DEVELOPMENT OF REAL/SYNTHETIC TIME HISTORIES TO MATCH SMOOTH DESIGN SPECTRA

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SUMMARY

In modern nuclear plant design, there is a definite trend toward time history type analysis of equipment and structural components to determine the peak seismic response and loads. However, the site response spectra remain as the only acceptable description of expected ground motion during a seismic event. Thus, it has become necessary to develop time history accelerograms compatible with smoothed response design spectra established for a site. This paper describes a new computerized technique for generating time history accelerogram traces from existing seismic records to match the smoothed site response spectra.

Our method of analysis is particularly attractive in that it minimizes man-machine interaction inherent in previous attempts to generate time histories. The program utilizes existing seismic traces represented in the frequency domain as opposed to the time domain through the use of Fourier transform techniques. By manipulation of both the amplitude and phase of the Fourier transform representation in accordance with criteria formulated from the comparison of a computed response spectra to the smoothed design response spectra, successive new time histories are generated having response spectra converging to the smooth design response spectra.

In addition to matching the smooth design response spectra, an additional constraint is imposed on generated time histories in that the peak acceleration of a time history is also established for the site in the form of the zero period ground response. This contraint is also accounted for automatically in the program by revising the time history at each iteration to eliminate peaks greater than the peak site ground response.

The authors have utilized the program successfully to match smoothed design response spectra of various damping ratios to generate seismic time history site accelerograms used in the development of floor response spectra and equipment analysis for several nuclear facilities.

1.0 INTRODUCTION

In the development of an artificial earthquake time history compatible with a site design ground response spectrum, two criteria should be considered:

1. The time history generated response spectrum (THRS) must match as closely as possible and remain higher than the smoothed design response spectrum (SDRS).
2. The artificial time history must have a peak ground acceleration equal to that defined for the site and must have frequency and duration characteristics comparable to time histories recorded at sites with similar foundation conditions.

The latter part of the second criterion is admittedly not a necessary condition for the generation of an artificial time history since, from a mathematical point of view, any time history which gives rise to a THRS satisfying the first criterion could be used to determine peak structural response. However, it has been the experience of the authors that artificially generated time histories having characteristics comparable to recorded seismic accelerograms are more readily acceptable to designers of nuclear facilities.

2.0 ANALYSIS

By definition, a response spectrum is the envelope of the peak response of a single degree of freedom oscillator of varying natural frequency over the frequency range of interest. Thus, for a seismic accelerogram given by a(t), the response is given by the solution of the single degree of freedom equation

$$\ddot{X} + 2\zeta\omega_n\dot{X} + \omega_n^2 X = -a(t) \qquad (1)$$

Where X = the displacement of the oscillator relative to the ground displacement

ζ = the damping coefficient as percent of critical damping.

ω_n = the natural frequency of the oscillator.

The solution of Eq. (1) is given by Duhamel's integral,

$$X(t) = -\omega_n \int_o^t a(\tau)\sin\omega_n(t-\tau)d\tau \qquad (2)$$

and the absolute response is given by

$$Y(t) = X(t) + U_g(t) \qquad (3)$$

where $U_g(t)$ is the ground displacement found by integrating the gound acceleration $\ddot{U}(t)$ twice with respect to time. Since stresses induced in a structure resulting from dynamic response during a seismic event are dependent upon the relative deformation of the structure to the ground, the relative displacement given by Eq. (2) above is the response of interest. Thus, the displacement response spectrum is given by

$$THRS(\omega_n,\zeta) = \underset{0<t<T}{Max}\left\{-\omega_n \int_o^t a(\tau)\sin\omega_n(t-\tau)d\tau\right\} \qquad (4)$$

where the interval 0 to T is greater than or equal to the range of definition of the accelerogram.

