A FINITE ELEMENT MESH GENERATION PROGRAM FOR ARBITRARY TWO- AND THREE-DIMENSIONAL STRUCTURES

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SUMMARY

The general mesh generation program BERGEN was conceived to meet an increasing demand to analyse the complex engineering structures associated with nuclear plant design. Two-dimensional mesh generation programs have been previously developed for axi-symmetric structures using isoparametric mapping techniques. The program described here extends the mapping technique to produce three-dimensional meshes for non-axisymmetric structures.

The basic assumption of the program is that an arbitrary structure can be thought of as an assemblage of discrete macroblocks. For convenience the shape of the macroblock is taken to be quadrilateral for two-dimensional structures and hexahedral for three-dimensional structures. In each case the macroblock can have sides or surfaces which vary in a linear, quadratic or cubic manner. By specifying selected nodal co-ordinates on the boundaries of the macroblocks and employing the isoparametric mapping technique the quadrilaterals and hexahedrals are transformed into a unit square and a unit cube respectively. Knowing the number of elements and the element type in a macroblock, interpolation of the scaled macroblock is performed and related to the actual macroblock in order to determine the intermediate nodal co-ordinates.

To provide for non-uniform meshes the interpolation of the scaled macroblock can vary in a linear, quadratic or any ordered manner. The element topology for each macroblock is deduced from prior knowledge of the element type and the number of elements comprising the mesh. An automatic facility is included in the program to join macroblocks at a common edge or surface. With careful discretisation of a structure into macroblocks a complete mesh can be generated using most of the finite elements available in the BERSAFE system of structural analysis.

Examples are given for two- and three-dimensional problems showing the capabilities of the program. The time taken to produce the mesh for the complex bifurcation joint and to complete a finite element stress analysis run took less than one working week. By existing methods the time taken by an experienced user would have been well in excess of four weeks.

1. INTRODUCTION

The finite element technique has become justifiably popular for the static and dynamic analysis of general structures. Using the latest generation of computers, with their vast available store, research workers have been able to handle far more complicated engineering structures than was possible a few years ago. Consequently, much time and effort has been spent in developing suitable elements to analyse these structures. However, the problems of formulating the necessary data has received little attention. To overcome these problems the general mesh generation program BERGEN [1] was designed such that:-

1. it should be easy to use.
2. it should be capable of generating meshes for two and three-dimensional structures using most of the elements available in BERSAFE [2].
3. it should be possible to allow for non-uniform meshes.
4. element numbering should lead to computational efficiency.
5. it should be possible to adequately represent curved edges or surfaces.
6. a nodal field parameter be available to the user.
7. intervention by the user should be kept to a minimum.

By assuming that a structure can be discretised into macroblocks, all the above requirements have been fulfilled. Figure 1 shows the type of finite elements, all available in BERSAFE, which can be automatically generated using BERGEN. These elements fall into two categories: the first assumes a quadrilateral macroblock for two-dimensional analyses and the second assumes an hexahedral macroblock for three-dimensional analyses. Non-uniform meshes are allowed within a macroblock and an automatic facility is included for connecting adjacent macroblocks together.

2. THEORY

Using the method given by Zienkiewicz et al. [3], isoparametric transformations are assumed which relate the actual geometry of the macroblocks to a scaled unit macroblock. Details of the isoparametric mapping of co-ordinates is given by Ergatoudis et al. [4]. To illustrate the method consider the two-dimensional macroblock of EP16 elements shown in Figure 2. The transformation of the (x, y) cartesian co-ordinates to the scaled co-ordinates (ξ, η) is given by

$$x = a_1 + a_2\xi + a_3\eta + a_4\xi\eta + a_5\xi^2 + a_6\eta^2 + a_7\xi^2\eta + a_8\xi\eta^2$$

$$y = a_9 + a_{10}\xi + a_{11}\eta + a_{12}\xi\eta + a_{13}\xi^2 + a_{14}\eta^2 + a_{15}\xi^2\eta + a_{16}\xi\eta^2$$

or in matrix form $\{u\} = [P]\{\alpha\}$ ...(1)

