SENSITIVITY OF FATIGUE LIFE PREDICTIONS TO APPROXIMATIONS IN THE REPRESENTATION OF METAL CYCLIC DEFORMATION RESPONSE IN A COMPUTER-BASED FATIGUE ANALYSIS MODEL

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SUMMARY

As part of a general program to develop methods allowing results of simple tests to be applied to the fatigue and analysis of complex structures subjected to random loading, a computerized model has been developed to simulate material cyclic deformation and fatigue behaviour. When a non linear analysis for local stress and strain at stress concentrators is incorporated in this model, it accurately predicts the fatigue life of randomly loaded notched components. Input data required for the model are quantitative descriptions of such basic features of material behaviour as memory of prior deformation, cyclic hardening and softening, cyclic mean stress relaxation, and fatigue life. The stepwise calculations utilize a non linear analysis for deformation at geometrical notch roots.

A quantitative evaluation of the assumptions used in the model was obtained from a comparison of the results of several types of simulations with actual specimen results. The prerequisite material information used in the model was obtained from constant strain amplitude tests of smooth specimens and constant load amplitude tests of notched specimens. Both specimen types were machined from a boron steel plate. Tests in which random straining sequences were applied to smooth specimens and random loading sequences to notched specimens were then performed using a small on-line digital computer to control an electro-hydraulic testing machine. Fatigue predictions for the two sample types were then made for these straining or loading sequences using the simulation model.

Several simplifications in the representation of material behaviour were effected and the sensitivity of fatigue predictions to these simplifications evaluated. Smooth specimen evaluations were carried out both with and without cyclic mean stress relaxation and cyclic softening and with two different fatigue damage parameters. Similar simplifications were evaluated in the fatigue simulation for notched specimens and, in addition, both a constant value of the fatigue concentration factor K_f and a value of K_f which was a function of applied nominal stress were used in the non linear "Neuber's Rule" analysis for notch deformation.

Results show that the essential ingredients of a simulation of structural fatigue are:

1. A fatigue damage parameter which incorporates the effect of mean stress.
2. A good mathematical description of the hysteresis loop shape.
3. An accurate deformation solution for the stress raisers.

Other factors which were less important to the simulation are:

1. The simulation of cyclic mean stress relaxation had little effect on life predictions for the random loading and straining patterns examined. However, in some deterministic histories, accurate representation of mean stress relaxation is known to be critical.
2. Elimination of cyclic softening in the material deformation simulation had little effect on the life predictions, provided a steady state description of material deformation was employed.

1.0 INTRODUCTION

Economical design of aircraft, automotive and pressure vessel structures requires that peak operating stresses at stress raisers reach levels where inelastic material deformation and finite fatigue life are encountered. Consequently, much of the research aimed at developing fatigue analysis methods (for this kind of structure) has concentrated on plastic analyses of notches or stress raisers, cyclic plasticity and plastic fatigue of metals. Key features of the response of metals to cyclic plasticity, including memory of deformation history, cyclic hardening or softening and relaxation of mean stress or cyclic creep have been incorporated in mathematical models [1,2,3].

Fatigue analysis models suitable for digital computation have been developed from the combination of such material deformation models with a nonlinear analysis for local stress and strain at a notch or stress raiser and a damage accumulation procedure [1,2,3]. In each of these models, the material simulation programme interacts with the notch analysis to transform inputs in the form of loads or nominal stresses into local stress-strain histories. Fatigue damage is then assessed in terms of this local stress-strain history and the fatigue life predicted. The suitability of this kind of model for design is, of course, conditioned in a large measure by the simplicity and availability of input information.

Although emphasis was placed on accurately describing each of the features of the fatigue process during development of the various fatigue models, one of the more important developments possible once an adequate mathematical fatigue model is available is the assessment of the sensitivity of fatigue life predictions to the accuracy with which various parameters describing the process are represented. A number of approximations often used by designers to describe such parameters are examined in this paper and a quantitative estimate is made of their influence on fatigue life predictions. This paper investigates the sensitivity of fatigue life predictions to a number of approximations often used by designers in describing various features of the fatigue process.

