

CHARACTERISTIC PARAMETERS OF RESPONSE OF PLATES IN CONTACT

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SUMMARY

Problems involving the contact of flat surfaces occur frequently in engineering practice. Typical examples are flanged connections of pipes, foundations, layered soil, etc. Available methods of solutions generally neglect, usually without justification, the complicated character of stress and displacement fields occurring in structures near the contact region. Components in contact can be often approximated by thick plates of corresponding shape. However the knowledge of the real stress and deformation states in real thick plates in contact is still very limited.

The paper presents results of an experimental study of the response of circular plates of various thicknesses to symmetrical punch loads.

The purpose of the study was to investigate the actual response of circular plates in contact to loads, to establish functional relations between the load, the geometric and material parameters of plates in contact and the geometric parameters of response, as contact area and axial deformation, etc. The more general objective was to determine which parameters are significant and which might be neglected in the development of theories for flat contact problems.

The following parameters of response were considered of major interest from both theoretical and engineering points of view: the contact area, the contact stress distribution, the deformation of face surfaces, the deformation of end surfaces, and the change of overall dimensions.

The paper presents the theory and technique of investigation, the typical relations established, and the discussion of results.

Results show that the diameter of contact boundary is a nonlinear function of loads when loads are below a certain value. Above this value the contact area remains practically constant in the load range higher than the nonlinear range. In this range the contact area depends essentially on the ratio (plate thickness)/(punch size), and does not depend on the plate diameter if the plate diameter is above a certain value which depends on the plate thickness and the punch size. The contact area depends on Poisson's ratio but not on Young's modulus of the contacting bodies if they are made of the same material. Some parameters of the deformation of the contacting bodies are proportional to the applied load.

Functional relations between the load, the major geometric and material parameters and the major parameters of response have been established. These relations present a picture of physical phenomenon under observation given in a simple form, which seems to be sufficiently comprehensive for design purposes and for development of mathematical models serving as basis for new analytical solutions.

1. Introduction

The present knowledge of contact problems is limited to certain selected cases as Hertz contact problem, Boussinesq's contact problem, or problem of a rigid punch acting upon an elastic semispace or upon a plate on rigid or elastic foundation [1].

Existing solutions for problems of plates (or beams) on an elastic half-space are based on Kirchhoff's plate theory (or simple beam theory). Both Winkler's and Boussinesq's theories have been used for the half-space to couple the surface tractions and the normal displacements. The solutions which have been obtained analytically are restricted to problems involving infinite plates subjected to axisymmetric loads [2-9]. Recently, some solutions of finite size plates on an elastic half-space have been developed [10,11]. However, the plates are assumed to be bonded to the elastic half-space. Moreover, the assumptions of the thin plate theory are obviously poorly satisfied for contact problem [12].

A solution is given by Laermann [13] for the problem of plates (or beams) on an elastic foundation. The plates are not considered to be bonded to the half-space. The solution is based on thin plate theory (or simple beam theory) and a continuous half-space. The problem is solved numerically and checked against experimental results to find the range in which thin plate theory is sufficiently accurate for design purposes.

Since existing solutions are based on simple plate theory, they cannot predict the three-dimensional nature of the stress or displacement fields which may be significant in regions near loads, where there is contact between a plate and a half-space or between plates. This is illustrated by results of some experimental investigation of flanged connections of pipes and pressure vessels [14,15,16].

An analysis of flat faced frictionless contact problem based on the classical theory of elasticity is given by Dundurs and Stippes [17]. This analysis predicts that the contact area is independent of the load level, and Young's modulus if the contacting bodies are made of the same material. However, from their results, it is not possible to determine the position of the boundary of the contact zone.

Experimentally, it has been found that the contact region depends on the load up to a certain level, but is constant for greater loads [13,18].

The contact problem presented in this paper is the mechanical response to loads of plates in flat face contact. This kind of problem is very often met in engineering practice; typical examples are: foundations, layered soil, and structures connected by bolts, etc.

The objectives of the presented investigation were to study the actual response of plates and beams in contact to loads, to generalize the experimental results and to provide a physical background for development of theories. Particular objectives were to determine which parameters are significant and which may be neglected in the future development of theories, and to establish the fundamental relations between contact area and conveniently normalized functions of all major parameters which could also be used as basic functions in numerical analysis for design purpose.

Accordingly, the paper presents basic data on contact area, linear and angular displacements for a wide range of loads, and of material and geometric parameters.

