THERMAL STRESSES IN CLAD FUEL ELEMENTS
PART I. CYLINDRICAL FUEL ELEMENTS
PART II. SPHERICAL FUEL ELEMENTS

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ABSTRACT

Equations for the thermal stresses developed in internally and externally clad cylindrical elements with radially symmetric temperature distribution are presented. For illustration, temperature and thermal stress distributions for four fuel element configurations (three cylindrical and one flat plate) with uniform heat generation in the fuel and no heat generation in the cladding are pictured and compared. Some conclusions of practical significance are drawn from this comparison.

Equations are presented for the thermal stresses developed in externally clad hollow spheres with radially symmetric temperature distribution. The cases of clad solid and hollow spherical fuel elements, having uniform heat generation in the fuel and no heat generation in the cladding, are used for illustration. The temperature and thermal stresses developed in the clad hollow element are calculated on deterministic and probabilistic bases (taking into account the range of all variables concerned). A comparison of the two sets of values is made with particular reference to reliability in design.

PART I CYLINDRICAL ELEMENTS

1. INTRODUCTION

Cylindrical elements are used in many reactors, principally in fuel elements. A number of such fuel elements are solid with an external cladding. Similar hollow fuel elements have certain advantages but present a more difficult fabrication problem. There are other applications in which a cylindrical vessel may be composed of two or more materials, e. g., one provides strength while the other provides corrosion resistance.

In operation there may be thermal gradients which set up thermal stresses. These stresses arise not only from thermal gradients but also from differences in physical properties such as modulus of elasticity, Poisson's ratio, and coefficient of thermal expansion, and from the specific geometry of the element. The equations for thermal stresses in a cylinder of a single material are available in the literature (1, 2, 3, 4, 5, 6). The equations for thermal stresses in externally clad cylindrical elements are also available (7). Part I presents equations for thermal stresses in an internally and externally clad cylindrical element composed of any three arbitrarily chosen materials with a radially symmetric temperature distribution.

2. BASIC EQUATIONS

Consider a hollow cylinder, as in Fig. 1, in which it is assumed that the fuel bearing material and both cladding materials are homogeneous, isotropic, and elastic. Additional assumptions are: (1) the modulus of elasticity (E), Poisson's ratio (μ), and the coefficient of thermal expansion (α) are independent of temperature over the range involved; (2) the temperature increment (T) above some arbitrary base temperature is positive; (3) the temperature is a function of radius (r) and independent of tangential and axial location; (4) there is a completely integral bond between the fuel and cladding; (5) there are no surface or body forces; (6) the element is straight; and (7) the element is "infinitely" long. The last of these assumptions implies that the equations are applicable at any axial position farther than about 1 to 2 diameters from the ends of a cylinder of any length.

Application of the general equations of stress equilibrium and the conditions of strain compatibility (1, 2) under the stated assumptions leads to the general equations for thermal stresses in annular elements as given below

In the inner annulus:

Radial Stress
$$\frac{\sigma_r}{E_A} = -\frac{\alpha_A}{r^2(1-\mu_A)}\int_{r_i}^{r} rT_A dr + (1 - \frac{r_i^2}{r^2})\left(\frac{C_1 + \mu_A C_2}{(1+\mu_A)(1-2\mu_A)}\right) \quad (1)$$

Tangential Stress
$$\frac{\sigma_\theta}{E_A} = \frac{\alpha_A}{r^2(1-\mu_A)}\int_{r_i}^{r} rT_A dr + (1 + \frac{r_i^2}{r^2})\left(\frac{C_1 + \mu_A C_2}{(1+\mu_A)(1-2\mu_A)}\right)$$
$$-\frac{\alpha_A T_A}{1-\mu_A} \quad (2)$$

Axial Stress
$$\frac{\sigma_z}{E_A} = \frac{2\mu_A C_1 + (1-\mu_A)C_2}{(1+\mu_A)(1-2\mu_A)} - \frac{\alpha_A T_A}{1-\mu_A} \quad (3)$$

In the middle annulus:

Radial Stress
$$\frac{\sigma_r}{E_B} = -\frac{\alpha_B}{r^2(1-\mu_B)}\int_{r_1}^{r} rT_B dr + \frac{C_3 + \mu_B C_2}{(1+\mu_B)(1-2\mu_B)} - \frac{C_4}{r^2(1+\mu_B)} \quad (4)$$