When T in Eq. (4) is greater than the range of definition of the acceleration time history, residual vibration effects are included in the response spectrum. If one sets T equal to the range of the acceleration time history, one includes only the forced vibration effects in the transient spectrum. Mathematical rigor would require that residual vibration effects be included by extending the range T sufficiently far beyond the end of the seismic time history to allow at least one period of vibration at the lowest natural frequency of the structure. However, the decay inherent in seismic traces is such that, in practical application, residual vibration effects are usually of little or no consequence to the response spectrum.

Given the displacement response spectrum defined by Eq. (4), the corresponding acceleration and velocity response spectra can be obtained from:

$$S_{v(\omega_n,z)} = \max_{0<t<T} \left\{ \dot{X}(t) \right\} \tag{5}$$

and

$$S_{a(\omega_n,\omega)} = \max_{0<t<T} \left\{ \ddot{X}(t) \right\} \tag{6}$$

However, seismic response spectra are typically defined in terms of the pseudo velocity and pseudo acceleration spectra

$$PS_v(\omega_n,\zeta) = \max_{0<t<T} -\omega_n^2 \int_o^t a(\tau)\sin(\omega_n(t-\tau)d\tau \tag{7}$$

and

$$PS_a(\omega_n,\zeta) = \max_{0<t<T} \left\{ -\omega_n^3 \int_o^t a(\tau)\sin(\omega_n(t-\tau)d\tau \right\} \tag{8}$$

which are consistent with the displacement spectrum in the sense that the peak pseudo velocity and pseudo acceleration occur at the same point in time as the peak of the displacement, which is not necessarily the case for the true velocity and displacement.

Given a smoothed design response spectrum (SDRS) and an acceleration time history a(t), we may define the difference between the design spectrum and time history generated response spectrum (THRS) corresponding to a(t) as

$$E(\omega,\zeta) = THRS(\omega,\zeta) - SDRS(\omega,\zeta) \tag{9}$$

Owing to the definition of a response spectrum, it is not possible to define a closed-form mathematical transformation from the time history a(t) to the response spectrum. Similarly, there is no one-to-one correspondence between response spectra and time histories in the sense that more than one time history can theoretically produce the same response spectrum. Thus, the process of fitting a time history to a response spectrum must be a numerically iterative procedure.

3.0 PROGRAM METHOD

In order to force the THRS to match the SDRS at discrete frequencies ω_i, an actual recorded seismic time history is used as a first approximation for the first iteration. The initial time history $a_0(t)$ is represented in frequency space at discrete frequencies using the Fourier Transform,

$$A_0(\omega) = \int_{-\infty}^{\infty} a_0(t) e^{i\omega t} dt \tag{10}$$

with the time history given by the inverse Fourier Transform,

$$a_0(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A_0(\omega) e^{-i\omega t} d\omega \tag{11}$$

Having represented the time history by its Fourier Transform, a new time history $a_1(t)$ may be generated by adjusting the Fourier Transform,

$$A_1(\omega) = R(\omega)\ A_0(\omega) + \Delta(\omega) \tag{12}$$

in which case the new time history becomes

$$a_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A_1(\omega) e^{-i\omega t} d\omega \tag{13}$$

In Eq. (11), the revised time history Fourier Transform is obtained from the previous Fourier Transform using both a multiplicative function of ω, $R(\omega)$ and an additive function $\Delta(\omega)$. Both functions are used because experience cited in the literature[1] has shown that the use of an additional function represented by $\Delta(\omega)$ in frequency space is better for the case where $E(\omega,\zeta)<0$, in which case the computed spectrum must be raised; whereas, a multiplicative function is more appropriate when $E(\omega,\zeta)>0$, or when it is necessary to lower the computed response spectrum.

3.1 Comparison of THRS and SDRS

A good representation of the original seismic trace can be obtained with one-thousand terms in the Fourier Transform. Figure 1 illustrates the 1940 El Centro N-S accelerogram as recorded; whereas, Figure 2 shows the inverse Fourier Transform representation.