2. Parameters of the Physical Model

The systems investigated consisted of sets of two or three plates of various thicknesses, made of the same materials, loaded symmetrically by two identical punches of various sizes.

The geometry of the systems studied is shown in Figure 1.

Since the investigation was carried out within the limits of the classical theory of linear elasticity and viscoelasticity, only load, geometric and material parameters were considered.

The parameters of response considered are: contact area, contact stress distribution deformation of both surfaces, deformation of end surfaces and change of overall dimensions. (Contact stress distribution is not presented in this paper, see authors' other publications [18,19,20].)

The independent parameters and their ranges are:

1) Geometric parameters

- a) thickness of plate, h , varied from 3 mm to 45 mm
- b) diameter of plate, d , varied from 80 mm to 200 mm
- c) size of punch, b , varied from 8 mm to 50 mm.

These ranges of parameters were chosen in order to cover most cases of plates in contact which occur in engineering practice. For example,
- by varying the thickness of top plate/total thickness of plates ratio from 0.031 to 0.5, one modelled all possibilities between a thin plate on an half-space and two identical plates in contact.
- by varying the punch size/plate thickness ratio from 0.066 to 8.5, one went from essentially a point load condition to a range where the local behaviour near the end of the punch and contact zone no longer depended on punch sizes.

2) Material parameters

Each set of plates and beams was made either of the same linear elastic material (glass) or of a linear viscoelastic material (high polymers such as plexiglass, polyester palatal P6, epoxy resin, polycarbonate and CR-39).

These materials have been chosen because their behaviour is well known, e.g., linear limit stresses, stress optical coefficient, etc., and because they apparent values of Young's modulus and Poisson's ratio cover a wide range. For instantaneous Young's modulus, E , the range is between 17.6 Kg cm^{-2} and 710 Kg cm^{-2} ; for Poisson's ratio, ν , it is between 0.16 and 0.428.

To investigate the influence of the relative rigidities of the punch and plate (or beam), both steel and plexiglass punches were used in combination with the various plate materials already listed. This gave values of $E_{\text{punch}}/E_{\text{plate}}$ between 0.046 and 120.

3) Loading parameters

Axial loads applied, P , through punches varied from 0 to 6,000 Kg. Maximum average contact stress under punch varied from 0 to 50% of linear limit stress value. The resulting average contact stresses between plates were kept below 30% of the linear limit stress value.

3. Theory and Technique of Experiments

Typical experimental arrangement for loading of plates is shown in Figure 2.

Contact area was determined by using Fresnel relations for intensities of transmitted and reflected radiation. Separation of surfaces being in initial contact was measured by using simple interference methods (Newton rings). Since the yellow sodium light ($\lambda = 589.3 \text{ nm}$) was used, the increase of fringe number by one corresponds to increase of distance between plates by 0.295 micron.

Scheme of measuring system is presented in Figure 3.

Deformation of outer surfaces of plates and change of plate diameters at various distances from surface was measured using potentiometric and inductive transducers.

Surface strains were determined by using resistance strain gages in eight points oriented radially and tangentially. The gage length is approximately 2 mm with initial resistance 120 ± 0.5 ohms. The bridge excitation voltage was kept within 0.3 volt.

4. Experimental Results and their Interpretation

To obtain a comprehensive set of data on the influence of loads, geometric and material parameters on the response of plates in flat face contact more than 200 sets of experiments were performed.

A typical example of the load-contact area relations obtained for a wide variety of plate and punch geometries is presented in Figure 4. Results show that the contact area is essentially constant except below a load level designated by P_0 . The average contact stress value corresponding to the load P_0 is between 1% to 5% of the linear limit stress of the model materials used.

An example of the type of deformations produced in a pair of identical plates is shown in Figure 5.

Figure 5a presents the typical vertical displacements of outer and inner plate surfaces. Within the region of contact, the change in the thickness can be seen to be considerable and should definitely be accounted for in any theoretical analysis. However, the change in thickness is insignificant in the region beyond the contact zone. It is interesting to note that the only point where the vertical displacement is zero is almost directly above the edge of the contact zone.

The in-plane dimensional changes are significant for this type of structural component; an example is presented in Figure 5b. The displacements in plane are of the same order of magnitude as the displacement of the plates in a direction normal to their mid-plane. This type of deformation could be of major interest if the plates were connected with other structural members such as shell.