Tangential Stress
$$\frac{\sigma_\theta}{E_B} = \frac{\alpha_B}{r^2(1-\mu_B)}\int_{r_1}^{r} rT_B dr + \frac{C_3 + \mu_B C_2}{(1+\mu_B)(1-2\mu_B)} + \frac{C_4}{r^2(1+\mu_B)}$$
$$-\frac{\alpha_B T_B}{1-\mu_B} \quad (5)$$

Axial Stress
$$\frac{\sigma_z}{E_B} = \frac{2\mu_B C_3 + (1-\mu_B)C_2}{(1+\mu_B)(1-2\mu_B)} - \frac{\alpha_B T_B}{1-\mu_B} \quad (6)$$

In the outer annulus:

Radial Stress

$$\frac{\sigma_r}{E_C} = -\frac{\alpha_C}{r^2(1-\mu_C)}\int_{r_2}^{r} rT_C dr + \left(1 - \frac{r_o^2}{r^2}\right)\left(\frac{C_5 + \mu_C C_2}{(1+\mu_C)(1-2\mu_C)}\right) + \frac{\alpha_C}{r^2(1-\mu_C)}\int_{r_2}^{r_o} rT_C dr \tag{7}$$

Tangential Stress

$$\frac{\sigma_\theta}{E_C} = \frac{\alpha_C}{r^2(1-\mu_C)}\int_{r_2}^{r} rT_C dr + \left(1 + \frac{r_o^2}{r^2}\right)\left(\frac{C_5 + \alpha_C C_2}{(1+\mu_C)(1-2\mu_C)}\right) - \frac{\alpha_C}{r^2(1-\mu_C)}\int_{r_2}^{r_o} rT_C dr - \frac{\alpha_C T_C}{1-\mu_C} \tag{8}$$

Axial Stress

$$\frac{\sigma_z}{E_C} = \frac{2\mu_C C_5 + (1-\mu_C)C_2}{(1+\mu_C)(1-2\mu_C)} - \frac{\alpha_C T_C}{1-\mu_C} \tag{9}$$

These equations contain five constants (C_1, C_2, C_3, C_4, and C_5) which are functions of physical properties, geometry, and temperature distribution. For the case in which the three materials do not have common values of physical properties, the expressions for the five constants have many terms and it is convenient to write them in terms of the material and geometry factors defined below.

$K_1 = 1 + \mu_A$

$K_2 = 1 + \mu_B$

$K_3 = 1 + \mu_C$

$K_4 = 1 - \mu_A$

$K_5 = 1 - \mu_B$

$K_6 = 1 - \mu_C$

$K_7 = 1 - 2\mu_A$

$K_8 = 1 - 2\mu_B$

$K_9 = 1 - 2\mu_C$

$K_{10} = r_1^2 - r_1^2$

$K_{11} = r_2^2 - r_1^2$

$K_{12} = r_o^2 - r_1^2$

$K_{13} = r_i^2 + r_1^2(1 - 2\mu_A)$

$K_{14} = r_o^2 + r_2^2(1 - 2\mu_C)$

Even with these K factors, however, the expressions for the five constants are very cumbersome. It is therefore convenient to group physical properties, temperature distribution, and K factors in the following manner.

$$a = \frac{\mu_A K_{10}}{K_1 K_7}$$

$$b = \frac{\mu_B E_B K_{11}}{E_A K_2 K_8}$$

$$c = \frac{\mu_C E_C K_{12}}{E_A K_3 K_9}$$

$$d = \frac{1}{2}\left(\frac{K_4 K_{10}}{K_1 K_7} + \frac{E_B K_5 K_{11}}{E_A K_2 K_8} + \frac{E_C K_6 K_{12}}{E_A K_3 K_9}\right)$$

$$e = \frac{K_{14}}{K_9}$$

$$f = \frac{\mu_C r_o^2}{K_9}$$

$$g = \frac{K_{13}}{K_7}$$

$$h = \frac{\mu_A r_i^2}{K_7}$$

$$j = \frac{2E_B r_2^2 K_5}{E_C K_2 K_8}$$

$$k = \frac{1}{K_9}\left(\frac{K_{12}}{K_3} - \frac{E_B K_{14}}{E_C K_2}\right)$$