2.0 FATIGUE ANALYSIS MODEL

The fatigue analysis model used in this investigation can be characterized by three major subsections consisting of 1) cyclic deformation behaviour, 2) a notch analysis, 3) cycle counting and damage accumulation. The inter-relation of these three elements is illustrated in Figure 1 as part of the overall analysis logic.

Cyclic deformation phenomena can again be classified into several basic mechanisms (Figure 2) and are modelled by separate algorithms in the computer model.

(a) Hysteresis loop shape is the shape of a closed path of the stress-strain locus during inelastic straining. This shape is described by a nonlinear function, the origin of which is at one point of stress-strain reversal and which passes through the next point of reversal. An adequate mathematical representation of the shape is provided in the computer model by the slopes and lengths of a series of arbitrarily small line segments fitted to the shape of the hysteresis loop.

(b) Memory (Figure 2) during cyclic straining, refers to the tendency of a metal, upon returning to the strain at which a previous stress-strain path was interrupted to follow a continuation of the previous stress-strain path rather than an extension to the current path. In the present model this effect is simulated by a series of rules which manipulate the loop shape line segments.

(c) Cyclic hardening or softening appear as an increase or decrease respectively, in the stress range of the hysteresis loop during cycling between fixed strain limits. To simulate either of these effects, an expression is introduced in the algorithm which changes the coefficients of the hysteresis loop shape equation as inelastic deformation proceeds.

(d) Cyclic mean stress relaxation (Figure 2) appears as a shift towards the zero axis of a hysteresis loop's mean stress during inelastic straining. The mechanism responsible for this stress relaxation will also cause changes in mean strain if the metal is cycled between unequal fixed stress limits. An equivalent effect is attained in the model by means of a forcing function which alters the instantaneous slope of the hysteresis loop in such a manner that the mean stress of a loop under strain controlled conditions will gradually shift towards zero.

A second subsection of the fatigue analysis model utilizes a variation of a notch analysis developed by Neuber [4] in the form

$$\Delta\sigma \; \Delta\varepsilon = (K_f \; \Delta S)^2/E \qquad (1)$$

As detailed in references [1,5], application of this transformation equation is accomplished in the following manner. At any point in the random load history of a notched component, let the nominal stress, local stress and local strain have the values S_1, σ_1 and ε_1, respectively. The computer simulation programme reads in a change in nominal stress, ΔS, computes the right-hand side of eq. (1) and draws a hysteresis loop shape with the origin at $(\sigma_1, \varepsilon_1)$ until the product of $\Delta\sigma . \Delta\varepsilon$ satisfies the equation. This procedure is repeated for each reversal in sequence to transform nominal stress histories into local stress-strain histories.

The third subsection resolves the individual hysteresis loops from the random stress-strain history using a "Rainflow" counting procedure [6], and sums fatigue damage. Two damage parameters were evaluated in this work. One is the Coffin-Manson plastic strain versus life parameter [7,8] and the other uses a parameter $\sigma_{max} . \varepsilon_a$ [9] which accounts for the presence of mean stress. In the smooth specimen fatigue simulations, damage was summed linearly for each parameter and failure predicted when the summations reached unity. Notched component damage was evaluated in a similar manner after the local stress-strain behaviour was calculated using the notch algorithm.

3.0 EXPERIMENTAL PROGRAMME

Computer simulation fatigue life predictions were compared with actual test data for high strength "Boron" steel with the chemical composition given in Table I. The as-received condition of the steel was equivalent to SAE 10B22 modified steel, quenched and tempered to R_c = 35 hardness. The ½ inch plate was retempered at 1020°F for three hours to an average core hardness of R_c = 21. Smooth and notched plate specimens were machined with the axis of the specimens parallel to the rolling direction of the plate. Post-machining treatment consisted of high speed grinding and diamond paste polishing in the longitudinal direction.