The strain components of the outer surfaces of plates, given for polar coordinates, are presented in Figure 5c. The results show that the circumferential strain components are always positive. It means that the plate is expanded outwards under load. The radial strain components near the punch have positive values, but their values decrease drastically with the increasing distance from the center of plate, and become negative at a short distance beyond the contact region. This implies that the strain state within the contact region is strongly three dimensional and is very complicated. However, in the region outside of the contact region, the strain values decrease as the distance from center increases, and the values of the radial strain components approach one third of the values of the circumferential strain components. According to results presented in Figure 5c, when the b/h_T ratio decreases, the strain values corresponding to $\Delta p = 1000$ Kg in the outside region are practically identical. The whole deformation state shows that the complicated three dimensional response of the plates in contact is highly localized.

In general, the results shown in Figure 5a, b, c, d are consistent. Furthermore, these results give a clear picture of the deformation state of the plates in flat face contact. A schematic picture of the deformation state can be therefore presented as it is shown in Figure 5e.

As Figure 4 shows, the contact area depends on the punch size and the thickness of each plate of the contacting assembly. In order to present the results in a convenient and general form, the punch size, b , and the contact region, c , have in most instances been expressed in non-dimensional form as a fraction of the total thickness of the contacting assembly (h_T).

The dependence of the normalized contact parameter c/h_T on the normalized size of loading punches b/h_T for two and three plates of various h_1/h_T ratios is given in Figure 6a and 6b.

These figures give results for most ratios of plate dimensions which could be of interest. For the system of two plates, the ratio of plate thickness varies from 1:1 ($h_1/h_T = 0.5$) to 1:10.7 ($h_1/h_T = 0.085$). For the system of three plates, the top and bottom are identical and their ratio with respect to the middle one varies from 1:0 ($h_2 = 0$) to 1:30.2 ($h_1/h_T = 0.31$).

In both instances results are presented for increasing punch sizes until the $(c/h_T)/(b/h_T)$ ratio became linear. This change is to be expected since, as b increases, the "edge deformation field" near one side of the punch does not overlap with the "edge deformation field" at the opposite side of the punch.

The presentation of generalized relations as in Figures 6a and b is very useful, except when the thickness h_1 is very small in comparison with h_T , i.e., h_1/h_T approaches zero. In this case the Figure 6 will not conveniently give an accurate prediction of contact area and it is advisable to present the results in a different form. The relation of $\tan \theta$ as a function of b/h_1 for various h_1/h_T are presented on Figures 7a and 7b.

$$\tan \theta = \tan \theta(b/h_1)$$

$$\text{where } \tan \theta = \frac{c - b}{2h_1}$$

for two and three plates.

The functional relations presented in Figures 6 and 7 seem to allow to make some useful generalizations, as:

- (1) It is noticeable, from the upper two curves of the $c/h_T = c/h_T(b/h_T)$ relations shown in Figure 6, that the value of c/h_T (or c) is practically constant for $0 < b/h_T < h_1/2h_T$ (or $0 < b < h_1/2$). This implies that a "Saint-Venant-like" effect occurs if $b/h_1 < 1/2$. Extrapolation of the curves to the limit, i.e., to $b/h_T = 0$, gives an estimate of the c/h_T that would result from a point load. Such a result would be of interest to those seeking an analytic solution to this problem.
- (2) If plate diameter d is larger than the contact region c , the contact area can be expressed as follows:

$$c = b + h_1 \tan \theta \text{ for any constant } h_1/h_T.$$

As shown in Figure 7 the value of $\tan \theta$ decreases as b/h_1 increases and approaches a constant value when $b/h_1 \geq 5$. In such a case the above relation simplifies to:

$$c = b + \text{constant when } b/h_1 \geq 5.$$

The dashed lines in Figure 6 are based on this.

- (3) As shown in Figure 6, the upper limit for c/h_T occurs when $h_1/h_T = 0.5$, that is for two identical plates in contact. The lower limit occurs when $h_1/h_T \rightarrow 0$ for $h_T = \text{constant}$.

In this case, Figure 6 shows that $c \rightarrow b$. Alternatively, the lower limit could occur for $h_1/h_T \rightarrow 0$ when $h_T \rightarrow \infty$. In this case the functional relations shown in Figure 6 are very inconvenient to use. On the other hand, Figure 7 shows that $\tan \theta = \tan \theta(b/h_1)$ is practically unchanged when h_1/h_T is varied from 0.183 to 0.0475. Moreover, there is no reason to expect any significantly different results for the range $0.0475 > h_1/h_T \geq 0$. Consequently, a designer could determine the contact region for any set of geometric ratios he might encounter from either Figure 6 or Figure 7.