Substituting into eq. (1) the cartesian co-ordinates (x_1, y_1; x_2, y_2; . . . x_8, y_8) and the corresponding scaled co-ordinates (ξ_1, η_1; ξ_2, η_2; . . . ξ_8, η_8) gives in matrix form

$$\{\delta\} = [C]\{\alpha\} \quad ...(2)$$

Solving eq. (2) for $\{\alpha\}$ gives

$$\{\alpha\} = [C]^{-1}\{\delta\} \quad ...(3)$$

and the cartesian co-ordinates from eq. (1) in the form

$$\{u\} = [P]\,[C]^{-1}\,\{\delta\} \qquad \ldots(4)$$

If $\{\phi\}$ is either the x or y co-ordinate and $\{\bar{\phi}\}$ the cartesian nodal values of either the x or y co-ordinates respectively, then eq. (4) becomes:-

$$\{\phi\} = [P]\,[C]^{-1}\,\{\bar{\phi}\} \qquad \ldots(5)$$

which for the above example can be written as:-

$$\{\phi\} = [1,\ \xi,\ \eta,\ \xi\eta,\ \xi^2,\ \eta^2,\ \xi^2\eta,\ \xi\eta^2] \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & -1 & 0 & 0 & 4 & 0 & 0 & 0 \\ -3 & 0 & 0 & -1 & 0 & 0 & 0 & 4 \\ 5 & -1 & -3 & -1 & -4 & 4 & 4 & -4 \\ 2 & 2 & 0 & 0 & -4 & 0 & 0 & 0 \\ 2 & 0 & 0 & 2 & 0 & 0 & 0 & -4 \\ -2 & -2 & 2 & 2 & 4 & 0 & -4 & 0 \\ -2 & 2 & 2 & -2 & 0 & -4 & 0 & 8 \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{Bmatrix} \qquad \ldots(6)$$

Obviously, a field parameter such as temperature t can be defined by eq. (5) with corresponding nodal values for $\bar{t}$.

Eq. (5) gives a unique mapping of the actual macroblock to the scaled macroblock. From the number of elements in the scaled ξ, η co-ordinate directions of a uniform mesh and the element type, the actual nodal co-ordinates can be determined by substituting the linearly interpolated values of (ξ, η) into eq. (5). For a non-uniform mesh quadratic or higher ordered interpolation is carried out along the ξ or η co-ordinate directions indicated by the user. Knowing the number of elements in the ξ, η co-ordinate directions the element topology can be deduced. The element and node numbering progresses along the scaled side of the macroblock with the minimum number of elements thus producing the minimum bandwidth in a front solution technique Irons [5].

To extend the analysis to other elements requires the definition of a suitable polynomial matrix [P] and the inversion of the resulting [C] matrix given in eq. (4). Clearly, the polynomial assumption given by eq. 1 allows the the boundaries of the macroblocks to assume at most a parabolic shape. For a mesh of EP8 elements the polynomial matrix is:-

$$[P] = [1,\ \xi,\ \eta,\ \xi\eta] \qquad \ldots(7)$$

which allows only a linear boundary to the macroblocks.

For the EZ60 element, which can accommodate parabolically curved surfaces, an arbitrary hexahedral macroblock is scaled to a unit cube as shown in Figure 3. In this case $\{\phi\}$ of eq. (5) is either the x, y or z co-ordinate and $\{\bar{\phi}\}$ the actual nodal values of either the x, y or z co-ordinates. The polynomial matrix [P] is of the form:-

$$[P] = [1,\ \xi,\ \eta,\ \zeta,\ \xi\eta,\ \xi\zeta,\ \eta\zeta,\ \xi\eta\zeta,\ \xi^2,\ \eta^2,\ \zeta^2,\ \xi^2\eta,\ \xi^2\zeta,\ \eta^2\zeta,\ \xi\eta^2,\ \xi\zeta^2,\ \eta\zeta^2,\ \xi^2\eta\zeta,\ \xi\eta^2\zeta,\ \varepsilon\eta\zeta^2] \qquad \ldots(8)$$

The FS12 element uses the same polynomial matrix as the EP16 elements. However, $\{\phi\}$ of eq. 5 is either the x, y or z co-ordinate and $\{\bar{\phi}\}$ the actual nodal values of either the x, y or z co-ordinates.

A tagging system is employed on the quadrilateral and hexahedral macroblocks which informs the program that a particular edge or surface forms a common boundary with an adjacent macroblock. Appropriate action is then taken on tagged edges or surfaces to overwrite the current node number if that node has previously been defined.