The design spectrum SDRS is input at N discrete frequencies, and the THRS is computed at each of the frequencies ω_j; j = 1, 2, 3...N. The design spectrum is then compared to the pseudo acceleration spectrum resulting from the initial time history and two error functions generated as follows:

$$E(\omega_j,\zeta) = THRS(\omega_j,\zeta) - SDRS(\omega_j,\zeta) \tag{9}$$

The first term is the difference between the computed response spectrum and the design spectrum. The second error function $R(\omega_j)$ is defined by

$$R(\omega_j,\zeta) = \frac{THRS(\omega_j,\zeta)}{SDRS(\omega_j,\zeta)} \quad (14)$$

Based on the assumption that the Fourier Spectrum, defined by

$$F(\omega) = (C(\omega)^2 + D(\omega)^2)^{1/2} \quad (15)$$

where $C(\omega)$ and $D(\omega)$ are respectively the real and imaginary parts of the Fourier Transform of the time history, is similar in shape to the response spectrum $THRS(\omega,\zeta)$: the Fourier Transform of the input history is modified to produce the next iterative time history representation.

The procedure used in modifying the time history varies, dependent upon whether the THRS is greater or less than the SDRS at a frequency point ω_j.

3.2 THRS Greater Than SDRS $E(\omega,\zeta)>0$

From the authors' experience, as well as experience cited in the literature,[1] it has been found that when the THRS exceeds the SDRS, it is necessary to pass the time history through a local suppressing filter in the range of ω_m, where the THRS exceeds the SDRS. Thus, when $E(\omega_m)>0$ and $R(\omega_m)>1$, the time history is filtered locally by scaling both the real and imaginary parts of the Fourier Transform according to

$$A_1(\omega_k) = \frac{1}{R(\omega_m)} A_0(\omega_k) \quad (16)$$

for $k = m-p$ to $m+p$ where the region ω_{m-p} to ω_{m+p} defines the band of frequencies surrounding the point ω_m at which the SDRS is given and the THRS evaluated, between ω_{m-1} and ω_{m+1}, as shown on Figure 3.

In other words, this scaling technique for modifying the input transform applies discrete step suppressing filters to the input time history in bands around the frequencies at which the THRS exceeds the SDRS. Since both the real and imaginary parts of the Fourier Transform are modified according to Eq. (14), there is no phase modification to the input time history.

3.3 THRS Less Than SDRS $E(\omega,\zeta)<0$

At frequencies ω_m at which the THRS falls below the SDRS, such that $E(\omega_m,\zeta)<0$, it is necessary to raise the THRS. For low values of ζ, the response of a single degree of freedom system of natural frequency ω_m to harmonic excitation of frequency ω_m, is peaked in the neighborhood of the resonance at ω_m, so that the effect on the response spectrum of the addition of a harmonic signal of frequency ω_m is local to the region near ω_m. Thus, for the case of the THRS below the SDRS, a signal of the form

$$g(t) = -E(\omega_m)e^{-\lambda t}\sin\omega_m t \quad (17)$$

is added to the original time history. The $e^{-\lambda t}$ decay factor is added so that the Fourier Transform of $g(t)$ exists and is given by

$$G(\omega) = \left[\frac{1}{2} \left\{ \frac{\omega+\omega_m}{\lambda^2+(\omega_m+\omega)^2} + \frac{\omega_m-\omega}{\lambda^2+(\omega_m-\omega)^2} \right\} + \frac{\lambda i}{2} \left(\frac{1}{\lambda^2+(\omega_m-\omega)^2} - \frac{1}{\lambda^2+(\omega_m+\omega)^2} \right) \right] E(\omega_m) \quad (18)$$

Therefore, the Fourier Transform of the original time history is modified to produce a new time history according to

$$A_1(\omega_j) = A_0(\omega_j) - \sum^{N_m} G(\omega_j, \omega_m) \quad (19)$$

where there were a total of N_m points, ω_m, at which the THRS was below the SDRS. Combining Eqs. (14) and (17), a new Fourier Transform $A_1(1)$ is obtained, which when inverted yields a new time history with both amplitude and phase modified,

$$A_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A_1(\omega)\, e^{-i\omega t} d\omega \quad (18)$$

Prior to computing a new response spectrum corresponding to $a_1(t)$ above, the time history is modified such that there is no peak greater than the peak ground acceleration defined as part of the design criteria along with the SDRS. This modified time history a'(t) then is used to compute a THRS and the modification process again applied to yield the next iteration.