$$\ell = \frac{\mu_B E_B r_2^2}{E_C K_2 K_8} + \frac{\mu_C K_{12}}{K_3 K_9} - \frac{\mu_C E_B r_o^2}{E_C K_2 K_9}$$

$$m = \frac{2 r_1^2 K_4}{K_7}$$

$$n = r_1^2 \left(\frac{E_B K_1}{E_A K_2 K_8} + 1\right)$$

$$p = \left(\frac{E_B K_1}{E_A K_2} - 1\right)$$

$$q = r_1^2 \left(\frac{\mu_A}{K_7} - \frac{\mu_B E_B K_1}{E_A K_2 K_8}\right)$$

$$v = \frac{\alpha_A}{K_4}\int_{r_i}^{r_1} r T_A\,dr + \frac{E_B \alpha_B}{E_A K_5}\int_{r_1}^{r_2} r T_B\,dr + \frac{E_C \alpha_C}{E_A K_6}\int_{r_2}^{r_o} r T_C\,dr$$

$$X = \frac{K_2\alpha_B}{K_5}\int_{r_1}^{r_2} rT_B dr + \frac{K_3\alpha_C}{K_6}\int_{r_2}^{r_o} rT_C dr$$

$$Y = \frac{K_1\alpha_A}{K_4}\int_{r_i}^{r_1} rT_A dr$$

$$Z = (1 - \frac{E_B K_3}{E_C K_2}) \frac{\alpha_C}{K_6}\int_{r_2}^{r_o} rT_C dr$$

In terms of these groups and a common factor, Δ, the constants in the thermal stress equations are:

$$\Delta C_1 = -k(q + ph)(bX + r_2^2 V) + k(qr_1^2 - nh)V$$
$$+ [j(q + ph) + \ell(n + pr_1^2)](cX - eV)$$
$$+k(n + pr_1^2)(fV - dX) - bkqY$$
$$+ (be + cr_2^2)[(q + ph)Z + p\ell Y] - pk(bf + dr_2^2)Y$$
$$+ C[(jg + n\ell)Y - (qr_1^2 - nh)Z] - dknY$$
$$+ (de - cf)[(n + pr_1^2)Z - jpY]$$

$$\Delta C_2 = apkr_2^2 Y + aknY$$
$$+ ae[jpY - (n + pr_1^2)Z] + ak(n + pr_1^2)X$$
$$+ bkmY - (be + cr_2^2)(m + gp)Z$$
$$+k(m + gp)(bX + r_2^2 V) - c[(gn - mr_1^2)Z + mjY]$$
$$+ k(gn - mr_1^2)V + j(m + gp)(eV - cX)$$

$$\Delta C_3 = ak(q + ph)X + akqY$$
$$- ae[(q + ph)Z + p\ell Y] + afpkY$$
$$+ k(gq - mh)V + \ell(m + gp)(cX - eV)$$
$$+ k(m + gp)(fV - dX) - c[(gq - mh)Z - m\ell Y]$$
$$- dmkY + (de - cf)(m + gp)Z$$

$$\Delta C_4 = -akqr_2^2 Y - ak(qr_1^2 - nh)X$$
$$- ae[(n\ell + jq)Y - (qr_1^2 - nh)Z] + afnkY$$

$$- k(gq - mh)\ (bX + r_2^2 V)$$

$$+ (be + cr_2^2)\ [(gq - mh)Z - m\ell Y]$$

$$+ mk(bf + dr_2^2)Y + [j(gq - mh) + \ell(ng - mr_1^2)]\ (cX - eV)$$

$$+ k(ng - mr_1^2)\ (fV - dX) + (de - cf)\ [(ng - mr_1^2)Z + jmY]$$

$$\Delta C_5 = -ar_2^2\ [(q + ph)Z + p\ell Y] + a\ [(qr_1^2 - nh)Z - (n\ell + jq)Y]$$