The influence of the plate diameter on the contact region is presented for a case of two identical plates (see Figure 8), loaded through punches with a b/h_T ratio of 1.03. The results show that c is essentially independent of d provided d exceeds the value of c predicted by Figure 6. Similar results can be obtained for plates of different thicknesses.

The results shown in Figures 6 and 7 are related to values of d which exceed the value of c by at least 10%.

The experimental results show that contact area is independent of Young's modulus, but influenced by the different Poisson's ratios. The influence of Poisson's ratios on contact response is presented in Figure 9 in the form:

$$\tan \theta = \tan \theta(v) \text{ for } b/h_T = \text{constant and } P \geq P_0.$$

The results shown in Figure 9 are related to loading through a steel punch. The results were slightly changed when the steel punch was replaced by a plexiglass punch. Consequently, the relative rigidities of the punch and plates does not seem to influence significantly the magnitude of the contact zone.

5. Conclusion

A study of over 200 sets of plates in flat contact shows conclusively that existing plate or beam theories cannot adequately describe the stress and deformation states which develop in the punch-contact zone region.

The study showed that an initial non-linear behaviour is a characteristic pattern of flat-faced contact problems. Above this initial load level, the response is linear; numerical or theoretical analysis which is based on the assumption of a continuum with a perfectly flat surface in the contact region should correspond to this fact.

The experimentally determined behaviour (deformations, stresses, etc.), may provide the information which is needed to develop and check more comprehensive theories for flat-faced contact problems.

If the applied load level is above the non-linear range, the following specific conclusions can be drawn:

- (1) The contact area is independent of
 - (a) level of applied load
 - (b) interfacial friction [19]
 - (c) Young's modulus for linear elastic materials, or apparent instantaneous modulus of linearly viscoelastic materials
 - (d) diameter of plates, or length of beams if they exceed a value which depends on the ratio, (punch size)/(plate thickness). This results is true for plates of arbitrary shape.

The contact area depends on

- (a) the ratio (punch size)/(plate thickness), and

(b) Poisson's ratio. However, the contact area is not influenced significantly by variations of the Poisson's ratio between 0.16 and 0.38.

The stresses and deformations of the contacting bodies are proportional to the level of applied loads.

- (2) Functional relations between contact area and the major parameters have been established and presented in Figures 6 and 7. These relations seem to cover all cases including the extremes of a point load and an infinitely large punch. The upper limit and the lower limit of the functional relations for $h_1/h_T = 0.5$, i.e., two identical plates and $h_1/h_T \rightarrow 0$, i.e., plates on an elastic half-space are also given. It is suggested that these relationships can be applied as basic elements for the development of a comprehensive theory of the general response of plates and beams in contact.
- (3) For plates in contact, the change in plate thickness within the contact region is not negligible. However, this change is insignificant outside the contact region. The stress state within the contact region seems to be strongly three-dimensional, but this three-dimensional effect is drastically reduced outside the contact region. The critical stress state is highly localized.

