BOUNDING CREEP SOLUTIONS FOR A STRESSED PLATE SUBJECTED TO VARIABLE SURFACE TEMPERATURE

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SUMMARY

The analysis of structures which are subjected to both variable thermal and mechanical loading and which suffer creep and plastic deformation remains among the more intractable problems of structural analysis. Although the individual aspects of the problem are reasonably well understood the solution of non trivial components taxes the capacity of even the largest computers. The authors have attempted, in the last few years, to devise methods of analysis that provide sufficient information for design purposes but obviate the need for complex analysis. The basic theory and some simple applications have recently been described by the authors:

— F.A. Leckie and A.R.S. Ponter, "Theoretical and Experimental Investigation of the relationship between plastic and creep deformation of structures". Conference on the Foundations of Plasticity. Warsaw, 1972. *Archives of Mechanics*, Vol. 24, No. 3, pp. 419-437, 1972.

— A.R.S. Ponter, "Deformation, displacement and work bounds for structures in a state of creep and subject to variable loading". *Appl. Mechanics, Trans. ASME,* Vol. 39, 1972.

In the present paper it is demonstrated how the theory can be applied to determine bounds on creep deformations of a uniform plate subjected to constant average in-plane stress and a fluctuating temperature through its thickness. The material is described by a non-linear Maxwell law with an activation energy temperature dependence. The effects of variable temperature on material and thermal expansion are included in the problem. Two solutions are computed which provide bounding solutions in a certain defined sense and correspond to the actual solution when the time scale of temperature fluctuations are either long or short compared with the characteristic time scale of the material. These bounding solutions are easily calculated and provide considerable insight into the ratchetting mechanisms which can occur.

It is found that the effects of thermal expansion can considerably increase the rate of deformation of the plate, but that the effect of thermal softening of the material can reduce this effect considerably. Thus, in any analysis, both phenomena must be realistically included, and ignoring the effect of temperature and material behaviour will provide excessively pessimistic results.

The calculations described in the paper provide the first step towards an understanding of the effects of variable temperature on creeping structures and indicate that comparatively simple calculations may provide the most relevant design information.

1) INTRODUCTION

The analysis of structures which suffer creep deformations and are subjected to variable thermal and mechanical loading remains amongst the more intractable problems of structural mechanics. The interaction between the nonlinear creep response of the material and presence of varying temperature fields has received comparatively little attention in the technical literature. Two effects are present, the material behaviour changes rapidly with temperature and the thermal expansion of the material causes incompatible volume changes. Either one or both of these effects are ignored in many of published creep analysis and the relevance of such calculations remains difficult to assess in general terms.

The possibility of a more detailed understanding of thermal-creep interaction is provided by some theoretical results recently derived by Ponter (1). A structure composed of an elastic/time-hardening creeping/plastic material was considered, which was subject to a history of applied loading, applied displacement and inelastic strain. A number of theoretical results were derived, including bounds on the energy dissipated by the material in the formation of inelastic (creep and plastic) strains for load levels below the plastic shakedown limit. For loading and thermal histories which were cyclic, the bounds were expressed in a particularly simple form, involving the creep energy dissipation associated with two equilibrium histories of stress. It was shown that these two bounds corresponded to the actual behaviour of the structure when the cycle time was very long and very short compared with a characteristic time of the material at a mean stress level. The two stress distributionsgave the asymptotic stress histories and provided the corresponding average displacement rates of the structure. The two bounds will be discussed in more detail in the following section but it may be mentioned here that consideration of typical time scales involved in many practical creep problems (2,3) indicate that cycle times are generally very short compared with characteristic material times and the upper bound analysis is likely to closely approximate the actual stress history.

The two stress histories given by this theory and the corresponding displacement rates they predict are likely to provide a strong indication of the behaviour of the structure for finite cycle times and indicate the relative importance of the various phenomena involved. In this paper we discuss the analysis of a problem of general interest, the creep of a plate subject to constant biaxial tension and surface temperatures which vary in time. We study in detail a particular material and loading discussed by Miller (6), Bree (4,5) and Anderson (7) which occurs in the design of nuclear fuel cans. These authors were primarily concerned with the plastic deformations, although Bree (5) and Anderson (7) included creep but excluded the effects of temperature on creep rates. The pattern of thermal loading considered cannot be considered severe and appears to be typical of the load levels and temperature differences which occur in many components which operate at high temperatures.

