DESIGN METHODS FOR DUCTILE STRUCTURES SUBJECTED TO RELATIVELY LARGE CYCLIC DEFORMATIONS

S. KRISHNASAMY

Research Division, Ontario Hydro, Toronto, Ontario, Canada

A.N. SHERBOURNE

Department of Civil Engineering, Faculty of Engineering,
University of Waterloo, Waterloo, Ontario, Canada

SUMMARY

In classical design methods based upon elastic analysis, the load at which first yield occurs in a structure is usually considered as its limiting strength. It has been proven, however, that structural systems made of ductile materials possess considerable reserve strength even after the yield limit has been reached. This is due to the plasticity of the material which allows a favourable redistribution of stress in the structure.

Simple plastic theory, which utilizes the ductility of materials, is essentially based upon the concept of a set of proportional loads which affect the collapse of a structure. It has also been established that a structure may fail under suitably varying or cyclic loading conditions even though the load is less than the plastic collapse value. The cyclic loading may cause the structure to fail either by "incremental collapse" or "alternating plasticity".

The theories advanced for analyzing structural response have generally ignored changes in material behaviour under cyclic loading despite definite evidence to the contrary. It is essential, therefore, that any structural design method should include this change.

In reality, cyclic loading is random in nature and is not necessarily always well defined. It may be possible, with the help of modern testing and computing techniques, to study the behaviour of a specific structural system under a given loading condition; an understanding of the basic response of structural elements is, however, essential for the development of a more general theory. An attempt is here made to develop such a design method for simple structural components under well defined cyclic loads; it might be possible to apply the method for more general loading conditions based upon this study of simple structural elements. In these circumstances cyclic loading can be divided into two cases involving: (i) deflection control and (ii) load control. Each case, in turn, can be further classified into three categories covering (a) full reversal (b) repeated and (c) partial reversal.

In this paper a study of the behaviour of structural components under deflection control is presented. Results from tests on beams under pure bending are employed to predict the deformation response and life of simple structural components (cantilever beams). The predictions are compared with experiments. The performance of cantilever beams under each of the above mentioned categories of loading is also discussed. Design charts are prepared for simple beams under various combinations of cyclic loading.

Introduction

The importance of studying the response of structural components under low cycle fatigue conditions has been established by the authors in earlier publications [1,2]. A theory has been formulated to predict the behaviour of structural elements under fully as well as partially reversed deflection controls and fully reversed load control. Cyclic moment-curvature models, which are discrete functions of instantaneous cyclic history, were derived from basic tests and employed in predicting the behaviour of structural components. The theoretical predictions were compared with experiments. A layout of the general program and the work carried out by the authors are shown in Tables 1 and 2.

The experiments [3] on structural components proved, in general, that a metal subjected to low cycle fatigue testing initially either softens or hardens depending upon its reference state; it eventually settles down to a stable condition until cracks develop to such an extent that the load carrying capacity of the specimen is reduced. The stabilized state usually extends from 25% to 75% of the life to failure. The specimen (or component) "shakes down" to a "stable" state since it will be unaffected either by initial transient phenomena (residual stresses) or the presence of cracks.

Based on this observation, it would be sufficient to develop a design method for structural components subjected to cyclic loading from moment-curvature model formulated by employing values corresponding to stable condition. The moment-curvature model is derived from tests on beams under pure bending subjected to cyclic loading. This model is then applied to predict the load-deformation relationship of simple structural components under cyclic loading.

The design method will not be complete unless it incorporates a criterion for estimating life of structural components. Hence, a failure criterion, established by coupling strain range and number of cycles to failure from pure bending tests, is employed to predict the life of components.

The design method covers structural elements under i) deflection control with zero mean, ii) deflection control with non-zero mean and iii) load control with zero mean. The validity of this design procedure is shown by comparing theory with experiments.

Material and Testing

Normalized mild steel was used in preparing the test specimens. The chemical and mechanical properties of the material are given in Ref. [3].

The test set-up, testing procedure and specimens are described elsewhere in detail [3]. The type of loading imposed is described in Figure 1.