A constant strain amplitude, axially applied load, testing programme yielded monotonic and cyclic properties. Figure 3 superimposes the monotonic and cyclic stress-strain curves, and Figures 4 and 5 depict life data.

Input strain and load histories were synthesized using a pseudo-random number generation technique. This ensured that the details of the history imposed on the specimens could be duplicated exactly in the simulation programme without recourse to large unwieldy data tapes. Actual boron steel specimens were tested on an MTS servo-hydraulic testing machine controlled by a small on-line computer, which given certain bounds and the pseudo-random number sequence, selected strain or load levels at random and commanded the testing machine to impose the levels. A segment of a typical strain history, imposed on a smooth specimen is shown in Figure 6, along with the load response. Each specimen had a similar strain time history imposed on it, but the upper and lower bounds or limits of the history were altered for each test. Plate specimens were tested in the same fashion, except that nominal load was used as the control variable rather than strain. Fatigue life was measured as the total number of random load levels exerted prior to specimen separation.

By transferring the same pseudo-random number sequence and strain or load bounds to the computer simulation model, the identical strain or load histories were imposed on the model. Fatigue life predictions produced by the model were then compared to the actual test results.

4.0 FATIGUE LIFE PREDICTIONS WITH SIMPLIFIED VERSIONS OF THE FATIGUE ANALYSIS MODEL

A complete material model which accounted for the change in loop shape with cyclic softening and allowed for mean stress relaxation was first used to simulate the random strain history tests of smooth specimens. Calibration of the various features of the computer model was effected using test data from the constant amplitude strain-controlled test series. Hysteresis loop shapes were measured during the constant amplitude fatigue tests and the half cycles were superimposed on a plot of stress range versus plastic strain range measured from the point of last reversal of the stress-strain locus. The loop shapes, although measured from different tests, were selected at points in each test according to a particular deformation or fatigue damage criteria. That is, the shapes might all be measured at the same fraction of fatigue life or at equal values of accumulated plastic strain. Figure 7 illustrates such a superposition for boron steel loop shapes at cumulative plastic strain values of 10.0. An arbitrary loop shape equation is depicted in Figure 8. Cyclic softening was accomplished in the model by changing the coefficients of the equation according to a function of the cyclically accumulated plastic strain, the parameter which served as the selection criteria for the initial superposition of the loop shapes. The coefficient of the forcing function which determines the rate of mean stress relaxation [1,3] was selected from a calibration of the model with a test containing actual mean stress relaxation effects. Prediction results for the smooth specimen model which contained the above features are presented pictorially in Figure 9 and tabulated in Line A of Table II for both the plastic strain-life and the $\sigma_{max} \cdot \epsilon_a$ - life damage parameters. Predictions for the latter parameter are within a factor of two, while the former is slightly less accurate probably due to the lack of any allowance for mean stress.

With the removal of the cyclic softening behaviour, that is, by using the steady-state loop shape measured at ½ fatigue life, no significant deterioration in life predictions was incurred (Table II, Line C). If, however, the initial loop shape measured in the first few cycles is used throughout the simulation, a definite deterioration in predicted life occurs (Table II, Line D).

Inclusion or exclusion of cyclic mean stress relaxation caused indifferent changes in predicted fatigue life. This is partially attributed to the characteristics of the random strain-time history. Roughly equal numbers of large, intermediate and small amplitudes are present, and any mean stresses imposed by a change from a large to a small cycle are quickly washed out by other cycles which still contain relatively large amounts of plasticity. Given strain-time histories with a disproportionately small number of high level cycles, a greater importance would be placed on the cyclic mean stress relaxation feature.

The simulations of the randomly loaded notched plates show a corresponding sensitivity to the basic material deformation phenomena. Figure 10 and Table III demonstrate good predictions for both fatigue damage parameters. The plastic strain-life criterion which ignores mean stress effects again deteriorates for long life tests. Exclusion of cyclic softening and cyclic mean stress relaxation too, yield little difference in life predictions (Table III).