We find certain clear patterns of behaviour for this problem. We find that if the effect of temperature upon the material behaviour is ignored, but thermal expansion included, the theory predicts enormous ratchetting effects. On the other hand when both effects are included the difference between the bounding solution is greatly reduced, but are still sufficiently large to require consideration in design. The most severe condition occurs when the stresses due to the applied loads are small compared with the thermo-elastic stresses.

As the bounding solutions concentrate attention upon two fairly simple circumstances, the modes of behaviour involved can clearly be seen. It would appear that the upper bound solution provides a comparatively simple design calculation which is both conservative and is closest to the actual circumstances.

In the next section the theoretical results derived in (1) are described briefly. This is followed by a description of the method of analysis used in the plate problem and a discussion of numerical results.

2) BOUNDING CYCLIC SOLUTION

We are concerned with a material which suffers elastic strain, creep strains v_{ij} and plastic strains p_{ij}. The material is subject to temperature changes (and possibly other externally applied agencies, such as irradiation) which cause strains δ_{ij}. The total strain is given by

$$\varepsilon_{ij} = e_{ij} + v_{ij} + p_{ij} + \delta_{ij} \tag{1}$$

The elastic strains are given by

$$e_{ij} = C_{ijk\ell}\, \sigma_{k\ell} \tag{2}$$

where $C_{ijk\ell}$ is positive definite and fully symmetric. The creep strains are given by

$$\dot{v}_{ij} = \frac{\partial}{\partial \sigma_{ij}} \left\{ \frac{\phi^{n+1}(\sigma_{k\ell})}{(n+1)} \right\} \frac{\dot{v}_o}{\sigma_o^n} \tag{3}$$

where ϕ denotes a homogeneous function of degree one in $\sigma_{k\ell}$, and $\dot{v}_o$ denotes the uniaxial creep rate corresponding to the uniaxial constant stress σ_o. The rate of creep energy dissipation $\dot{D}^c$ is given by

$$\dot{D}^c = \sigma_{ij}\, \dot{v}_{ij} = \phi^{n+1}(\sigma_{k\ell}) \frac{\dot{v}_o}{\sigma_o^n} \tag{4}$$

The plastic strains p_{ij} are given by a perfectly plastic model associated with a convex yield function $f(\sigma_{k\ell}) = 0$,

$$\dot{p}_{ij} = \dot{\mu} \frac{\partial f}{\partial \sigma_{ij}} \quad , \quad f = 0 \text{ and } \dot{\sigma}_{ij} \frac{\partial f}{\partial \sigma_{ij}} = 0$$

$$\dot{p}_{ij} = 0 \quad , \quad f < 0 \text{ or } f = 0 \text{ and } \dot{\sigma}_{ij} \frac{\partial f}{\partial \sigma_{ij}} < 0 \tag{5}$$

We consider a body with volume V and surface S. The surface is subjected to applied loads $P_i(x_j,t)$ over part S_T, and applied displacements $U_i(x_j,t)$ over the remainder. Within the volume a temperature field $\theta(x_j)$ acts which induces inelastic strains and we assume some functional dependence of $\dot{v}_o(t)$ upon θ. The particular form we adopt in our calculation is

$$\frac{\dot{v}_o}{\sigma_o^n} = k e^{\frac{-\Delta H}{R\theta}}$$

where ΔH denotes an activation energy, R the universal gas constant and k a material constant.

In terms of these quantities we may define certain stress histories. The linearly elastic stress distribution for the stated problem is denoted by $\hat{\sigma}_{ij}$ and a related stress history σ^*_{ij} is defined by

$$\sigma^*_{ij}(x_k,t) = \hat{\sigma}_{ij}(x_k,t) + \bar{\rho}_{ij}(x_k) \tag{6}$$

where $\bar{\rho}_{ij}$ denotes an arbitrary time constant residual stress field in equilibrium with zero applied loads on S_T. The stress history σ^*_{ij} may be recognized as that associated with the lower bound shakedown theorems.

The "stationary state" creep solution σ^s_{ij} is defined as the solution to the stated problem under the simplifying assumption $e_{ij} = p_{ij} = \delta_{ij} = 0$ and $U_i = 0$ on S_u. Therefore σ^s_{ij} provides the purely viscous solution with rigid supports. The temperature field enters into the problem only through the functional dependence of $\dot{v}_o$ upon θ.