Moment-curvature Relationships

The response of beams under pure bending subjected to i) strain cycling with zero mean, ii) strain cycling with non-zero mean and iii) moment cycling with zero mean are shown in Figures 2, 3 and 4 respectively. It has been established [4] that if the maximum strain exceeds 2.5%, the influence of axial extension will be significant and the moment values have to be adjusted accordingly. Hence the maximum strain experienced by the beams in all these tests are limited not to exceed 2.5%.

In the case of fully reversed strain cycling (strain cycling with zero mean strain), there was no mean moment experienced by the beams (Figs 1 & 2). In the case of strain cycling with non-zero mean, the beam is subjected to a mean moment as shown in Figures 1 and 3. The mean moment which is initially significant becomes relatively small after about 10% to 25% of the life of failure depending upon strain range and mean strain imposed on the beam. A typical test result is shown in Figure 3.

The cyclic variation of strain in the beams under fully reversed moment is shown in Figures 1 and 4. When the moment range is held constant, initially the material either softens or hardens, as seen in the case of strain cycling, and then settles down to a "stable" condition. However, unlike the case of strain control tests, the material "creeps" under constant moment range cycling as shown in Figure 1. This, hereafter, will be referred to as "cyclic creep".

It is important to note the difference between strain and moment cycling tests. Since no "cyclic creep" occurs in the case of strain cycling with zero mean, maximum and minimum moment amplitudes are equal in magnitude. But, in the case of strain cycling with non-zero mean, there is a mean moment present, the amount of which depends upon strain range and mean strain. However, it is ignored in this study since it is relatively small compared to moment amplitude (half the moment range). Thus a single curve coupling moment and strain range will be sufficient to represent the material behaviour.

Due to the presence of "cyclic creep" in the case of moment cycling, maximum and minimum strain amplitudes are not equal in magnitude. However, the value of "cyclic creep" becomes very insignificant when the beam "shakes down" to "stable" state. Hence, the moment and strain range values corresponding to stable state are sufficient to construct the moment-curvature relationship.

It can be seen from Figures 2, 3 and 4 that the stabilized state usually extends from 25% to 75% of the life to failure. Hence, moment curvature relations, Figure 5, can be constructed from strain and moment control tests by plotting the moment and the corresponding strain ranges pertaining to the half-life state for several different beams. These relationships can be represented mathematically as $\Delta\epsilon = a(m)^b$, $\Delta\epsilon$ and m being percentage strain range and non-dimensional moment respectively and a, b material constants. These curves are based on the assumption that the cross-section of the beam remains constant until significant crack develops to reduce substantially the sectional area. Also shown in Figure 5 is the combined curve fitted to strain cycling and moment cycling tests. The figure demonstrates that the beams, irrespective of whether they are under strain or moment control, "shakedown" to nearly the same stable state.

In Figure 6, the combined curve is shown along with experimental points and 95% confidence limits; also shown is the fitted mean curve for fully reversed (or zero-mean) strain cycling data only. The combined curve can be employed in predicting the response of structural components. However, the difference between the combined curve and the mean curve for (zero-mean) strain cycling data is not very significant; hence, the latter has been employed to predict structural response instead of the combined curve. The strain control tests are easier than moment control tests to carry out. Hence, it is attempted here to see whether the stable state behaviour of structural components under deflection as well as load control can be predicted fairly accurately, by moment-curvature relation derived from strain cycling tests alone. Figure 7 shows the moment-curvature characteristic from fully reversed strain cycling tests, along with 95% confidence limits.

Analysis of Structural Components

It has been shown[3] that cantilever beams also shake down to a "stable" state after an initial period of softening or hardening.

The moment curvature relationship (Fig 7) is now employed in predicting the stable state load-deformation behaviour of cantilever beams. The analytical method to derive the load-deformation behaviour is described elsewhere [1] in detail and will not be included here. The theory is seen to correlate well with experiments (Fig 8).

Failure of Structural Components

Many investigations have been carried out in the past to examine the failure of metals under low endurance fatigue conditions. These investigations have established the existence of a relationship coupling the value of cyclic life, N_f, and true plastic strain range. In this investigation, a failure criterion is established from cycling straining tests on specimens under pure bending. This failure criterion, in conjunction with "stable" moment-strain curves derived from cyclic pure bending tests, is used to predict the life of structural components under low endurance fatigue.