4.0 RESULTS

Figures 4 through 7 show artificial time histories generated using the computer program developed based on the above methods and the associated response spectra compared to the original design response spectra. In both cases, a 1,000 term Fourier Transform representation of the 1940 El Centro N-S seismic time history was used as the initial time history. The design spectra and, hence, the comparison points between the computed and design spectra, was given at 161 points in the frequency range from 0.4 to 25 Hz. Table I shows the points used to match the design spectra.

5.0 DISCUSSION

As shown on the figures, the computed response spectra for both time histories shown match the design response spectra reasonably well with the maximum deviations in the neighborhood of 40 to 50 percent, on the conservative side. As expected, the 5 percent damped spectrum is smoother and provides a closer match to the design spectrum than for the 2 percent spectrum. For both cases, the process was convergent, but became exceedingly slower with a tendency to oscillate around the SDRS as the THRS converged to the SDRS. When this occurred, computer time became excessive. It was found that convergence was improved by using a maximum number of points to represent the SDRS. However, the increased convergence rate is somewhat offset by the increased run time in the spectral computations and Fourier Transform modifications.

Also, it was found, that where several ground response spectra are defined for different damping ratios, it is unrealistic for a THRS for one damping ratio to match the SDRS for another damping ratio. It was observed that the THRS increasingly exceeds the SDRS for damping ratios greater than that for which the time history was generated and falls below the design spectrum for damping ratios less than the one at which the match was made.

LIST OF REFERENCES

[1] Nien-Chien Tsai, "Spectrum-Compatible Motions for Design Purposes," ASCE, Journal of Engineering Mechanics Division, Vol. 98, No. EM2, April 1972.

[2] Yang, R. C., Safeguard BMD System Development of a Waveform Synthesis Technique SAF-64, August 1970.

[3] Sneddon, I. N., Fourier Transforms, McGraw-Hill Book Company, Inc., New York, 1951.

TABLE I

LIST OF FREQUENCY POINTS WHERE THE TIME HISTORY RESPONSE SPECTRUM HAS BEEN COMPUTED

Point No.	Frequency (Hertz)	Point No.	Frequency (Hertz)	Point No.	Frequency (Hertz)
1	.400	55	3.800	111	15.000
2	.420	56	4.000	112	15.200
3	.440	57	4.200	113	15.400
4	.460	58	4.400	114	15.600
5	.480	59	4.600	115	15.800
6	.500	60	4.800	116	16.000
7	.520	61	5.000	117	16.200
8	.540	62	5.200	118	16.400
9	.560	63	5.400	119	16.600
10	.580	64	5.600	120	16.800
11	.600	65	5.800	121	17.000
12	.620	66	6.000	122	17.200
13	.640	67	6.200	123	17.400
14	.660	68	6.400	124	17.600
15	.680	69	6.600	125	17.800
16	.700	70	6.800	126	18.000
17	.720	71	7.000	127	18.200
18	.740	72	7.200	128	18.400
19	.760	73	7.400	129	18.600
20	.780	74	7.600	130	18.800
21	.800	75	7.800	131	19.000
22	.820	76	8.000	132	19.200
23	.840	77	8.200	133	19.400
24	.860	78	8.400	134	19.600
25	.880	79	8.600	135	19.800
26	.900	80	8.800	136	20.000
27	.920	81	9.000	137	20.200
28	.940	82	9.200	138	20.400
29	.960	83	9.400	139	20.600
30	.980	84	9.600	140	20.800
31	1.000	85	9.800	141	21.000
32	1.100	86	10.000	142	21.200
33	1.200	87	10.200	143	21.400
34	1.300	88	10.400	144	21.600
35	1.400	89	10.600	145	21.800
36	1.500	90	10.800	146	22.000
37	1.600	91	11.000	147	22.200
38	1.700	92	11.200	148	22.400
39	1.800	93	11.400	149	22.600
40	1.900	94	11.600	150	22.800
41	2.000	95	11.800	151	23.000
42	2.100	96	12.000	152	23.200
43	2.200	97	12.200	153	23.400
44	2.300	98	12.400	154	23.600
45	2.400	99	12.600	155	23.800
46	2.500	100	12.800	156	24.000
47	2.600	101	13.000	157	24.200
48	2.700	102	13.200	158	24.400
49	2.800	103	13.400	159	24.600
50	2.900	104	13.600	160	24.800
51	3.000	105	13.800	161	25.000
52	3.200	106	14.000		
53	3.400	107	14.200		
54	3.600	108	14.400		
		109	14.600		
		110	14.800		