Now consider the case when P_i, U_i and θ are all cyclic with period T. It was shown in (1) that the stress distribution asymptotes to a cyclic state (provided such a state exists) with period T and that the total inelastic and elastic work (excluding the work done by the thermal strain δ_{ij}) may be bounded by the creep energy dissipation association with σ^*_{ij} and σ^s_{ij}.

$$\int_0^T\int_V \dot{D}^c(\sigma^s_{ij})\, dV\, dt \leq \int_0^T\int_V \sigma_{ij}\,(\dot{\varepsilon}_{ij}-\dot{\delta}_{ij})\, dV\, dt \leq \int_0^T\int_V \dot{D}^c(\sigma^*_{ij})\, dV\, dt\,. \tag{7}$$

The stress distributions are subject to the yield conditions

$$f(\sigma^s_{ij}) \leq 0 \quad \text{and} \quad f\left(\frac{n+1}{n}\,\sigma^*_{ij}\right) \leq 0 \tag{8}$$

The latter condition implies that $\left(\frac{n+1}{n}\right) P_i(t)$ must lie below the shakedown limit for the structure.

The optimal upper bound has special significance and occurs when the accumulated creep strain over a cycle

$$\Delta^u v_{ij} = \int_0^T \dot{v}_{ij}(\sigma^*_{k\ell})\, dt \tag{9}$$

is kinematically admissible and therefore derivable from a displacement field $\Delta^u U_i(x_j)$. It was shown in (3) that the particular $\bar{\rho}_{ij}$ which satisfies this condition provides the asymptotic solution to the problem when the cycle time T is small compared with a characteristic time of the material (the time to accumulate creep strain equal to the elastic strain at some mean stress).

On the other hand the lower bound solution corresponds to the situation when the cycle time is very long and the perturbation due to change of thermal loading and the changes in elastic strain make a negligible contribution to the total deformation of the body. The creep strain $\dot{v}_{ij}(\sigma^s_{k\ell})$ is, by definition, kinematically admissible and the corresponding displacement rate $\Delta^L \dot{U}_i$ provides an accumulated displacement $\Delta^L U_i$ over a cycle.

The two solutions σ^*_{ij} and σ^s_{ij} provide the most extreme conditions which may occur in the body in the sense signified by inequality (7). The lower bound σ^s_{ij} corresponds to the most widely used solution for the problems and can be seen to always underestimate the energy dissipated within the body. The deviation from this lower bound solution depends upon both the time scales involved and the behaviour of the material under variable loading, which involves questions which are not answered by this theory. Williams and Leckie (2) have discussed this question in relation to the behaviour of a simple structure subjected to variable loading under isothermal conditions. Their conclusions indicate that the upper bound solution σ^*_{ij} may be expected to closely approximate the actual behaviour of the body for many structures, as typical cycle times are very short compared with the total lifetime of the structure. When such situations do not occur this upper bound solution provides the worst case in a certain defined sense. They also show how a time-hardening solution may be used to predict deformation rates for any other constitutive relationship, a procedure which may be adopted in conjunction with the bounding solutions.

The Plate Problem

We consider the problem of a plate (Fig. 1) of thickness 2h subject to a uniform state of mean stress (p_x, p_y), with respect to fixed axis in the plane of the plates central surface, which result from a distribution $\sigma_x(z)$, $\sigma_y(z)$ through the plate thickness.

The lower side of the plate $z = -h$ is subjected to a temperature $\theta_1(t)$ and the upper scale $z = +h$ to a temperature $\theta_2(t)$. Changes in temperature are considered to take place sufficiently slowly for thermal transients to be negligible, so that the temperature distribution through the plate is given by

$$\theta(z,t) = \frac{(\theta_1 + \theta_2)}{2} + \frac{z}{h}\,\frac{(\theta_2 - \theta_1)}{2} \tag{10}$$

We assume that surface x = constant and y = constant suffer only rigid body displacement, thereby simulating the condition which occurs, for example, in a thin walled tube under internal or external pressure $(p_x = 2p_y)$ or a sphere under internal or external pressure $(p_x = p_y)$. The details of the computation and the non-dimensional variables adopted are given in the full paper. Attention was confined to temperature cycles of the form shown in Fig. 2 where the surface temperatures are maintained constant within time intervals λT and $(1-\lambda)T$.