Failure Criterion from Pure-bending Tests

A failure criterion is established, as shown in Figure 9, from strain as well as moment control tests. A more defined failure criterion than the usual complete fracture of the specimen is necessary since a structural component in an actual situation may be rendered unserviceable due to excessive growth of cracks, refusal of further load, or increased deflection, prior to complete fracture.

Consequently, the failure criterion adopted is that of final reduction in bending moment in the case of strain control tests and increase in strain range when the specimens are under moment control.

Based upon the above failure criteria, the cycles to failure, N_f, for various tests were found from experiments.

Design of Structural Components under Cyclic Loading

The method developed above can be used in predicting the behaviour and life of cantilever beams subjected to deformation as well as load control. The procedure adopted is as follows:

(i) The "stable" moment-curvature relationship (Fig 7) developed earlier is applied to predict the "stable" load-deformation characteristic of cantilever beam.

(ii) Values of bending moments (yielding the maximum strain range in the beam) corresponding to a "stable" state are computed from the theoretical load-deformation characteristic in Figure 8.

(iii) The strain ranges corresponding to the "stable" state are then calculated from Figure 7 and then are employed to predict the life of cantilever beams from Figure 9.

Figures 10 and 11 show theoretical and experimental lives of cantilever beams under deformation as well as load control. Prediction of lives of beams are made in two ways: one from the results of strain-control tests and the other from the results of moment-control tests.

It can be seen from Figures 10 and 11 that the failure criterion based upon moment-control tests yields an upper bound while the criterion derived from strain control tests provide a lower bound on test performance.

Discussion and Conclusions

A method is developed for the design of simple structural components subjected to a constant range of alternating deflection or load. A "stable" moment-strain range relationship is established by plotting values of moment amplitude and strain range pertaining to half-life of several different beams. It has been established that, irrespective of the type of control imposed on a beam, it shakes down to the same stable state. This moment-strain range characteristic, coupled with a failure criterion developed from pure-bending tests, is employed in predicting the "stable" response and lives of cantilever beams subjected to cyclic alternating load or deflection.

A reasonably good correlation has been found in the prediction of cyclic behaviour of cantilever beams subjected to deflection and load control.

The life predictions based upon failure criteria derived from strain-controlled and moment-controlled tests will yield lower and upper bounds, respectively, to the actual values.

Acknowledgements

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References

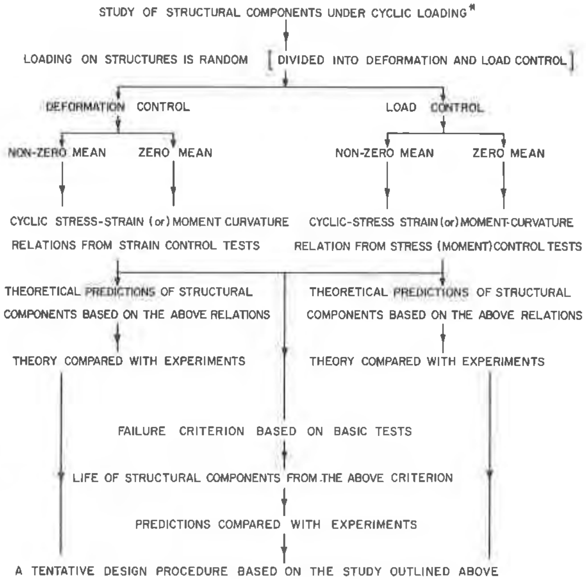
[1] Krishnasamy, S. and Sherbourne, A.N., "Mild Steel Structures Under Reversed Bending", RILEM Symposium on the 'Effects of Repeated Loading on Mild Steel Structures', Mexico City, 1966.

[2] Sherbourne, A.N. and Krishnasamy, S., "Mild Steel Beams Under Cyclic Alternating Deflections", Proceedings of IABSE, Volume 30-I, 1970.

[3] Krishnasamy, S. and Sherbourne, A.N., "Design of Mild Steel Structures Under Low-Endurance Fatigue Conditions", Proceedings of the First International Congress on Structural Mechanics in Reactor Technology, Berlin, 1971.

[4] Topper, T.H., "Cyclic Plastic Loading of Mild Steel", Ph.D. Thesis, Cambridge University, England, 1962.