$$- a\ [j(q + ph) + \ell(n + pr_1^2)]\ X + af\ [(n + pr_1^2)Z - jpY]$$

$$+ b\ [(gq - mh)Z - m\ell Y] - \ell(m + gp)\ (bX + r_2^2 V)$$

$$+ (bf + dr_2^2)\ (m + gp)Z - [j(gq - mh) + \ell(gn - mr_1^2)]\ V$$

$$+ d\ [(gn - mr_1^2)Z + jmY] + j(m + gp)\ (dX - fV)$$

where the common factor, Δ, is

$$\Delta = -akr_2^2(q + ph) + ak(qr_1^2 - nh)$$

$$- ae\ [j(q + ph) + \ell(n + pr_1^2)] + afk(n + pr_1^2)$$

$$+ bk(gq - mh) - \ell(be + cr_2^2)\ (m + gp)$$

$$+ k(bf + dr_2^2)\ (m + gp) - c\,[j(gq - mh) + \ell(ng - mr_1^2)]$$

$$+ dk(ng - mr_1^2) + j(de - cf)\ (m + gp)$$

Evaluation of these constants for a given case obviously requires considerable effort. The possibility of graphical presentation was considered and discarded as being of little help since the constants contain definite integrals of the form

$$\int_{r_1}^{r_2} r\ T\ dr$$

which cannot be evaluated until the temperature distribution for the specific problem is known.

For the case where both the inner and outer cladding are of the same material (which would be expected in a fuel element), there is obviously some simplification of the equations for the constants. The simplification is too small, however, to justify rewriting the equations.

If the annular element has only two sections, e. g., an externally clad pipe, the equations will simplify still further. The resulting equations are given in Reference 7.

In any of the configurations considered above, the writing and use of a computer program may ease the burden of calculation.

3. APPLICATION IN NUCLEAR FUEL ELEMENTS

Fuel elements in stationary-fuel nuclear reactors consist of nuclear fuel, either metallic or ceramic, in any one of several geometric configurations, clad or sheathed with a second material which provides containment of fission products and protects the fuel from corrosion by the coolant. The more common nuclear fuel elements are cylindrical, flat plate, or variations of these. Comparison of the thermal stresses developed in various geometries adds to understanding and is useful for analysis or design.

In making comparisons of some simple fuel element configurations, all assumptions made earlier apply. The physical properties of fuel and cladding used for this comparison are given in Table I. In all cases, a constant heat generation rate of 50,000 BTU/hr.-in.^{3} is assumed in the fuel with no heat generation in the cladding. It is assumed that the elements remain within the elastic range regardless of the magnitude of stress calculated and are unstressed at the uniform base temperature. All cases are discussed on a one-dimensional basis i. e., temperature and stresses are functions of radius in cylindrical elements or location in the thickness in flat plate elements.

Case I

Consider a cylindrical element in which the fuel is a solid cylinder ($r_i = 0$) rather than an annulus (Fig. 2). Equations for the stresses developed are obtained by simplifying the equations given above or in Reference 7. Assuming an interface radius (r_c) of 0.134 in with a cladding thickness of 0.020 in., the outside radius (r_o) is 0.154 in. The temperature and stress distributions for this element under the above assumptions are given in Fig. 3.

Case II

Consider a cylindrical element with fuel in the form of an annulus (Fig. 2). Equations for the stresses are obtained by simplifying the equations given above or from Reference 7. Using the same quantity of fuel as in Case I with an inside radius (r_i) of 0.120 in., the interface radius (r_c) is 0.180 in. Using the same cladding thickness of 0.020 in., the outside radius (r_o) is 0.200 in. The temperature and stress distributions for this element are shown in Fig. 4.

Case III

Consider a cylindrical element (Fig. 1) with annular fuel clad on the inside and outside. The stresses are given by the equations given above. If the same fuel annulus is used as in Case II, the inside radius (r_i) is 0.100 in., the inner interface radius (r_1) is 0.120 in., the outer interface radius (r_2) is 0.180 in., and the outer radius (r_o) is 0.200. Assuming the same surface temperature at the inside and outside radii, the temperature and stress distributions for this element are given in Fig. 5.