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References

- [1] Galin, L. A., "Contact Problems in the Theory of Elasticity", translated from Russian by Mrs. H. Moss, edited by I. N. Sneddon, Dept. of Math., North Carolina State College, 1961.
- [2] Dhaliwal, R. S., "Punch Problem for an Elastic Layer Overlaying an Elastic Foundation", International Journal of Engineering Science, Vol. 8, pp. 273-288, Pergamon Press, 1970.
- [3] Aleksandrov, V. M., "Asymptotic Solution of the Contact Problem for a Thin Elastic Layer", J. of Applied Mathematics and Mechanics, Vol. 33, 1969.
- [4] Lebedev, N. N., and Ufliand, Ia. S., "Axisymmetric Contact Problem for an Elastic Layer", J. of Applied Mathematics and Mechanics, Vol. 22, 1958.
- [5] Vorovich, I. A., and Ustinov, Iu. A., "On Pressure of a Die on a Layer of Finite Thickness", J. of Applied Mathematics and Mechanics, Vol. 23, 1959.
- [6] Pal'mov, V. A., "The Contact Problem of a Plate on an Elastic Foundation", J. of Applied Mathematics and Mechanics, Vol. 24, 1960.
- [7] Popov, G. Ia., "On a Method of Solution of the Axisymmetric Contact Problem of the Theory of Elasticity", J. of Applied Mathematics and Mechanics, Vol. 25, 1961.
- [8] Tsai, Y. M., "Stress Distributions in Elastic and Viscoelastic Plates Subjected to Symmetrical Rigid Indentations", Quarterly of Applied Mechanics 27, 3, Oct. 1969.
- [9] Weitsman, Y., "On the Unbonded Contact Between Plates on an Elastic Half-space", J. of Applied Mechanics, Transaction of ASME, Series E 36,2, pp. 198-202, June 1969.
- [10] Svec, O. J., and McNeice, G. M., "Finite Element Analysis of Finite Sized Plates Bonded to an Elastic Half-space", Journal of Computer Methods in Applied Mechanics and Engineering, Vol. 1, No. 3, Nov. 1972.
- [11] Svec, O. J., and Gladwell, G. M. L., "A Triangular Plate Bending Element for Contact Problems", Int. J. of Solids and Structures, Vol. 9, No. 3, March 1973.
- [12] Essemburg, F., Gulati, S. T., "On the Contact of Two Axisymmetric Plates", Journal of Applied Mechanics, June 1966.
- [13] Laermann, K. H., "Auflagerlänge von Balken auf elastisch nechiebigem Untergrund bei Berücksichtigung des Abhebens", Die Bautechnik, 5/1969.
- [14] Pindera, J. T., and Sze, Y., "Studies of Physical and Mathematical Models of Some Flanged Connections", Fourth International Conference on Stress Analysis, Cambridge (England), April 6-10, 1970, Proceedings.
- [15] Pindera, J. T., and Sze, Y., "Response to Loads of Flat-faced Flanged Connections and Reliability of Some Design Methods", Transactions of the CSME, Vol. 1, No. 1, March 1972.
- [16] Pindera, J. T., and Sze, Y., "Influence of the Bolt System on the Response of the Face-to-face Flanged Connections", to be presented at the Second International Conference on Structural Mechanics in Reactor Technology, Berlin, September 1973.
- [17] Dundurs, J., and Stippes, M., "Role of Constants in Certain Contact Problems", Journal of Applied Mechanics, Vol. 37, No. 4, 1970.
- [18] Pindera, J. T., and Sze, Y., "Response of Elastic Plates in Flat Contact", International Symposium on Experimental Mechanics, June 12-16, 1972, University of Waterloo, Canada. Proceedings.
- [19] Sze, Y., "Experimental Study of Model Similarity for some Flat Face Contact Problems", Doctoral Thesis, July 1972, University of Waterloo, Canada.
- [20] Sze, Y., and Pindera, J. T., "Determination of Stress Components in a Plane Stress State of some Flat Face Contact Problems using Scattered Light Technique", to be published.

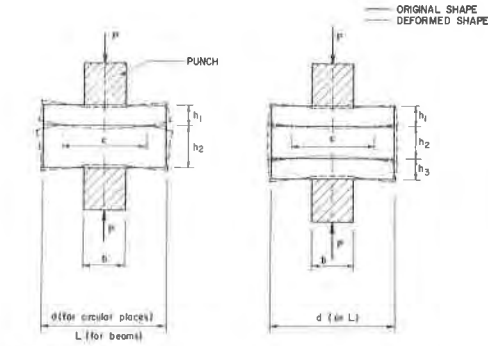


Fig. 1. I.T.P. 6 T.3.

Figure 1: Geometry of bodies in flat face contact under investigation.

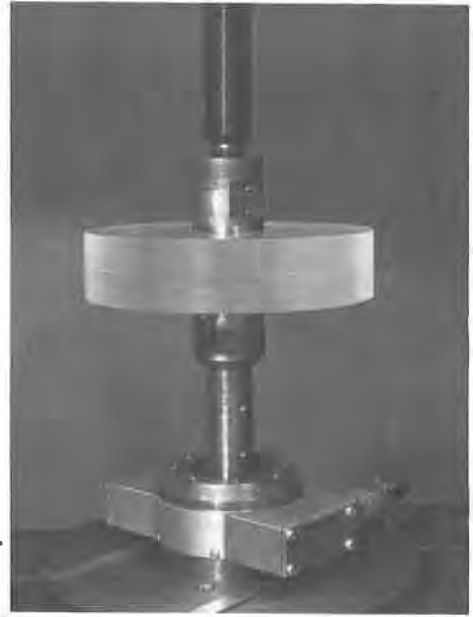


Figure 2: View of plates in loading system.