TABLE I



* LOADING APPLIED TO THE COMPONENTS IS SUCH THAT IT PRODUCES FAILURE DUE TO LOW-CYCLE FATIGUE

TABLE 2 (INDICATES THE WORK CARRIED OUT BY THE AUTHORS)

TYPE OF CONTROL	STRAIN CONTROL		STRESS CONTROL		MOMENT CONTROL	
RELATIONSHIPS AND SYSTEMS	ZERO MEAN	NON-ZERO MEAN	ZERO MEAN	NON-ZERO MEAN	ZERO MEAN	NON-ZERO MEAN
STRESS - STRAIN RELATIONSHIPS	FROM AXIAL "PUSH-PULL" TESTS					
MOMENT - CURVATURE CHARACTERISTICS	FROM PURE BENDING TESTS ALSO FROM "PUSH PULL" TESTS	FROM PURE BENDING TESTS			FROM PURE BENDING TESTS	
LOAD DEFORMATION RELATIONSHIPS	FROM MOMENT - CURVATURE RELATIONS ; COMPARED WITH EXPERIMENTS	FROM MOMENT - CURVATURE RELATIONS ; COMPARED WITH EXPERIMENTS			FROM MOMENT - CURVATURE RELATIONS ; COMPARED WITH EXPERIMENTS	
STRUCTURAL COMPONENTS ANALYSED	ONE, TWO AND THREE "HINGE" SYSTEMS	ONE AND TWO "HINGE" SYSTEMS			ONE "HINGE" SYSTEM	
COMPONENTS FOR WHICH LIFE PREDICTIONS MADE	ONE "HINGE" SYSTEM (COMPARED WITH EXPERIMENTS)				ONE "HINGE" SYSTEM (COMPARED WITH EXPERIMENTS)	

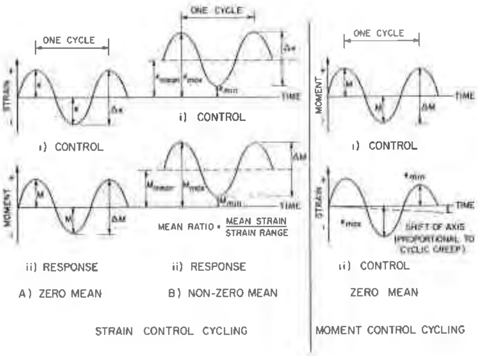


Fig 1. **Loading Scheme: Controlled Strain and Moment**

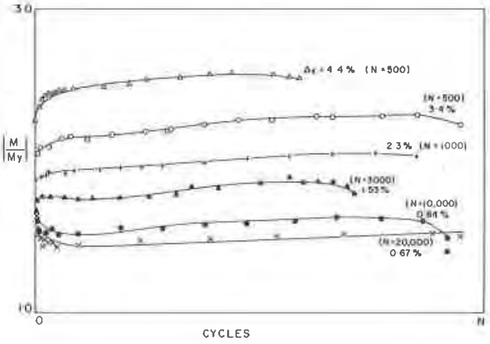


Fig 2. **Cyclic Variation of Moment under Constant (Zero Mean) Strain Range**

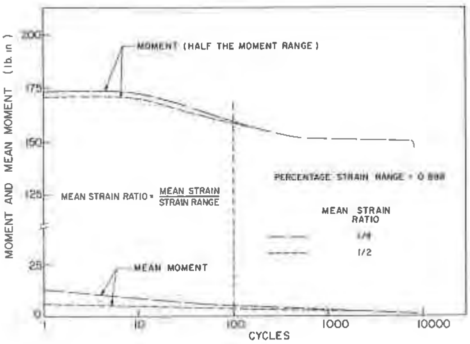


Fig. 3. **Cyclic Variation of Moment and Mean Moment under Constant (Non-Zero Mean) Strain Range (A Typical Test)**

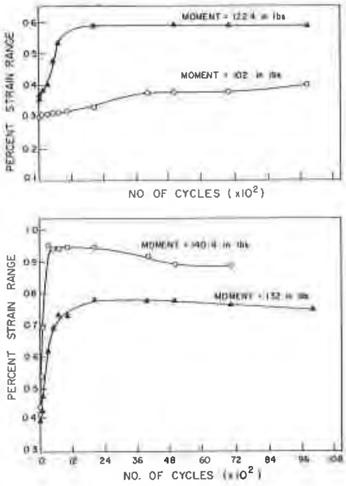


Fig. 4. **Cyclic Variation of Strain Range under Constant (Zero Mean) Moment Range**