For the simulations of the randomly loaded notched plates, the description of K_f was found to have the greatest effect on fatigue life predictions. Methods of describing this fatigue concentration factor have previously centered on the use of the long life value of K_f (generally assumed equal to K_t) as depicted in Figure 11. However, the data of Figure 11 also shows a variation of the concentration factor with fatigue life or nominal stress. Both the assumption of K_f equal to K_t and a value of K_f calculated as a function of the change in nominal stress, ΔS (Figure 12) were used to predict the fatigue life of randomly loaded notched plates. The results show that a determination of K_f without regard to the nominal stress range ΔS for each half cycle, will introduce large errors in predicted life. If the long life value of K_f is used for all stress ranges, life predictions become very conservative (Table III, Lines D and E). With the use of a value of K_f which is a function of ΔS, however, a significantly improved simulation is achieved (Table III, Line A).

5.0 CONCLUSIONS

Simulation results with several degrees of accuracy in estimation of cyclic material properties and the effect of the notch stress concentration show that the essential ingredients of a simulation of structural fatigue are:

1. A fatigue damage parameter which incorporates the effect of mean stress.
2. A good mathematical description of the hysteresis loop shape.
3. An accurate deformation solution for stress raisers (a variable fatigue concentration factor K in Neuber's Rule was used in the example in this paper).

Other factors which were less important to the simulation are:

1. The simulation of cyclic mean stress relaxation had little effect on life predictions for the random loading and straining patterns examined. However, in some deterministic histories, accurate representation of mean stress relaxation is known to be critical.
2. Elimination of cyclic softening in the material deformation simulation had little effect on the life predictions, provided a steady-state description of material deformation was employed.

6.0 ACKNOWLEDGEMENTS

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7.0 NOMENCLATURE

$\Delta\varepsilon$	true strain range
$\Delta\varepsilon_e$, $\Delta\varepsilon_p$	true elastic and true plastic strain range
$\Delta\varepsilon/2$, ε_a	true strain amplitude
$\Delta\sigma$	true stress range
$\Delta\sigma/2$	true stress amplitude
σ_{max}	true maximum stress per cycle or reversal
ΔS	nominal stress range
Δe	nominal strain range
K_t	theoretical stress concentration factor
K_f	fatitue concentration factor
R	number of reversals or half cycles
R_f	number of reversals to failure
$\Sigma\Delta\varepsilon_p$	summation of the magnitudes of all prior plastic strain
E	modulus of elasticity
R_c	Rockwell "C" hardness number

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TABLE I: CHEMISTRY OF HEAT OF BORON STEEL (% WT.)

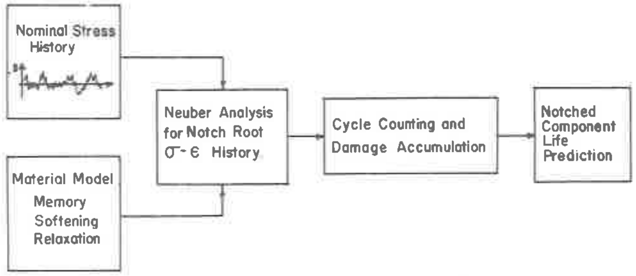
C	Mn	P	S	Si	Ni	Cr	Mo	Cu	Aℓ	Ti	Zr	B	Fe
0.20	1.32	0.009	0.010	0.27	0.01	0.02	< 0.01	0.01	0.05	0.050	< 0.005	0.00096	Remainder