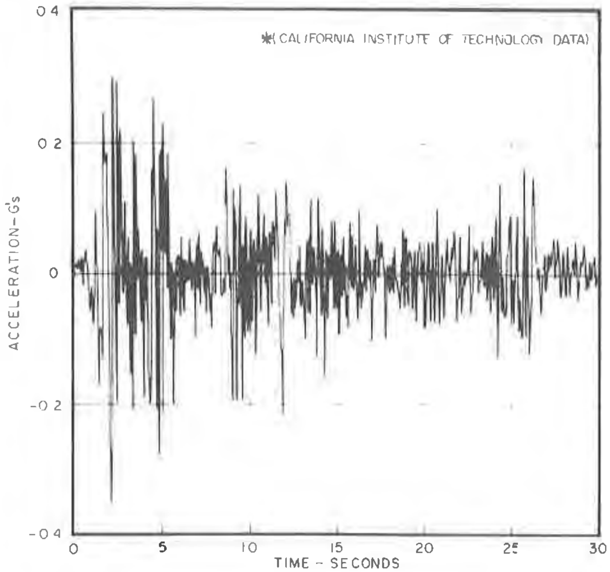


FIGURE 1 - ORIGINAL VERSION OF EL CENTRO 1940 N-S *

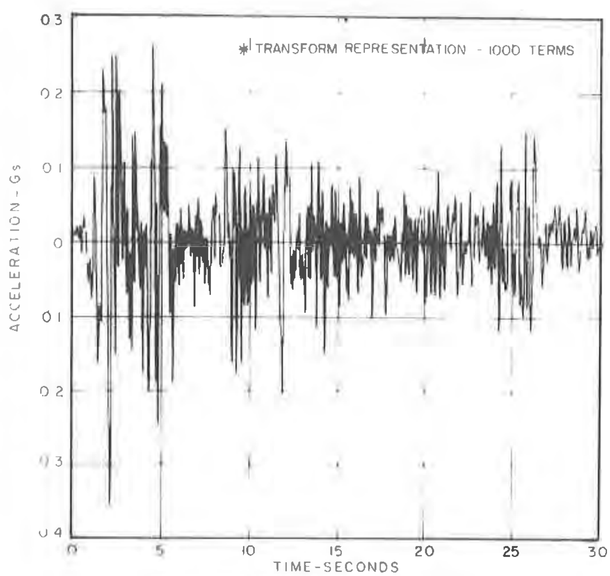
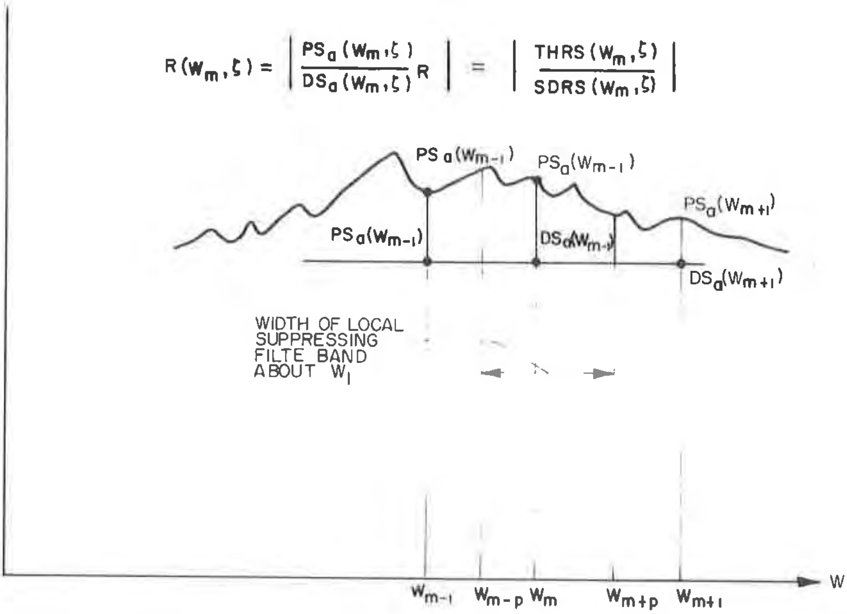