3. EXAMPLES

3.1 Pressure Vessel

The first example, shown in figure 4, is of a torispherical head of an axisymmetric pressure vessel. The vessel is discretised into three quadrilateral macroblocks each with a uniform mesh of EP16 elements. The macroblock input data necessary to completely define the geometry of the torispherical head is indicated on Figure 4. Automatic combination of the macroblocks is performed from the appropriate tagging data input with each macroblock.

3.2 Hole in a Plate

The second example illustrates the use of non-uniform meshes within a macroblock. The geometry of the plate is shown in Figure 5 together with the macroblock discretisation and the nodes to be read as data. A 4 x 4 mesh of EP16 elements is prescribed over each macroblock such that the elements of the first and second macroblock are closer to the hole in the plate. Note that the EP16 elements cannot exactly represent the circular hole. However the errors between the exact and generated co-ordinates of the nodes above the hole are negligible.

3.3 Bifurcation Joint

The third example shows the use of the hexahedral macroblocks for the analysis of non-axisymmetric structures. To save on computer time advantage is taken of the double symmetry of the bifurcation joint and only one quarter is analysed with the EZ60 elements. The discretisation into the five macroblocks together with the input node data, is shown in Figure 6 and the resulting mesh in Figure 7. The advantage of including an automatic facility to join the macroblocks is immediately evident with this problem. It may be necessary to adjust the co-ordinates of the nodes lying on the interpenetration of the two branches since a macroblock of EZ60 elements is only capable, at most, of modelling a smooth parabolically varying surface. Also, it should be noted that one macroblock cannot exactly represent a semi-circulator geometry. This fact was violated in this problem in order that the mesh shown in Figure 7 would be more meaningful.

3.4 Cooling Tower

The final problem considered is a cooling tower analysed with four macroblocks of the FS12 elements. The macroblock discretisation is shown with the resulting mesh in Figure 8. Note that two macroblocks in a semi-circle should be taken as the minimum discretisation for adequate geometric representation.

4. CONCLUSIONS

The limitation of using isparametric mapping is that certain curves, for example a circle, cannot be modelled exactly. However, in most cases the errors produced will be negligible. The likelihood of producing errors in the input data cannot be overlooked. Serious errors will be detected by the program. However, non-serious errors can remain undetected. Hence it is essential that all generated meshes should be drawn using a facility such as BERPLOT [6].

Discretisation into macroblocks is as much a skill as the discretisation of a structure into finite elements. Experience in both cases is the best guide.

ACKNOWLEDGEMENT

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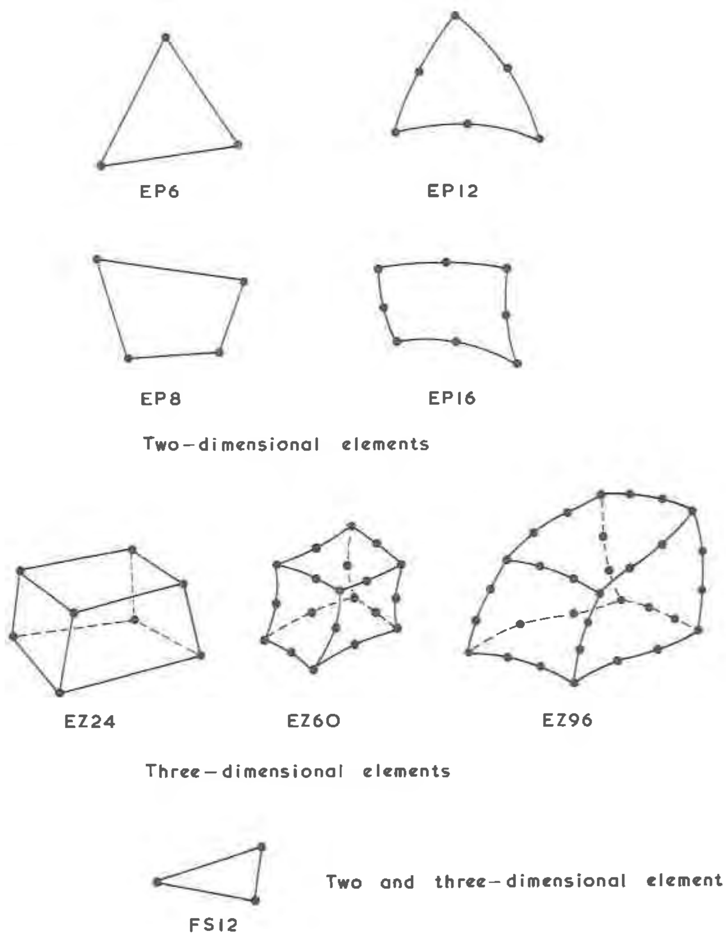


Figure 1. Elements generated by BERGEN.