TABLE II: SMOOTH SPECIMEN LIFE COMPARISONS

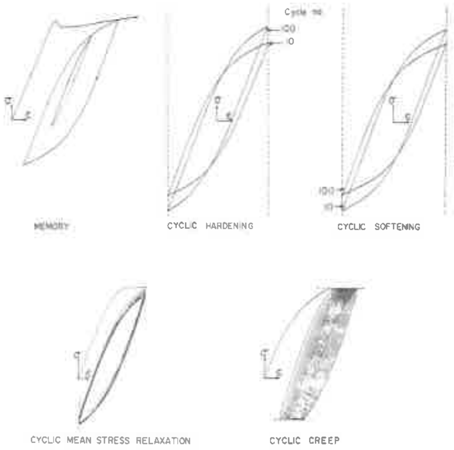
Simulation Type	Ratio of Predicted Life to Actual Life							
	$\sigma_{max}\cdot\varepsilon_a$ Damage Parameter Test Number				Plastic Strain-Life Damage Parameter Test Number			
	1	2	3	4	1	2	3	4
(A) With Cyclic Softening With Relaxation	0.88	1.21	1.45	0.54	1.19	1.50	1.69	0.39
(B) With Cyclic Softening Without Relaxation	0.88	1.20	1.41	0.47	1.19	1.50	1.67	0.34
(C) Without Cyclic Softening (i.e., Steady State Loop Shape) With Relaxation	0.90	1.23	1.43	0.47	1.18	1.48	1.65	0.34
(D) Without Cyclic Softening (i.e., Initial Loop Shape) With Relaxation	0.60	0.81	0.98	0.33	1.47	2.73	2.33	0.13
Actual Life	8,791	13,476	38,592	1,148,860	8,791	13,476	38,592	1,148,860

TABLE III: NOTCH SPECIMEN LIFE COMPARISONS

Simulation Type	Ratio of Predicted Life to Actual Life									
	$\sigma_{max}\cdot\varepsilon_a$ Damage Parameter Test Number					Plastic Strain-Life Damage Parameter Test Number				
	1	2	3	4	5	1	2	3	4	5
With Cyclic Softening With Relaxation With Variable K_f	2.39	2.48	1.57	1.41	1.47	2.87	2.27	0.685	1.07	0.364
With Cyclic Softening Without Relaxation With Variable K_f	2.38	2.37	1.44	1.37	1.54	2.84	2.32	0.710	1.05	0.380
Without Cyclic Softening (i.e., Steady State Loop Shape) With Relaxation With Variable K_f	2.37	3.01	1.52	1.48	1.61	2.75	2.12	0.665	0.985	0.349
With Cyclic Softening With Relaxation With Constant K_f	0.40	0.375	0.284	0.293	0.238	0.674	0.490	0.305	0.335	0.197
Without Cyclic Softening (i.e., Steady State Loop Shape) Without Relaxation With Constant K_f	0.411	0.376	0.290	0.296	0.246	0.657	0.477	0.292	0.327	0.192
Actual Life	7,807	33,720	672,432	242,900	2,234,000	7,807	33,720	672,432	242,900	2,234,880