FIGURE 2 - INVERSE FOURIER TRANSFORM REPRESENTATION OF EL CENTRO 1940 N-S *



W_{m-1}, W_m, W_{m+1} – COMPARISON POINTS AT WHICH DESIGN SPECTRA IS GIVEN

W_{m-p} TO W_{m+p} – POINTS USED IN FOURIER TRANSFORM IN BANDWIDTH ABOUT W_m

FIGURE 3: BAND WIDTH OF LOCAL SUPPRESSING FILTER ABOUT COMPARISON POINT W_m

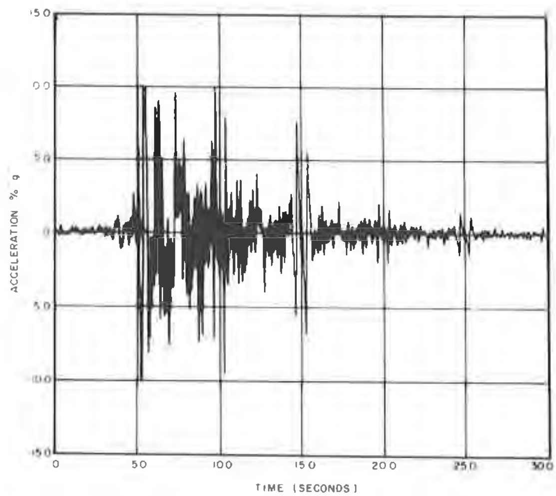


FIGURE 4 - ARTIFICIAL TIME HISTORY NO. I