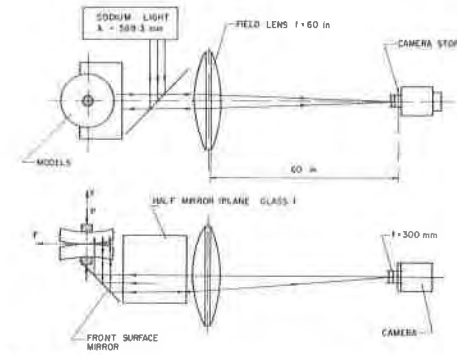


Fig. 3. I.T.P. 6 T.3.

Figure 3: Scheme of system for measuring contact area and interface surfaces separation using modulation of radiant power by reflection and interference.

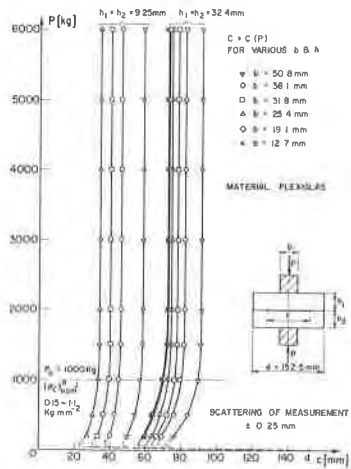


Fig. 4. I.T.P. 6 T.3.

Figure 4: Examples of dependence of contact area diameter on load for various sizes of loading punches.

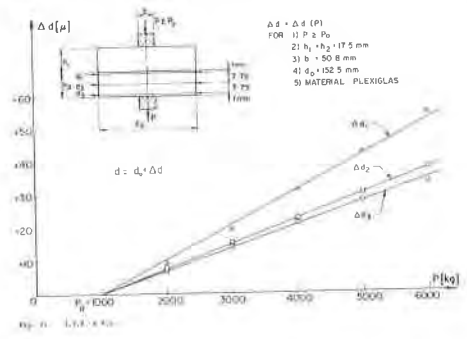
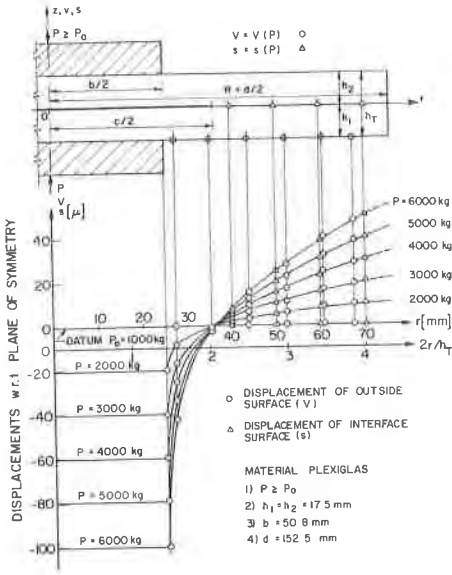


Fig. 5a J.T.P. & Y.S.

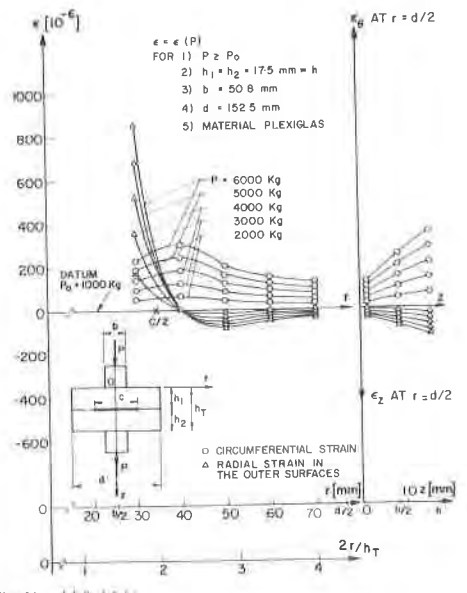
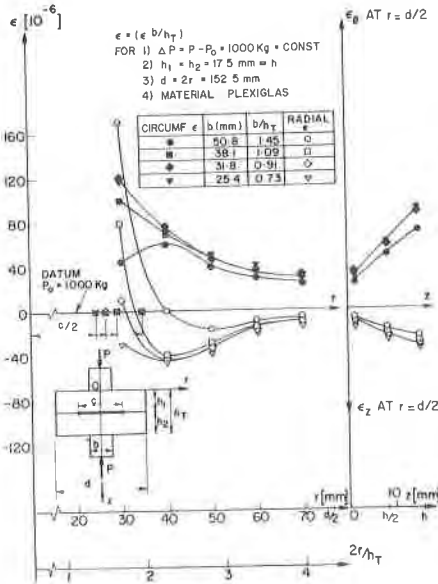


Fig. 5c J.T.P. & Y.S.