Case IV

Consider a flat plate element (Fig. 6). The stresses are given in Reference 8. By taking a fuel zone which is 0.060 in. thick with 0.020 in. thick cladding on each side, an element is obtained which is essentially the same as Case III except in the form of a flat plate rather than a cylinder. Assuming the same surface temperature on both sides of the element, the temperature and stress distributions are those shown in Fig. 7.

4. COMPARISON OF CASES

Upon comparing Case I and II, one notes that the only difference between the two elements is the fuel is a solid cylinder in Case I and an annulus in Case II. This means, of course, a somewhat larger diameter for the element in Case II. A comparison of temperatures indicates a maximum temperature in case II about one-half that in Case I. The use of the annulus reduces the radial stress at the inside radius to zero. The compressive tangential stress and the compressive axial stress at the inside radius are reduced to about 75% and 40%, respectively, of those at the center line in the solid fuel cylinder. At the outside radius, both the tangential and axial stresses (tensile) in Case II are reduced to about 60% of those in Case I. In other words, the simple change from a solid fuel region to a hollow one produces an appreciable decrease in temperature and thermal stress magnitudes for the same heat output and surface temperature. The corollary is that, for the same allowable stress, the hollow element of Case II can produce more heat than the solid fuel element of Case I.

Comparing Case III (with cooling at both inside and outside surfaces) with Case II (with cooling at the outside surface only) one would expect an appreciable decrease in maximum temperature and stresses. One finds that maximum temperature in Case III is decreased to about one-half that in Case II. The maximum compressive tangential and axial stresses for Case III are about 65 and 60%, respectively, of those for Case II. The maximum tensile tangential and axial stresses for Case III are about 45 and 50%, respectively, of those for Case II.

Comparing Case IV with Case III, one finds small decreases in temperature and stress magnitude in going from the annular to the flat plate element. The stresses in the flat plate which are equivalent to the tangential and axial stresses in the annulus become indistinguishable. There is no stress in the flat plate equivalent to the radial stress in the annulus.

5. SOME PRACTICAL CONSIDERATIONS

In calculating the stresses developed in the above four cases, "limitless" elasticity was assumed. This assumption may often be questionable, as, for example, in Case I. In this instance, magnitudes of the calculated stresses may well be greater than the yield strength or the permissible creep strength at elevated temperature. In order to avoid excessive stresses and resultant dimensional changes (or even failure) in the element, the alternatives are reduction of the heat generation rate or use of other configurations.

In all four cases discussed above, the temperature distributions shown are based on an increment above a uniform base temperature which was arbitrarily taken as that of the outside surface of the cladding. The stress distributions shown are based on the assumption of zero initial thermal stress at the uniform base temperature. In the case of a body composed entirely of one material, or a body composed of two or more materials having the same moduli of elasticity, Poisson's ratios, and thermal coefficients of expansion, this assumption would be justified. If any one material in the composite body has one or more of these three properties different from those of the other materials, then there can be appreciable thermal stresses in the body at the uniform base temperature. In a fuel element, for example, the properties of the cladding may be appreciably different from those of the fuel and thereby introduce rather large initial thermal stresses. These initial stresses may add to (or subtract from) the thermal stresses developed by the temperature gradient depending on the relative magnitudes of the physical properties of the two materials. For example, if the fuel and cladding differ in physical properties only in that the fuel has a greater coefficient of thermal expansion than the cladding, a uniform increase in temperature would introduce compressive stresses in the fuel and tensile stresses in the cladding. Superposition of these on the stresses in any one of the four elements considered above would mean still greater stresses leading to premature failure of the element.

In calculating temperature, an assumption of no temperature drop at the interface between fuel and cladding was made for the purpose of easier calculations. The equations are equally valid if there is a temperature drop at the interface, provided all the other assumptions are justified.

In calculating stresses in the fuel elements, integral bonding between fuel and cladding was assumed. If there is no integral bonding between them, but the stress normal to the interface is compressive, the equations are still applicable. If the stress normal to the interface is tensile, the equations do not apply directly. The fuel and cladding must then be treated as separate members.

The development of equations for thermal stress makes no distinction between steady state and transient temperature gradients. The equations were stated in a general form with the actual temperature distribution equation undetermined. Consequently, the equations are equally valid for either steady state or transient conditions provided one can, in some fashion, determine the temperature distribution.

