COMPUTER PREDICTION OF FATIGUE CRACK PROPAGATION UNDER RANDOM LOADING

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SUMMARY

In this paper, the results of two simulations on fatigue crack propagation under random loading are presented.

First, fatigue crack propagation under random loading is simulated based on linear fracture mechanics. From the results of axial load tests on double symmetric V-notched steel sheet specimens, fatigue crack growth rate \( \frac{d(2a)}{dN} \) may be expressed in the form,

\[
\frac{d(2a)}{dN} = C\Delta K^4 \quad (\Delta K > \Delta K_{th})
\]

\[
= 0 \quad (\Delta K \leq \Delta K_{th}),
\]

where \( C \) is a log-normally distributed random variable and \( \Delta K_{th} \) is the threshold stress intensity factor range below which cracks are considered to be non-propagating from the practical point of view. The procedure of simulation is obtained by reducing the present problem to that of obtaining the output response of a non-linear zero-memory system with feedback loop to random noise signal. Random loads with desired power spectral density are obtained by passing digitally generated white noise through numerical filters. As the increment of crack length at \( N \)th cycle is determined by the stress intensity factor range at that cycle, which is proportional to the product of the stress range at \( N \)th cycle and the square root of half the crack length at \( (N-1) \)th cycle, if we introduce the concept of \( \Delta K_{th} \), the effect of the sequence of load cycles can not be neglected. The advantage of numerical filter technique is in that the effect of loading sequence can be included into the analysis.

The numerical and experimental results are compared for narrow band random loading, which type of loading is closely related with random load fatigue of components of gas cooled reactors. Both results agree well except for very short cracks.

Second, elasto-plastic response at the crack tip is considered by the introduction of finite element method and fatigue crack propagation under random loading is simulated incorporating the effect of memory of prior load history. In the calculation, it is assumed that when the total absorbed hysteresis energy of elements equal a postulated critical value \( U^* \), separation occurs and the crack extends to the next element. Here, only numerical results are given for aluminum alloys, but the method is applicable to any kind of material.
1. INTRODUCTION

The aim of this paper is to simulate fatigue crack growth under random loading. In general, the process of fatigue crack propagation can be described by the block diagram as shown in Fig. 1. When input values, i.e., the initial crack length $2a_0$ and the range of an alternating load $\Delta P$ are given, the increment of fatigue crack growth $\Delta (2a)$ is obtained as the response of material at the crack tip, i.e., resistance of material against fatigue crack growth. Then $\Delta (2a)$ is feedback and added to $2a_0$, and the crack length at that cycle $2a_0 + \Delta (2a)$ is obtained. This process is repeated for many cycles. The present problem of random loading can be solved by randomizing $\Delta P$. Therefore, fatigue crack growth under random loading can be analyzed as the problem of transformation of random signals passing through a system with a feedback loop.

The content of this paper is shown schematically in Fig. 2. The term "zero-memory" means that the output $\Delta (2a)$ at $N^{th}$ cycle depends only on the input values $\Delta P$ and $2a$ at $N^{th}$ cycle and is otherwise independent of these values before and after $N^{th}$ cycle. Though analysis based on linear fracture mechanics is zero-memory type, it is of interest to note that the effect of the sequence of loads appears as will be mentioned in chapter 2. The term "with memory" means that the output $\Delta (2a)$ at $N^{th}$ cycle depends not only on the input values at $N^{th}$ cycle, but also on the values of preceding cycles. Especially in the case of random loading, the analysis of the history of stress and strain at the crack tip is quite important to make clear the phenomenon of acceleration and delay in crack growth. In this report, elasto-plastic response at the crack tip is obtained by finite element method for the case of stationary crack and for the case of propagating crack.

2. SIMULATION BASED ON LINEAR FRACTURE MECHANICS

2-1. THE CASE OF SINUSOIDAL LOADING

Based on linear fracture mechanics, fatigue crack growth rate can be described in the form

$$\frac{d(2a)}{dn} = C \times \Delta K^n.$$

Here, $C$ and $n$ are material constants. In the following analysis, $C$ is assumed as random variable to take into account the statistical dispersion of fatigue crack growth. According to the experimental results of fully reversed tests on double V notched sheet specimens conducted by Kitagawa et al. [1], fatigue crack growth rate for medium carbon steels can be described in the following form.
\[
\frac{d(2a)}{dN} = \begin{cases} \infty & \Delta K \geq \Delta K_{fc} \\ 0 & \Delta K_{th} < \Delta K < \Delta K_{fc} \\ 0 & \Delta K \leq \Delta K_{th} \end{cases} \quad \text{eq. (1)}
\]