Fig. 5d J.T.P. & Y.S.

Figure 5: Deformation state of two identical circular plates under loads.

- (a) Axial displacements of outer and inner plate surfaces with respect to plane of symmetry versus load.
- (b) Changes of outer diameters of plate versus load for three planes: middle plane of plate and two planes close to the upper and lower surface of plate.
- (c) Surface strain components versus load (circumferention strain and radial strain).
- (d) Surface strain components for various sizes of loading punches at constant load.

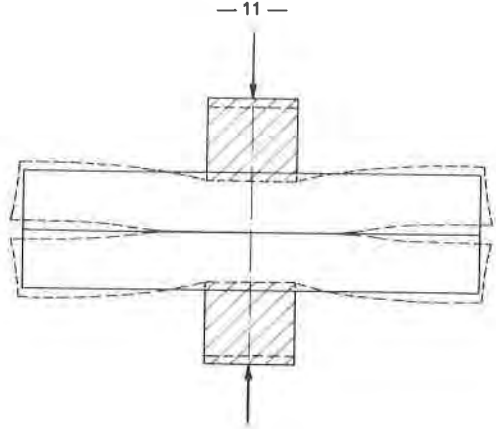


Fig. 5e STATE OF DEFORMATION OF CIRCULAR PLATES LOADED BY CIRCULAR PUNCHES. SCHEMATIC PRESENTATION

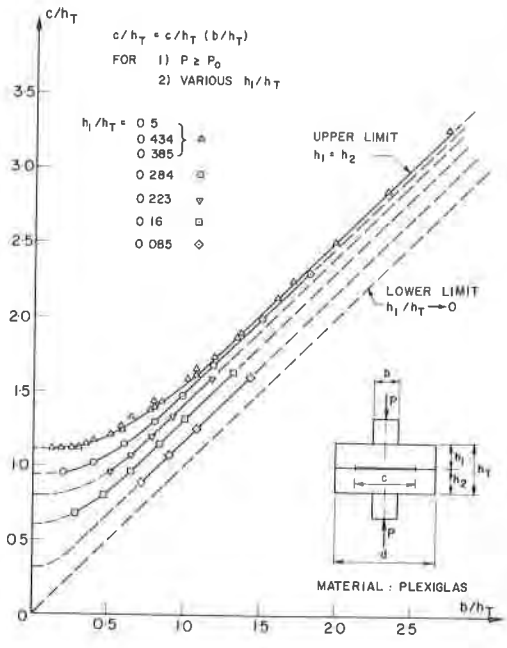


Fig. 6a J.T.P. & Y.S.

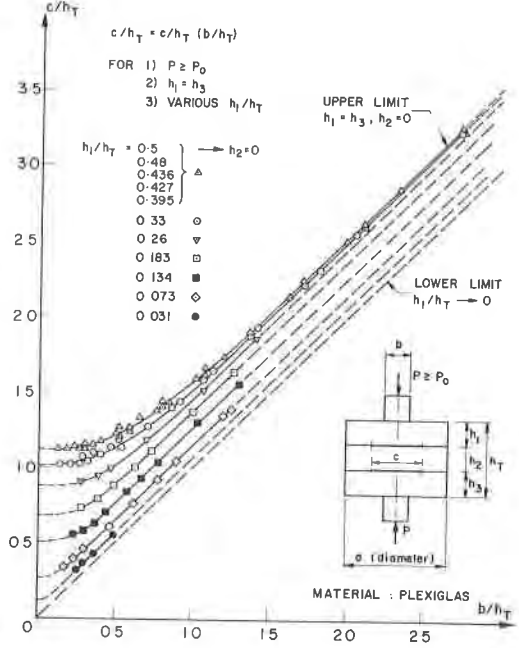


Fig. 6b J.T.P. & Y.S.

Figure 6: Generalized presentation of the normalized contact parameter versus normalized sizes of loading punches.