PART II SPHERICAL ELEMENTS

6. INTRODUCTION

Spherical elements are often made from two materials for use in reactor systems. These elements can be fuel elements e. g., the spherical graphite element (9). They can also be pressure vessels such as the Homogeneous Reactor Experiment No. 2 in which mild steel was used primarily for strength but was internally clad with stainless steel for corrosion resistance.

In spherical elements, as in annular, thermal stresses result from thermal gradients and from differences in physical properties and the specific geometry. Part II presents equations for thermal stresses in an externally clad spherical element composed of two arbitrarily chosen materials with a radially symmetric temperature distribution.

7. BASIC EQUATIONS

Derivation of the thermal stress equations for the clad sphere (Fig. 8) parallels that for the annular element with the same assumptions (through No. 5) applying. Under these conditions, the general equations for thermal stresses in clad spheres are:

In the inner sphere:

Radial Stress
$$\frac{\sigma_r}{E_A} = - \frac{2\alpha_A}{r^3(1 - \mu_A)} \int_{r_i}^{r} r^2 T_A dr + C_1\left(1 - \frac{r_i^3}{r^3}\right) \qquad (10)$$

Tangential Stress
$$\frac{\sigma_\theta}{E_A} = \frac{\alpha_A}{r^3(1-\mu_A)}\int_{r_i}^{r} r^2T_A dr + C_1\left(1+\frac{r_i^3}{2r^3}\right) - \frac{\alpha_A T_A}{1-\mu_A} \quad (11)$$

In the outer sphere:

Radial Stress
$$\frac{\sigma_r}{E_B} = -\frac{2\alpha_B}{r^3(1-\mu_B)}\int_{r_c}^{r} r^2T_B dr + C_2\left(1-\frac{r_o^3}{r^3}\right) + \frac{2\alpha_B}{r^3(1-\mu_B)}\int_{r_c}^{r_o} r^2T_B dr \quad (12)$$

Tangential Stress
$$\frac{\sigma_\theta}{E_B} = \frac{\alpha_B}{r^3(1-\mu_B)}\int_{r_c}^{r} r^2T_B dr + C_2\left(1+\frac{r_o^3}{2r^3}\right) - \frac{\alpha_B}{r^3(1-\mu_B)}\int_{r_c}^{r_o} r^2T_B dr - \frac{\alpha_B T_B}{1-\mu_B} \quad (13)$$

These thermal stress equations contain two constants, C_1 and C_2, which are functions of physical properties, geometry, and temperature distribution. For the purpose of shortening the expressions, let

$$K_1 = 1+\mu_A \qquad K_4 = 1-\mu_B$$
$$K_2 = 1+\mu_B \qquad K_5 = 1-2\mu_A$$
$$K_3 = 1-\mu_A \qquad K_6 = 1-2\mu_B$$

In terms of a common factor, Δ, the constants in the thermal stress equations are:

$$\Delta C_1 = 2\left[(2K_6r_c^3 + K_2r_o^3)\left(\frac{\alpha_A}{K_3}\int_{r_i}^{r_c} r^2T_A dr + \frac{E_B}{E_A}\frac{\alpha_B}{K_4}\int_{r_c}^{r_o} r^2T_B dr\right) - \frac{E_B}{E_A}(r_o^3 - r_c^3)\left(\frac{K_1\alpha_A}{K_3}\int_{r_i}^{r_c} r^2T_A dr + \frac{K_2\alpha_B}{K_4}\int_{r_c}^{r_o} r^2T_B dr\right)\right]$$

$$C_2 = 2\left[(r_c^3 - r_i^3)\left(\frac{K_1\alpha_A}{K_3}\int_{r_i}^{r_c} r^2T_A dr + \frac{K_2\alpha_B}{K_4}\int_{r_c}^{r_o} r^2T_B dr\right) + (2K_5r_c^3 + K_1r_i^3)\left(\frac{\alpha_A}{K_3}\int_{r_i}^{r_c} r^2T_A dr + \frac{E_B\alpha_B}{E_AK_4}\int_{r_c}^{r_o} r^2T_B dr\right)\right]$$

where the common factor, Δ, is:

$$\Delta = (r_c^3 - r_i^3)(2K_6r_c^3 + K_2r_o^3) + \frac{E_B}{E_A}(r_o^3 - r_c^3)(2K_5r_c^3 + K_1r_i^3)$$

These equations are for a hollow sphere. If the inner sphere is solid, as in a possible nuclear fuel element, the inner radius, r_i, is replaced by zero.