1 CONCEPTS INCORPORATED IN A COMPUTER SIMULATION OF FATIGUE



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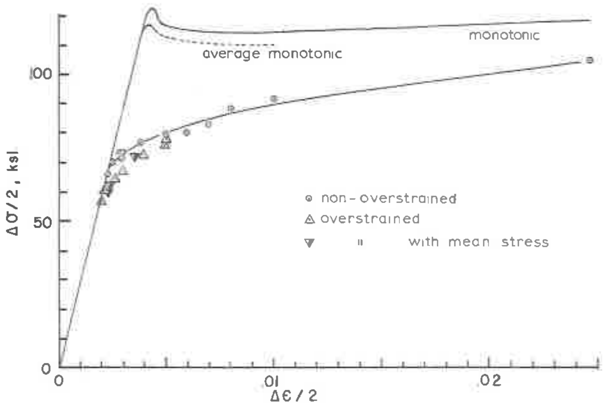
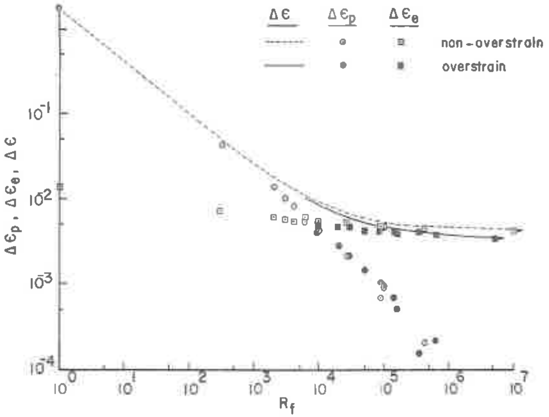
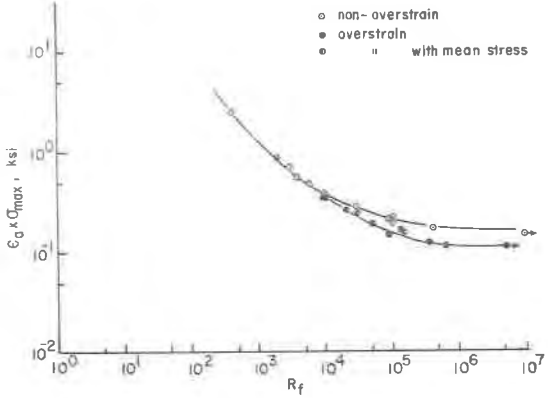


3 MONOTONIC AND CYCLIC STRESS-STRAIN CURVES OF BORON STEEL



4 STRAIN LIFE CURVES FOR BORON STEEL



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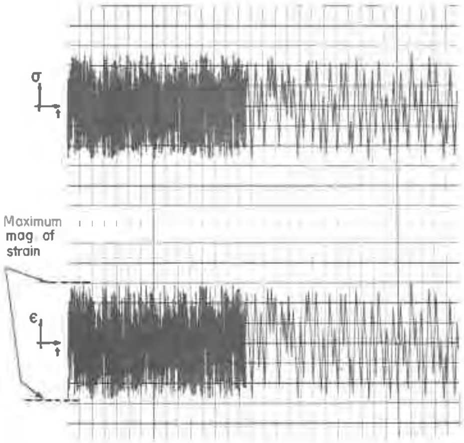

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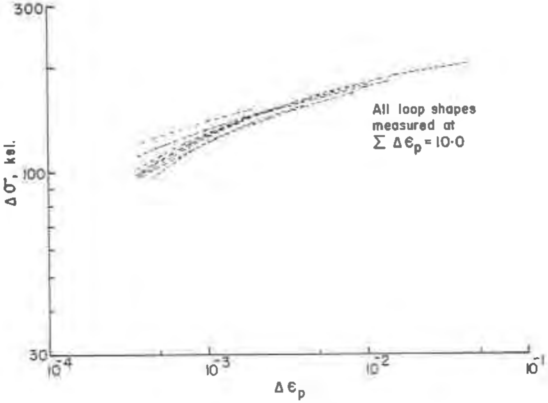
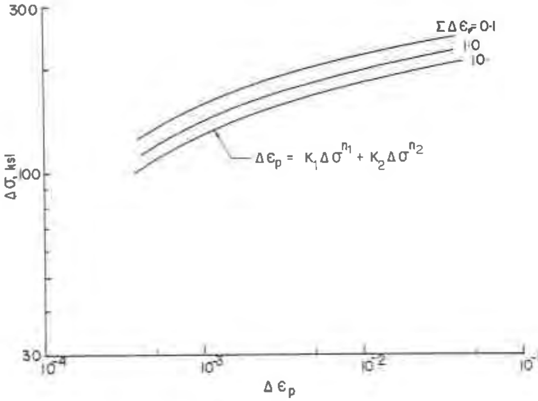
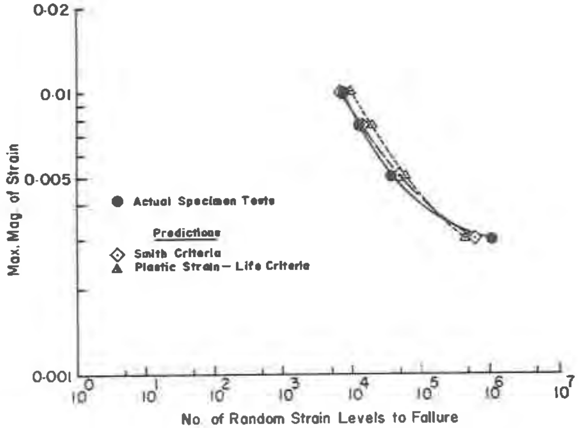