(a) Two circular plates for various ratios of thicknesses.

(b) Three circular plates for various ratios of thicknesses.

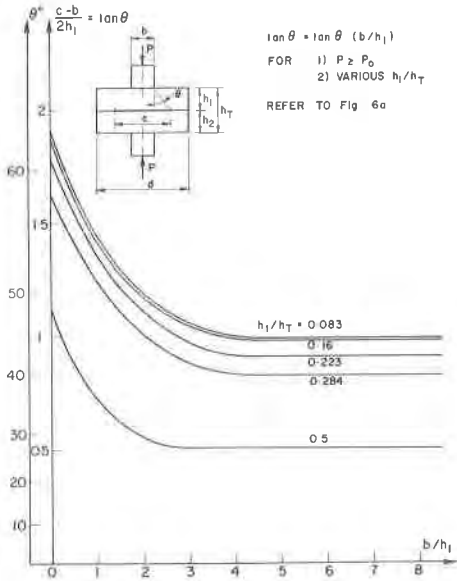


Fig. 7a J.T.P. & Y.S.

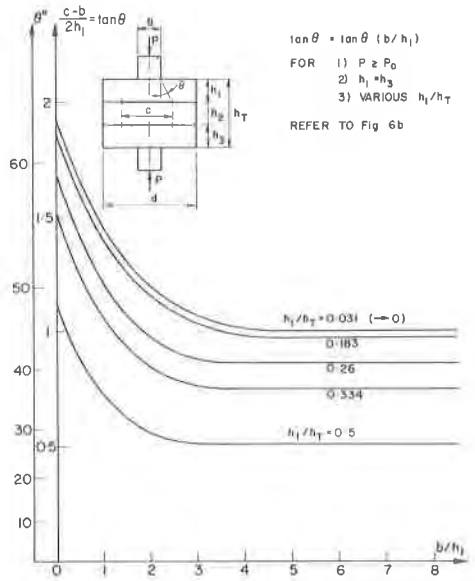


Fig. 7b J.T.P. & Y.S.

Figure 7: Generalized presentation of the normalized quantity $\tan \theta$ versus normalized sizes of loading punches.

- (a) Two circular plates for various ratios of thicknesses.
- (b) Three circular plates for various ratios of thicknesses.

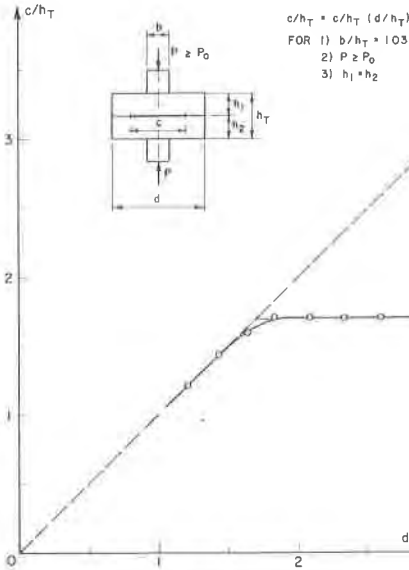


Fig. 8 J.T.P. & Y.S.

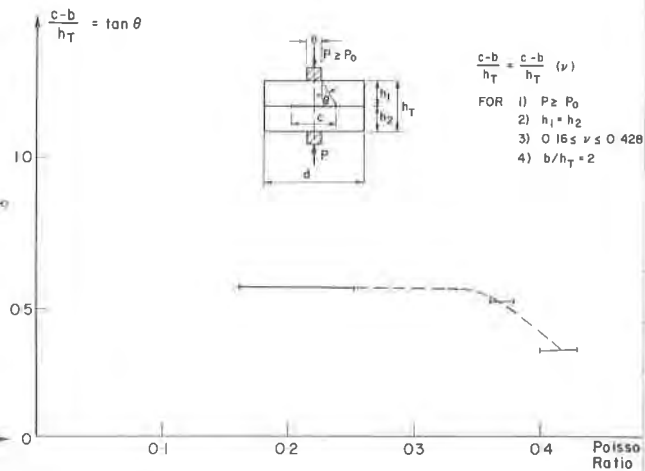


Fig. 9 J.T.P. & Y.S.

Figure 8: Generalized presentation of normalized contact diameter of plate for two identical circular plates.

Figure 9: Generalized presentation of the normalized quantity $\tan \theta$ versus Poisson's ratios for two identical circular plates.