8. APPLICATION IN NUCLEAR FUEL ELEMENTS

To demonstrate the use of these equations, consider the case of a spherical fuel element having a solid core $(r_i = 0)$ with an interface radius of 0.16 in $(r_c = 0.16)$ and a cladding thickness of 0.020 in $(r_o = 0.180$ in). The physical properties of fuel and cladding are given in Table 1. A constant heat generation rate of 50,000 BTU/hr-in^3 is assumed in the fuel with no heat generation in the cladding. The temperature and stress distributions for this element are shown in Fig. 9.

As a further illustration, consider a spherical fuel element having an inside radius of 0.120 in $(r_i = 0.120)$, an interface radius of 0.180 in $(r_c = 0.180)$, and a cladding thickness of 0.020 in $(r_o = 0.200$ in). Using physical properties for fuel and cladding given in Table 1 and assuming a constant heat generation rate of 50,000 BTU/hr-in^3 in the fuel with no heat generation in the cladding, the temperature and stress distribution for this element are those shown in Fig. 10.

A number of practical considerations were discussed in Part I for cylindrical elements. These same comments apply also to the case of spherical elements.

9. PROBABILISTIC CALCULATIONS

It is obvious that the temperature and stress distributions shown in Figs. 3, 4, 5, 7, 9, and 10 are based on deterministic calculations, i.e., the parameters used have single values which are effectively mean values. The use of a mean implies a range of values for individual parameters. If the measured values of these parameters are normally distributed, the effect of range can be expressed in terms of standard deviation. Since each parameter can vary in a random manner, independently of all the other parameters, the resulting temperature or stress can also be expected to have a range of values.

Assuming normal distributions for the parameters, the temperature and stresses will also be normally distributed. The Algebra of Normal Functions (10, 11) can be applied to calculate means and standard deviations for temperature and stress distributions based on means and standard deviations for the parameters. Using this Algebra with the values of the parameters given in Table 2, temperature and stresses were calculated for the case of the hollow sphere for which deterministic values are shown in Fig. 10. The probabilistic means, with the values within which ninety percent of the calculated results will fall, are shown in Fig. 11. The probabilistic means are not the same as the deterministic means although the differences are rather small in some situations. The calculation of standard deviation allows an indication of what range of values may be expected in the course of events when each parameter can be expected to have any random value within a given range.

Applying the maximum shear theory at the inside and outside radii with the yield strengths given in Table 3 gives some interesting results. At the outside radius, the safety factor is 3.83 but the reliability is 0.9999999, indicating one failure in ten million, on the average. At the inside radius, the safety factor is 1.82. This might be considered an acceptable value. The reliability, however, is only 0.84, i. e., on the average, sixteen out of one hundred would be expected to "fail". This appears to be intolerably high.

The apparent contradiction lies in the fact that use of a safety factor gives no real indication of whether a design is proper, over-designed, or under-designed. The key lies in the fact that the standard deviations of the stresses developed and of the materials used are included in the calculation to give a more meaningful design based on reliability. In this example, the low reliability is due to the relatively large standard deviations of the radial and tangential stresses. In other words, safety factor becomes less meaningful in design as the ranges of the parameters involved become larger.

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Table I

PHYSICAL PROPERTIES OF MATERIALS

	FUEL	CLADDING
Thermal Conductivity, k, BTU/hr.-ft.-^{o}F.	17	10
Modulus of elasticity, E, lbs./in.^{2}	25×10^{6}	12×10^{6}
Poisson's ratio, μ,	0.20	0.35
Coefficient of thermal expansion, α, ^{o}F.$^{-1}$	15×10^{-6}	3.5×10^{-6}