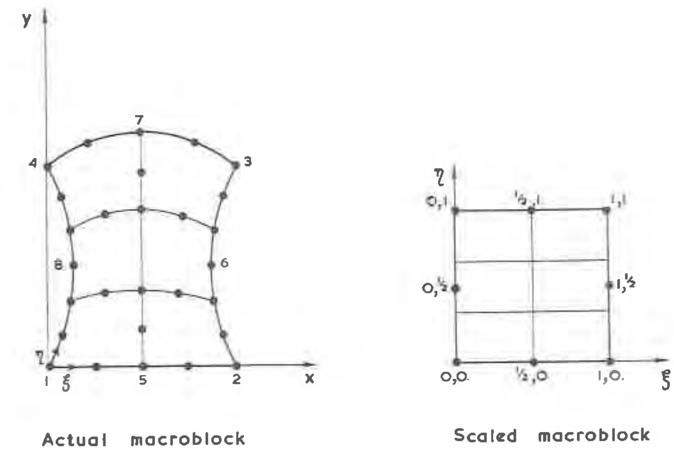
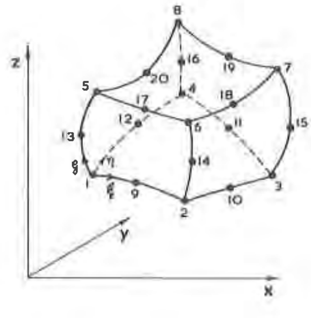
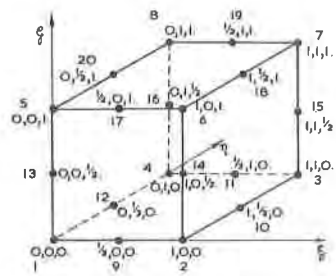


Figure 2. Two-dimensional transformation of a macroblock of EP16 elements.

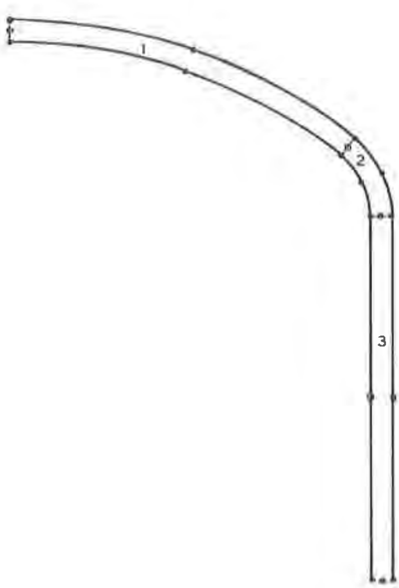


Actual macroblock



Scaled macroblock

Figure 3. Three–dimensional transformation of a macroblock of EZ60 elements.



