experimental load-displacement curves.

▲ Experimental A5    □ Experimental A7    ○ Theoretical

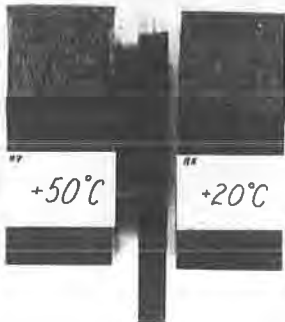


Fig. 6 Fracture surfaces for 4TCT specimen broken at 20°C and 50°C respectively (20Mo4) 4TCT.

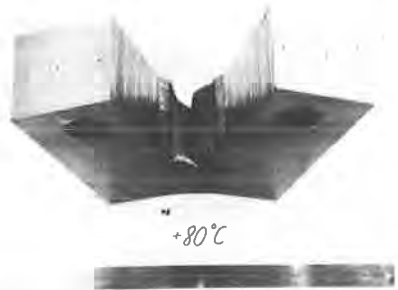


Fig. 7 4TCT specimen tested at +80°C (20Mo4).

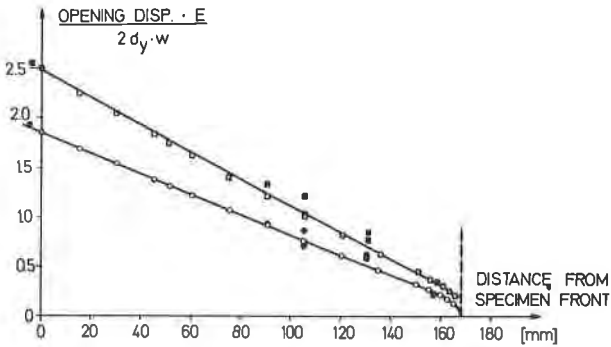


Fig. 8 Comparison between theoretical and experimental crack profiles for specimen A5.

■ ● Experimental □ ○ Theoretical ●○ Load of 31,7 MP  
 ■ □ Load of 40,7 MP

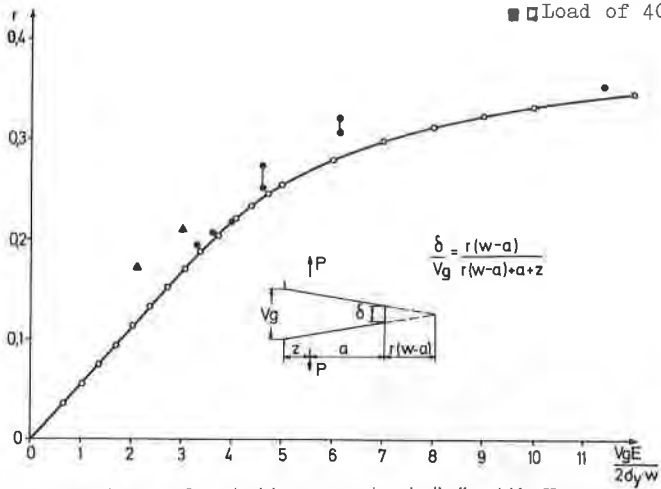


Fig. 9 Variation of rotation constant "r" with  $V_g$ .

▲ A5 ● A7 Experimental ○ Theoretical

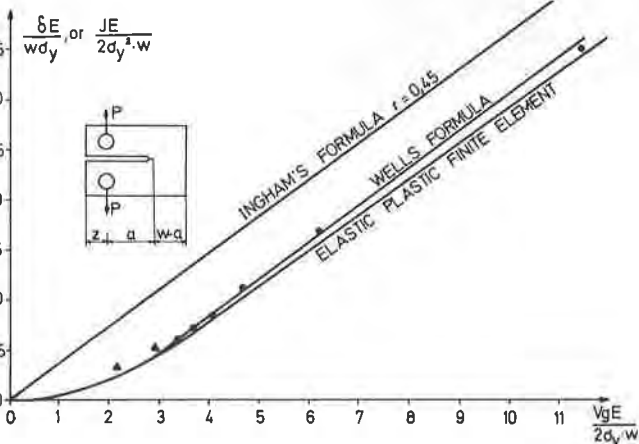


Fig. 10 Comparison between theoretically and experimentally determined COD. ▲ A5 ● A7 Experimental

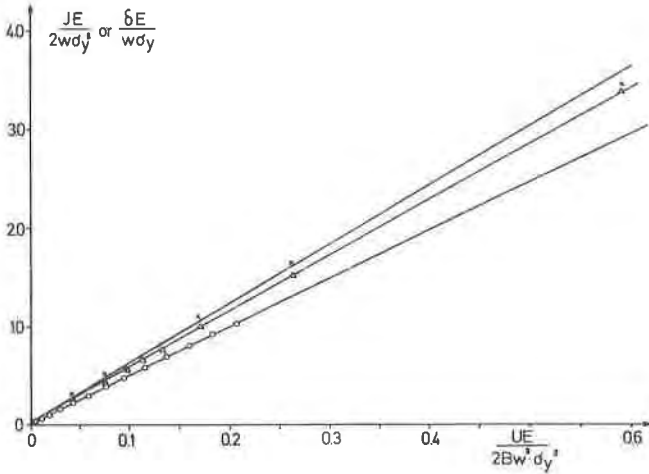


Fig. 11 Relationship between J or COD and external work U (theoretical and experimental).

- △ Experimental with  $\delta$  taken from Vg/COD calibration
- × Experimental with  $\delta$  taken from extrapolated profiles
- Theoretical

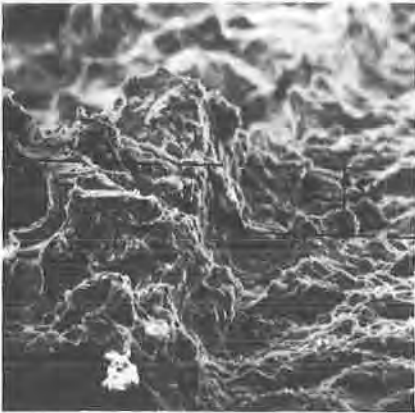


Fig. 12 Determination of COD after fracture using scanning electron microscope.

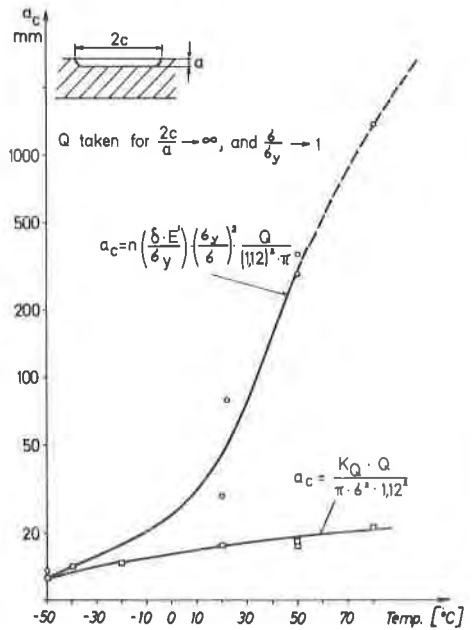


Fig. 13 Comparison of critical defect size as a function of temperature.