In terms of the temperature history we define a mean temperature θ_m as the constant temperature which produces the same accumulated strain over a cycle as the history of temperature on the central surface z = 0. Therefore

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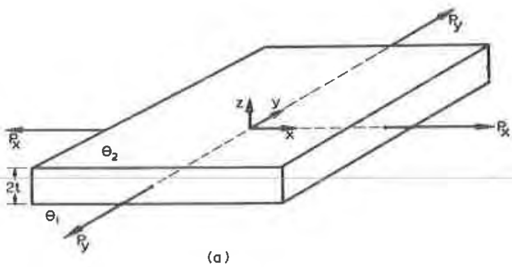
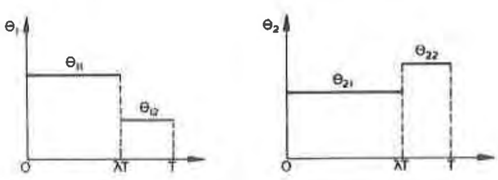


(a)



(b)

Fig. 1. Bounding solution for a plate subjected to variable surface temperature

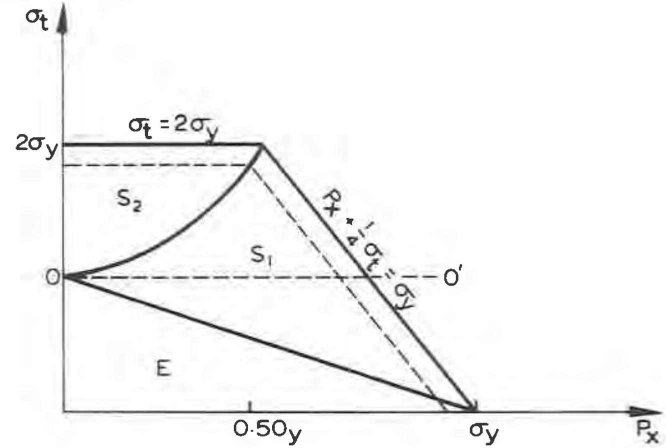


FIG. 2.

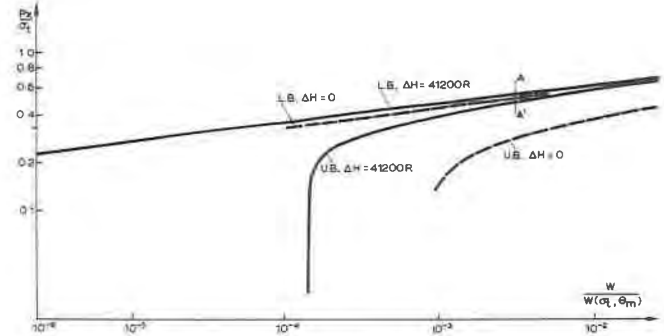


Fig. 3 Work Bounds

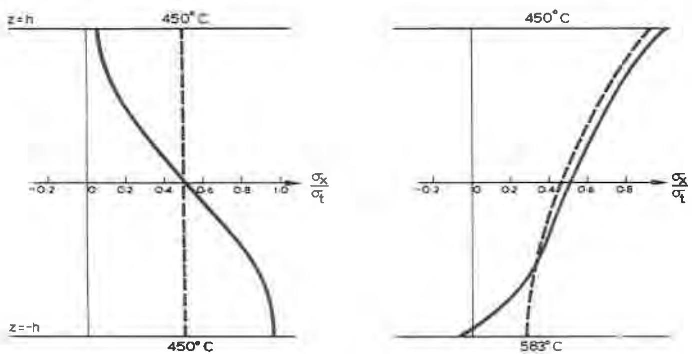


Fig. 4a Distribution of σ_x ($\frac{\Delta H}{R}$ = 41200° K P_x = 0.508 σ_t, ΔU_y = 0)

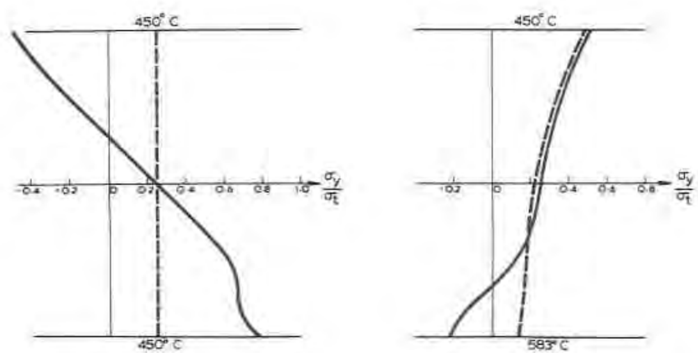


Fig. 4b Distribution of σ_y, $\frac{\Delta H}{R}$ = 41200 (P_x = 0.508 σ_t, ΔU_y = 0)