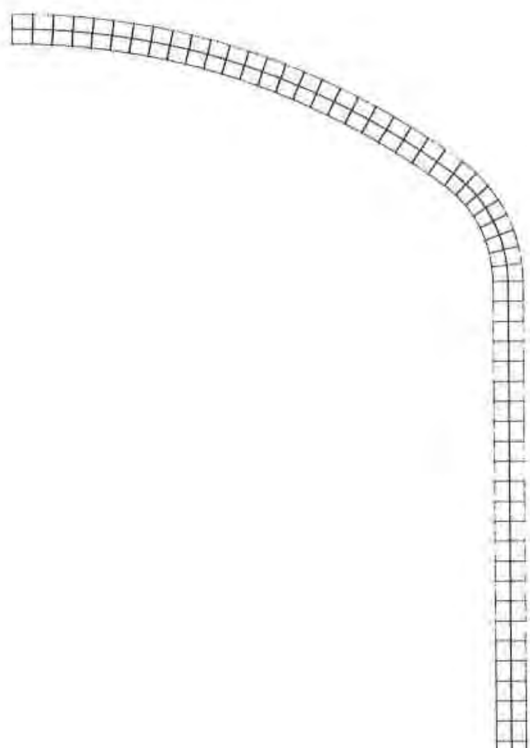


FIGURE 4, PRESSURE VESSEL