From the test results \(C\) may be assumed to be a log-normally distributed random variable with the mean value of \(4.06 \times 10^{-12}\) and the standard deviation of 0.13. In words, a test specimen fractures unstably if \(\Delta K\) exceeds \(\Delta K_{fc}\), and if \(\Delta K\) is below \(\Delta K_{th}\) crack growth rate becomes extremely so slow as to be considered non-propagating from the industrial viewpoint. The fourth power law holds for \(\Delta K\) values between \(\Delta K_{th}\) and \(\Delta K_{fc}\). The relationship of eq. (1) can be expressed by the block diagram as shown in Fig.3.

2-2. THE CASE OF RANDOM LOADING

In the case of random loading, fatigue crack growth behavior can be studied by randomizing the load \(\Delta F\). It can be noted from Fig.3 that the present problem can be reduced to the problem of the transformation of random signals passing through a zero-memory nonlinear system with a feedback loop. The same fatigue crack growth rate as Paris [2] calculated can be obtained if we ignore the feedback loop in Fig.3 and put \(\Delta K_{th} = 0\). But as the effect of the loading sequence appears by the presence of the feedback loop in Fig.3 even within the limits of linear fracture mechanics analysis, especially if we introduce \(\Delta K_{th}\) into the analysis, the feedback loop can not be ignored. Moreover, this feedback loop is also important in that the stress and strain distribution at the crack tip changes much with the extension of a crack in such small specimens as used in laboratory tests.

2-3. SIMULATION PROCEDURE

As a system with a feedback loop is difficult to solve analytically, simulation technique is adopted here for analysis. The procedure of analysis is shown in Fig.4. First, white noise is generated on a digital computer by a normal random number generating program subroutine, and this white noise is passed through a numerical filter and changed into random noise with desired power spectral density. Then the maxima and minima are picked up, and the load range and \(\Delta K\) value are computed. The increment of crack growth is then determined by eq.(1). In the analysis, the mean value of \(4.06 \times 10^{-9}\) is used instead of \(4.06 \times 10^{-12}\) to shorten the computation time.
RESULTS

Here as a fundamental example of random loading, narrow band random loading is analyzed which can be obtained by passing white noise through a lightly damped linear one degree of freedom system. The digitally generated narrow band random noise and its power spectrum are shown in Fig. 5 and Fig. 6 respectively. In the case of random loading, it is difficult to determine what kind of stress should be used for its magnitude, but in case of narrow band random loading, the stress magnitude can be uniquely determined because its extrema or ranges are Rayleigh-distributed. Here, simulation results and Kitagawa’s experimental results which have the same instantaneous r.m.s. values are plotted on the same graph (Fig. 7). The dot-dash line and solid line show experimental and simulation results respectively. The dotted line is obtained by moving the dot-dash line parallel with respect to the horizontal axis. Comparing the solid line with the dotted line, it can be seen that in region B where actual crack length is longer than 2 mm, i.e., 1 mm each from edge notches, fatigue crack propagation curve under narrow band random loading can be well predicted by the present simulation. The reason that the simulated result is more winding than the dotted line is because in the analysis the scatter of C in the equation \( \frac{d(2a)}{dn} = C \sqrt[3]{K^4} \) is obtained for many test specimens and includes various experimental errors, and thus is larger than the case where the scatter of C is obtained for a single test specimen, in other words, the scatter of C is due to the distribution of material resistance against crack growth. The discrepancy in region A between simulation and experimental result may be attributed to the assumption that below \( \Delta K_{th} \) a crack stops propagating while in reality even below \( \Delta K_{th} \) a crack grows though extremely slowly.

The above results are obtained for load control tests, but the same discussion holds good for displacement control tests. It is understood from the above analysis that fatigue crack growth at least under narrow band random loading can be simulated as the problem of random signals acting on zero-memory nonlinear system with a feedback loop. But narrow band random load has a strong correlation between succeeding peaks and troughs, thus possessing comparatively deterministic nature, and moreover Kitagawa’s experiments were carried out in the low stress region. Those are conceivably the reasons that fatigue crack growth can be predicted by zero-memory type simulation based on linear fracture mechanics. As the bandwidth of a random load widens,
the wave form becomes quite random. Although experimental results are not sufficient, it may be expected that the phenomenon of acceleration and delay in fatigue crack growth due to the sequence of load peaks become remarkable with increasing bandwidth. Therefore, though this simulation technique is in principle applicable to any random load with arbitrary bandwidth, it is more often than not necessary to treat fatigue crack propagation under random loading as the problem of random signals undergoing nonlinear transformation with memory, considering the elasto-plastic response at the crack tip, as will be described in the following chapter.