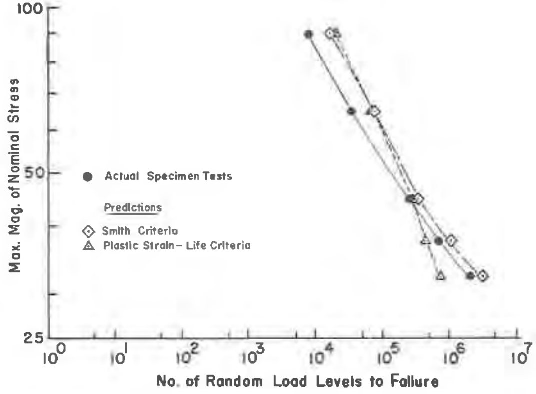
7 STRESS-STRAIN LOOP SHAPES FOR VARIOUS STRAIN AMPLITUDES



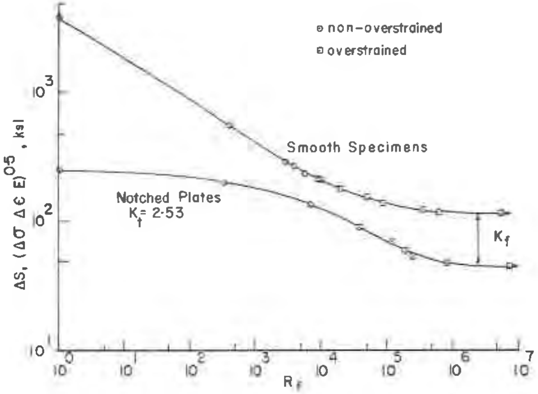
8 ANALYTICAL REPRESENTATION OF STRESS-STRAIN LOOPS FOR BORON STEEL



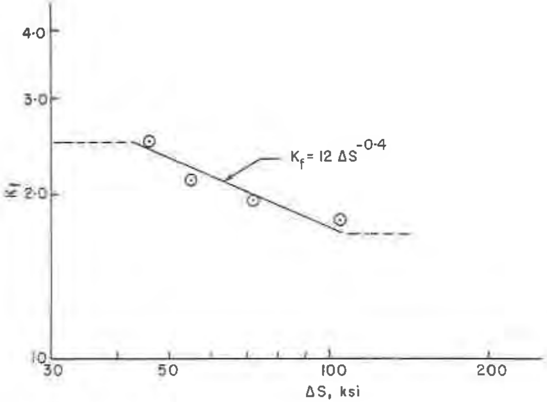
9 MAXIMUM MAGNITUDE OF STRAIN LEVEL VS. LIFE PLOT OF ACTUAL TEST RESULTS AND COMPUTER PREDICTIONS OF BORON STEEL SMOOTH SPECIMENS



10 MAXIMUM MAGNITUDE OF NOMINAL STRESS LEVEL VS. LIFE PLOT OF ACTUAL TEST RESULTS AND COMPUTER PREDICTIONS OF BORON STEEL NOTCHED PLATES



11 SMOOTH AND NOTCHED SPECIMEN FATIGUE LIFE RESULTS FOR BORON STEEL



12 FATIGUE NOTCH FACTOR AS A FUNCTION OF APPLIED NOMINAL STRESS.