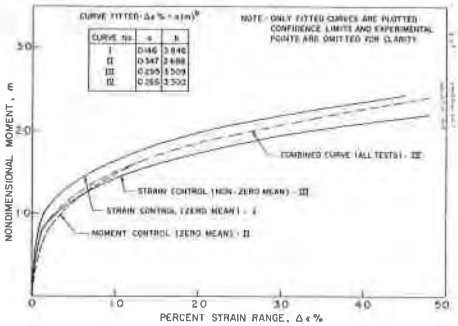


Fig. 5. **Stable Moment-Strain Range Relationships: From Strain and Moment Control Tests Along with Combined Curve**

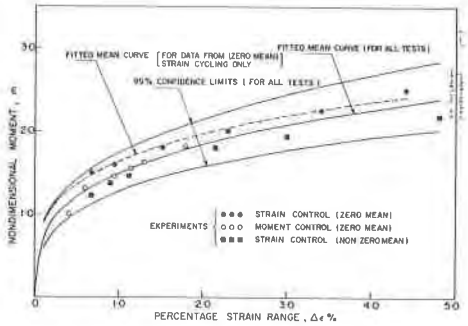


Fig 6. **Stable Moment-Strain Range Relationships: From Constant (Zero Mean) Strain Range Tests Along with Combined Curve**

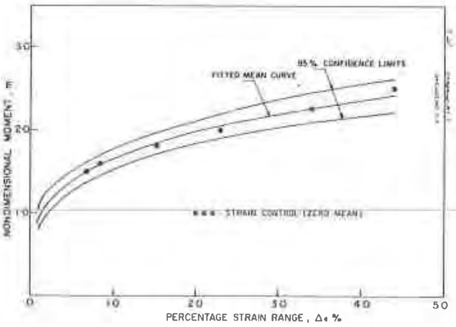


Fig. 7. **Stable Moment-Strain Range Relationship from Constant (Zero Mean) Strain Range Tests: Confidence Limits Included**

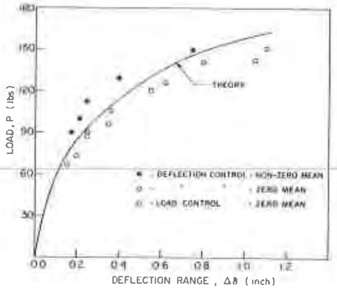


Fig. 8. **Load-Deflection Range Curve for Cantilever Beams: Theory and Experiment**

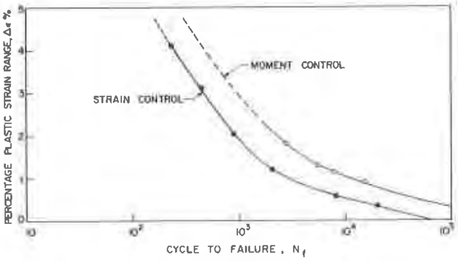


Fig. **9.** **Plastic Strain Range vs Cycles to Failure: From Strain and Moment Control Tests**

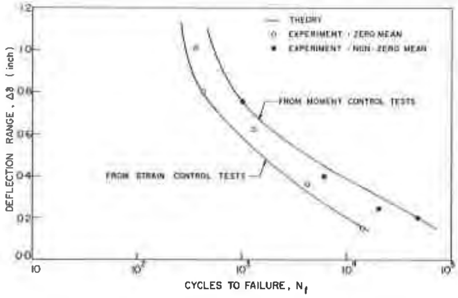


Fig. **10.** **Deflection Range vs Cycles to Failure for Cantilever Beams: Predicted and Experimental Values**

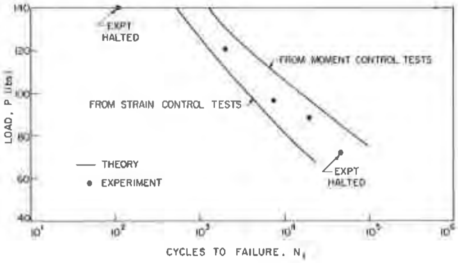


Fig. **11.** **Load vs Cycles to Failure for Cantilever Beams: Predicted and Experimental Values**