3. FINITE ELEMENT ANALYSIS

As the history of stress and strain at the crack tip is important in analyzing fatigue crack growth under random loading, elasto-plastic response at the crack tip must be considered. Here, as a first step toward simulating fatigue crack growth under random loading by finite element method, elasto-plastic analysis was made on a sheet specimen with a central transverse crack subjected to a tension-tension load. The size and geometry of the specimen is shown in Fig. 8, and the analysis was carried out on a quarter part of the specimen because of its symmetry. The finite element mesh used is shown in Fig. 9. The mechanical properties are shown in Table I. The tension-tension loads are shown in Fig. 10.

3.1. STATIONARY CRACK

First, elasto-plastic response was calculated for a stationary crack. Fig. 11 and Fig. 12 show the variation of plastic zone under constant and Lo-Hi tension-tension loads respectively. The shaded area is the cyclic plastic zone and the numbers indicated in the figure correspond to the numbering of peaks and troughs.

3.2. PROPAGATING CRACK

Next, finite element analysis was made introducing the condition for crack propagation. It is assumed here that a crack propagates when the hysteresis energy in the cyclic plastic zone is accumulated up to a critical value $\Delta \sigma$. When a crack of length $2a$ extends by the amount of $\Delta (2a)$, the tractions at the $\Delta (2a)$ part which have carried the load before crack propagation become zero. Therefore, analysis considering the condition for fatigue crack propagation is possible by eliminating incrementally the stress $\Delta \sigma$ working on the $\Delta (2a)$ part when the condition for propagation is satisfied, choosing the stress and strain state at crack length $2a$ as the initial condition. In finite element analysis, nodal reaction force at the crack tip was eliminated
incrementally. (Miyoshi [3])

4. SUMMARY

(1) The process of fatigue crack growth under narrow band random loading is treated as the problem of random signals working on the zero-memory nonlinear system with a feedback loop and is simulated based on linear fracture mechanics. The experimental results are fairly well predicted by this simulation.

(2) As a first step toward simulating fatigue crack growth under random loading, elasto-plastic response at the crack tip under a varying tension-tension load is obtained for a stationary crack and for a propagating crack.

REFERENCES

[1] KITAGAWA, H., FUKUDA, S., NISHIYAMA, A.

[2] PARIS, P.


LIST OF SYMBOLS

K - stress intensity factor

ΔK - stress intensity factor whose stress is expressed in range.

ΔK_{fc} = 2K_{fc,max} - fatigue fracture toughness

ΔK_{th} - threshold stress intensity factor whose stress is expressed in range

2a₀ - initial crack length

Δ(2a) - increment of crack length

2a - total crack length

N - number of repeated cycles of load range

\frac{d(2a)}{dN} - fatigue crack growth rate

ΔP - load range

W - width of test specimen

t - thickness of test specimen

L - length of test specimen

\sigma_y - stress in the loading direction

\epsilon_y - strain in the loading direction

\omega - plastic zone size in monotonic loading

\omega^p - plastic zone size in cyclic loading

U^p - hysteresis energy
### TABLE I. MECHANICAL PROPERTIES

<table>
<thead>
<tr>
<th>E</th>
<th>$\nu$</th>
<th>$\delta_v$</th>
<th>$H'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7200</td>
<td>0.33</td>
<td>50</td>
<td>70</td>
</tr>
</tbody>
</table>

$$\bar{\delta} = \delta_v + H' \bar{\delta}_p$$

**FIG. 1** BLOCK DIAGRAM OF THE PROCESS OF FATIGUE CRACK GROWTH

**FIG. 2** SCHEMATIC DESCRIPTION OF THE CONTENT

**FIG. 3** BLOCK DIAGRAM OF FATIGUE CRACK PROPAGATION
FIG. 4 COMPUTATIONAL PROCEDURE

FIG. 5 DIGITALLY GENERATED NARROW BAND RANDOM LOAD

FIG. 6 POWER SPECTRUM
FIG. 7  FATIGUE CRACK GROWTH CURVES

FIG. 8  GEOMETRY OF THE SPECIMEN

FIG. 9  FINITE ELEMENT IDEALIZATION
FIG. 10  LOADING PATTERNS

FIG. 11  CHANGE IN PLASTIC ZONE

FIG. 12  CHANGE IN PLASTIC ZONE