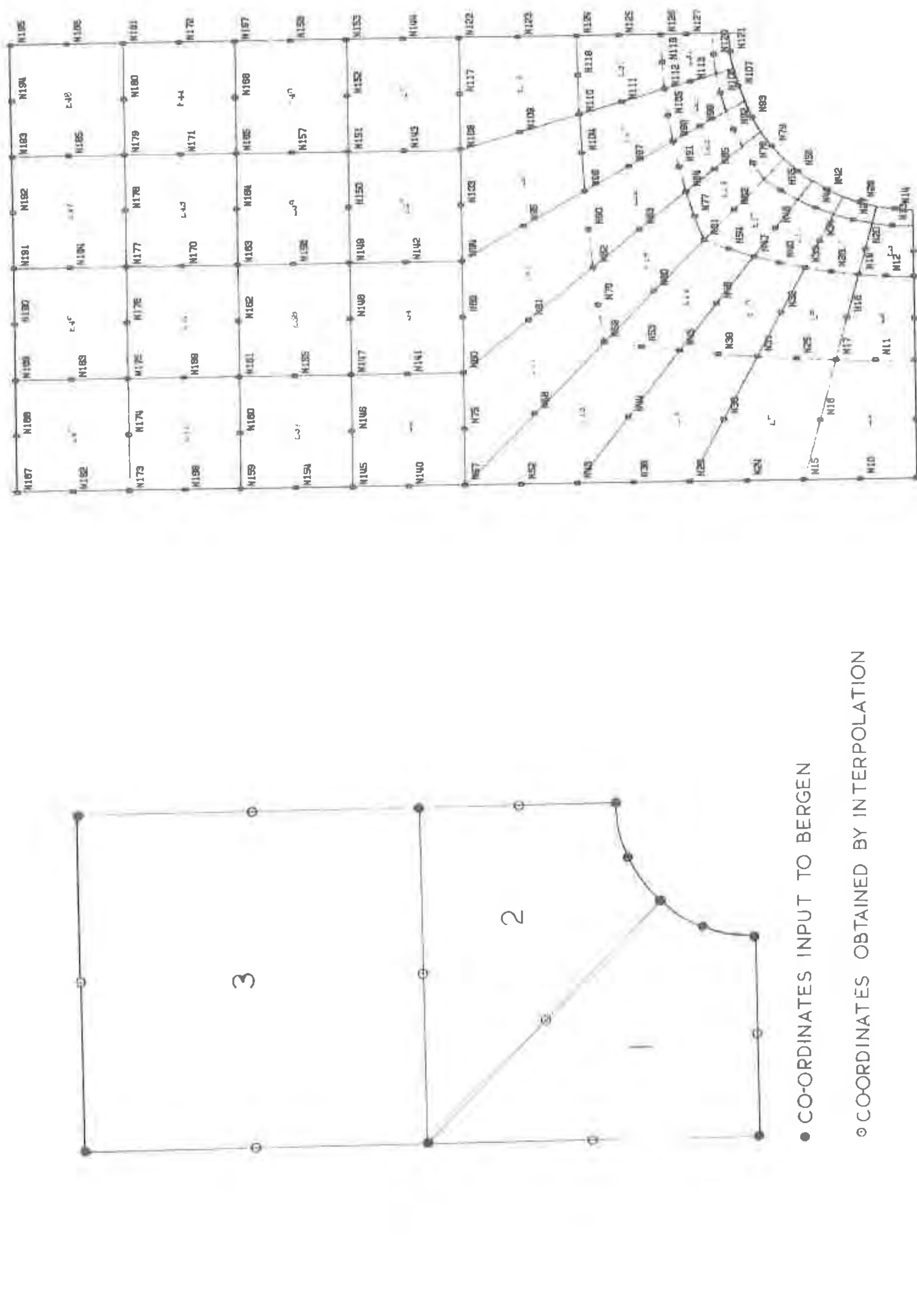
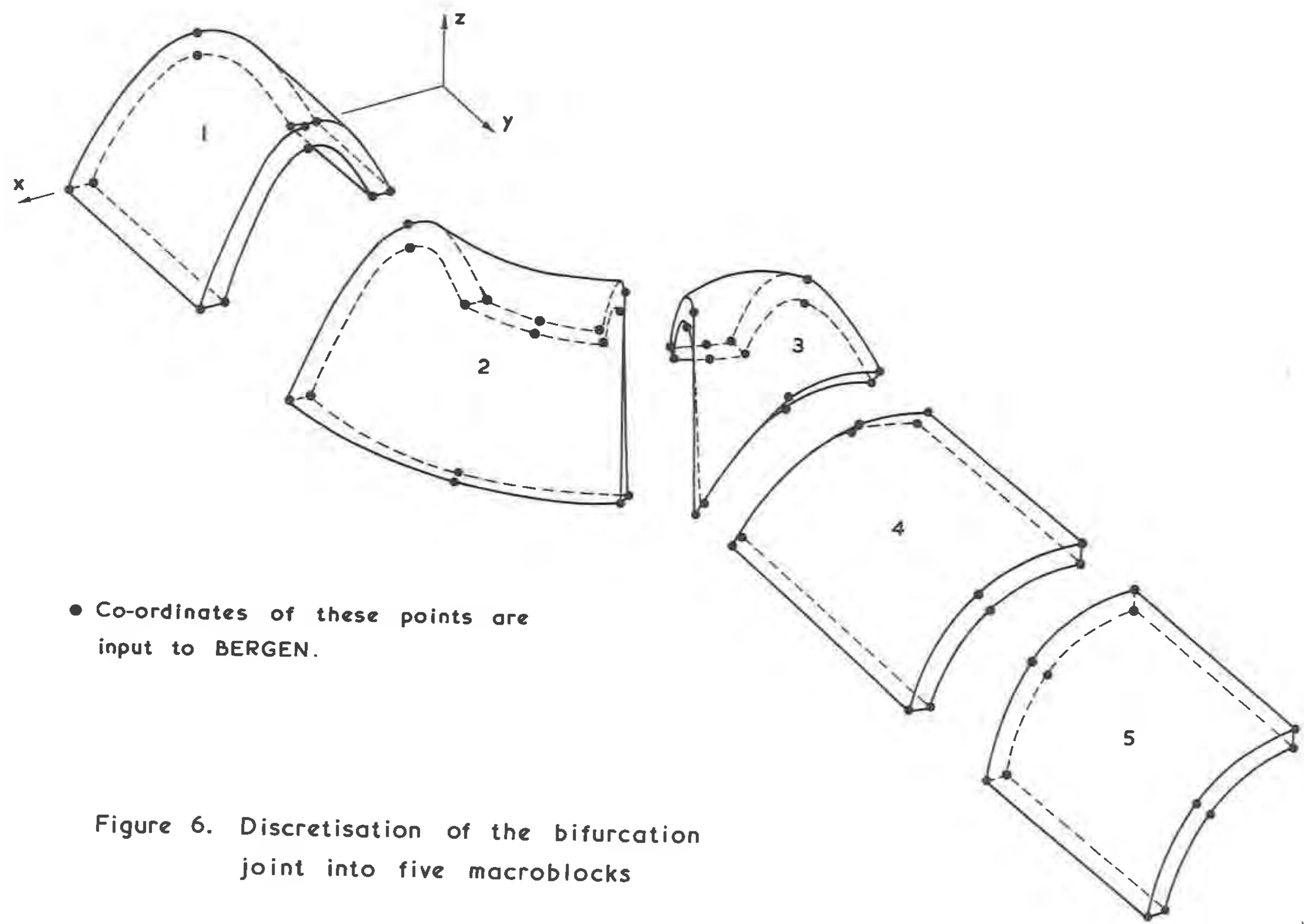