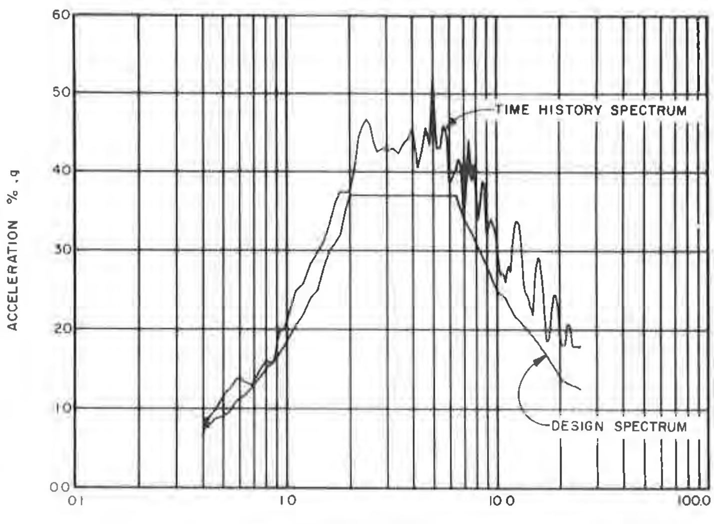
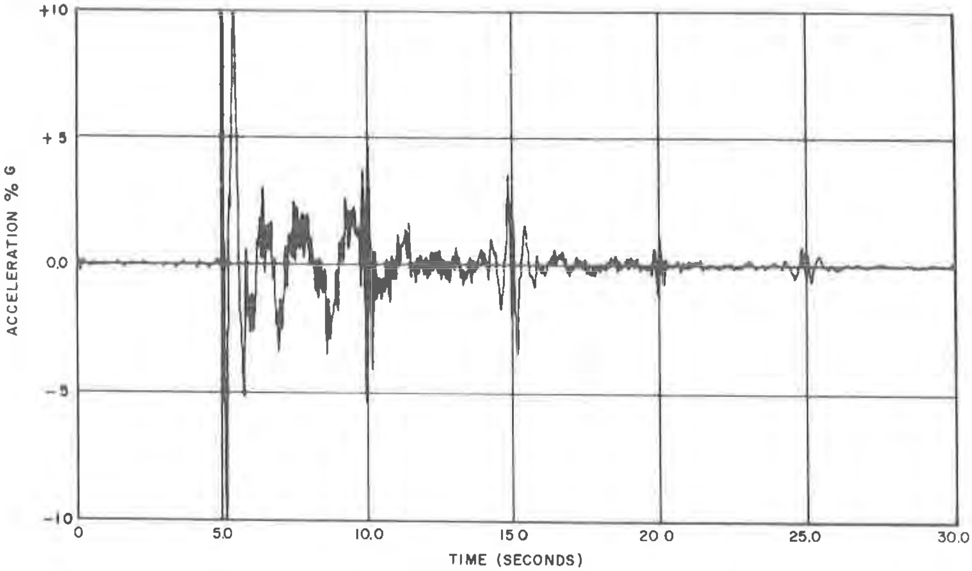


FIGURE 5 - RESPONSE SPECTRA FOR ARTIFICIAL TIME HISTORY NO. I
SSE - 2% DAMPING

K 1/5*



NOTE PEAK HORIZONTAL ACCELERATION IS EQUAL TO 0.10G

FIGURE 6 - ARTIFICIAL TIME HISTORY NO. 2

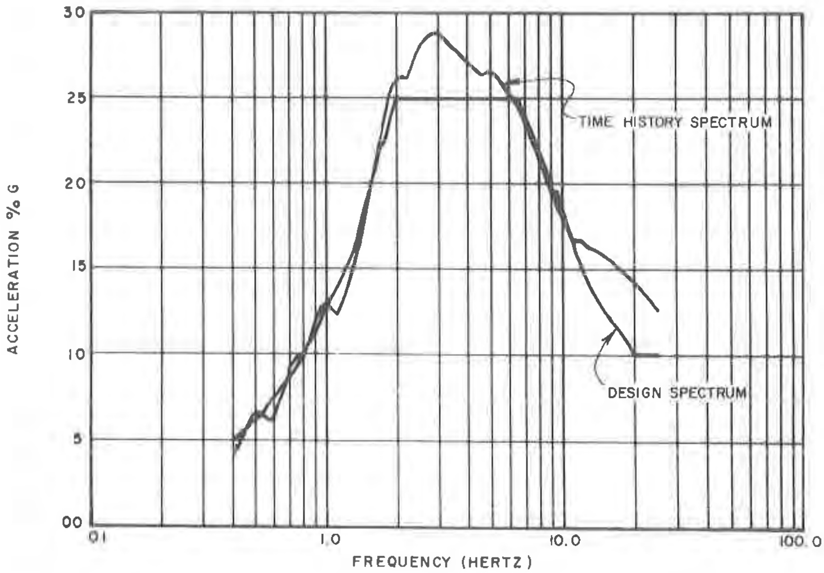


FIGURE 7 - RESPONSE SPECTRA FOR ARTIFICIAL TIME HISTORY NO. 2
SSE - 5% DAMPING