FIGURE 5, HOLE IN A PLATE



● Co-ordinates of these points are input to BERGEN.

Figure 6. Discretisation of the bifurcation joint into five macroblocks

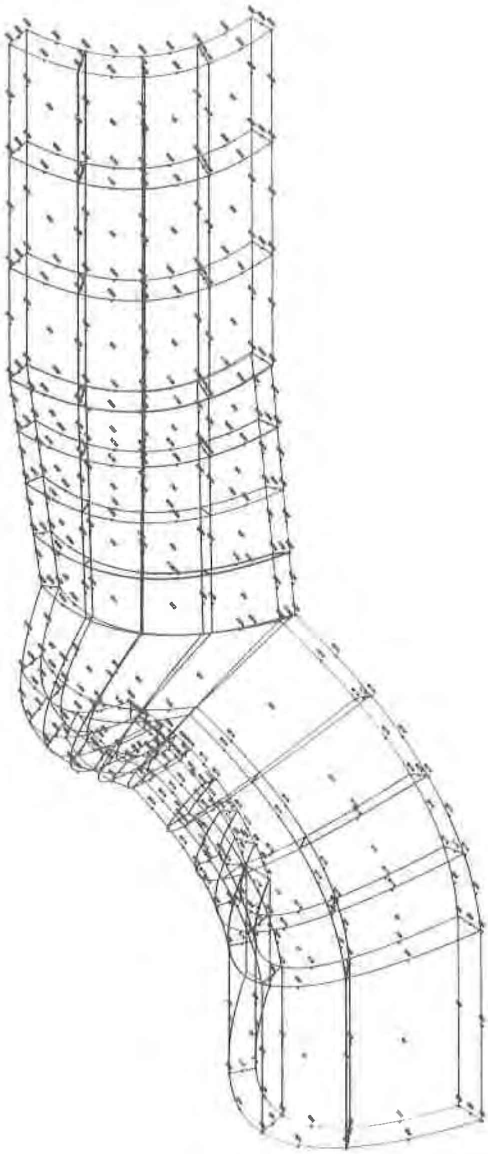
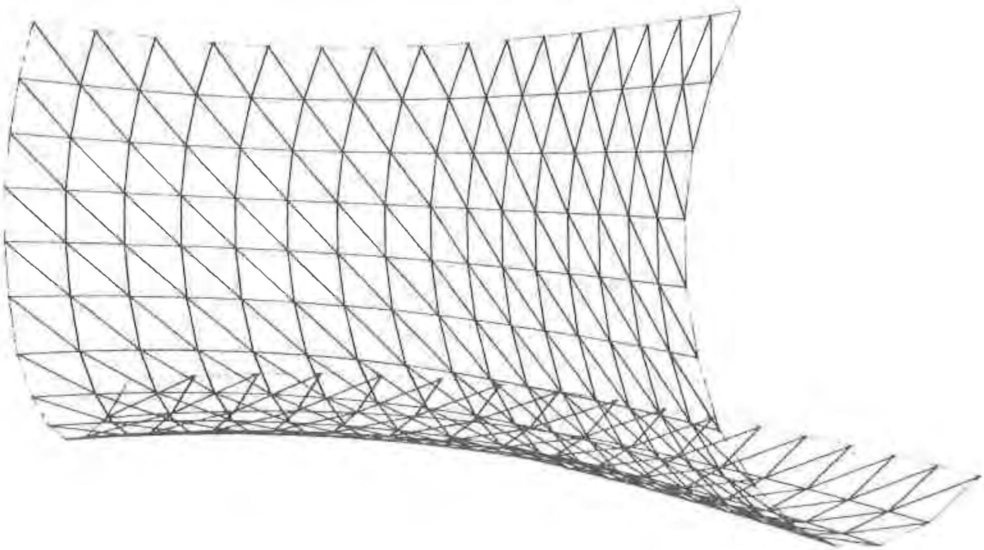


Figure 7, Bifurcation Joint



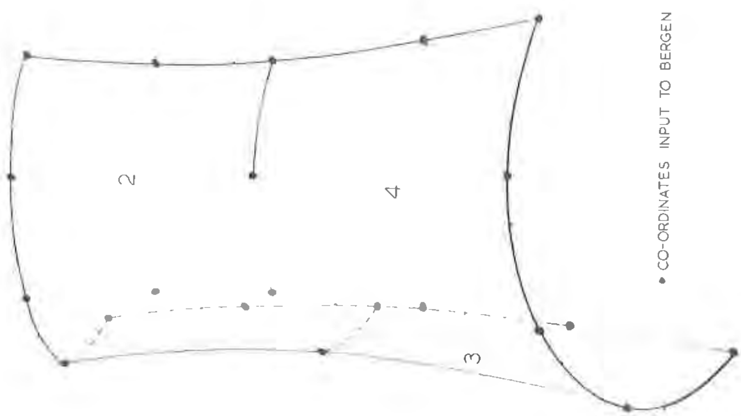


FIGURE 8, COOLING TOWER