Table II

DIMENSIONS OF PARAMETERS

	Fuel			Cladding		
	mean	standard deviation %	standard deviation	mean	standard deviation %	standard deviation
Thermal Conductivity, k BTU/hr-ft^{o}F	17	3	0.51	10	3	0.30
Modulus of Elasticity, E lbs/in^{2}	25×10^{6}	2	5×10^{5}	12×10^{6}	2	2.4×10^{5}
Poisson's Ratio, μ	0.20	2	0.0040	0.35	2	0.0070
Coefficient of thermal expansion, α ^{o}F^{-1}	15×10^{-6}	3	4.5×10^{-7}	3.5×10^{-6}	3	1.05×10^{-7}
Heat Generation, G BTU/hr-in^{3}	50,000	8	4000	0	0	0
Radius, r, in	r	1	0.01r	r	1	0.01r

Table III

TENSILE YIELD STRENGTHS

	Yield Strength (0.2% offset), psi	
	mean	standard deviation
Fuel	35,000	3000
Cladding	49,000	5500

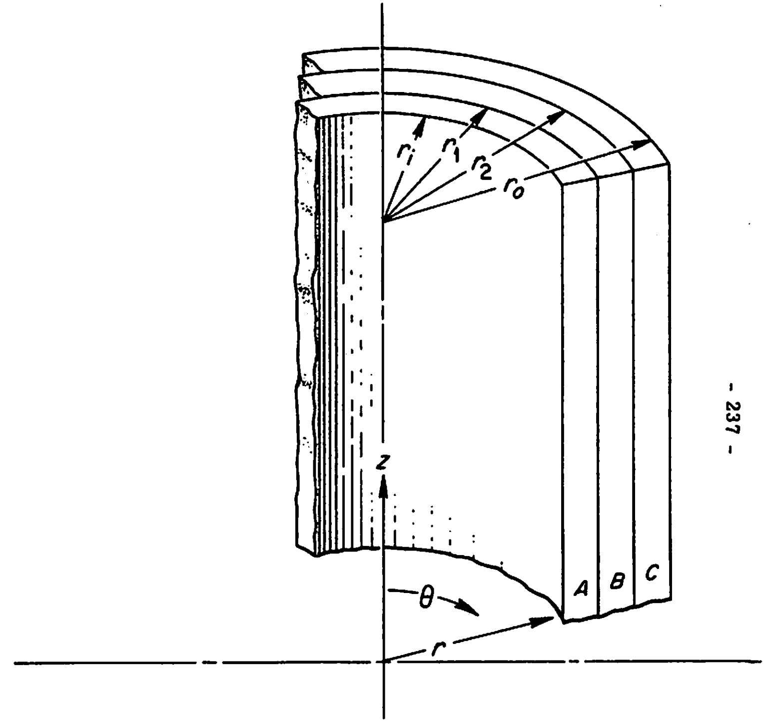

Fig. 1: Section of Internally and Externally Clad Cylindrical Element

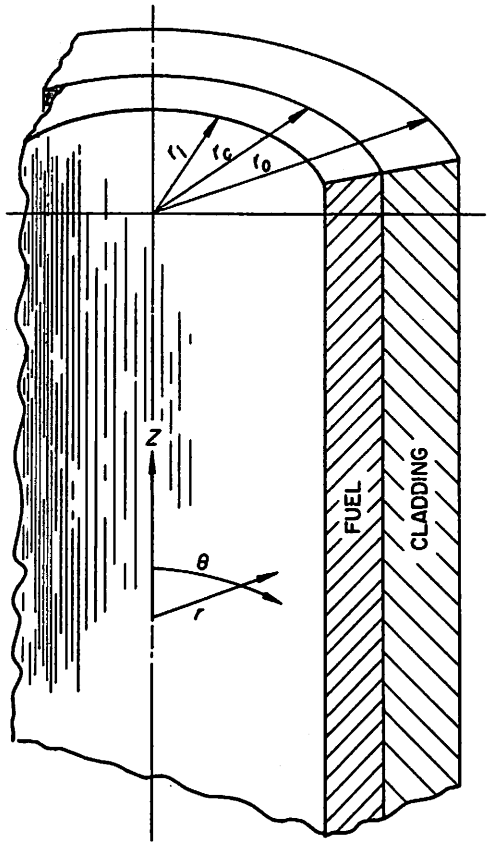

Fig. 2: Section of Externally Clad Cylindrical Element

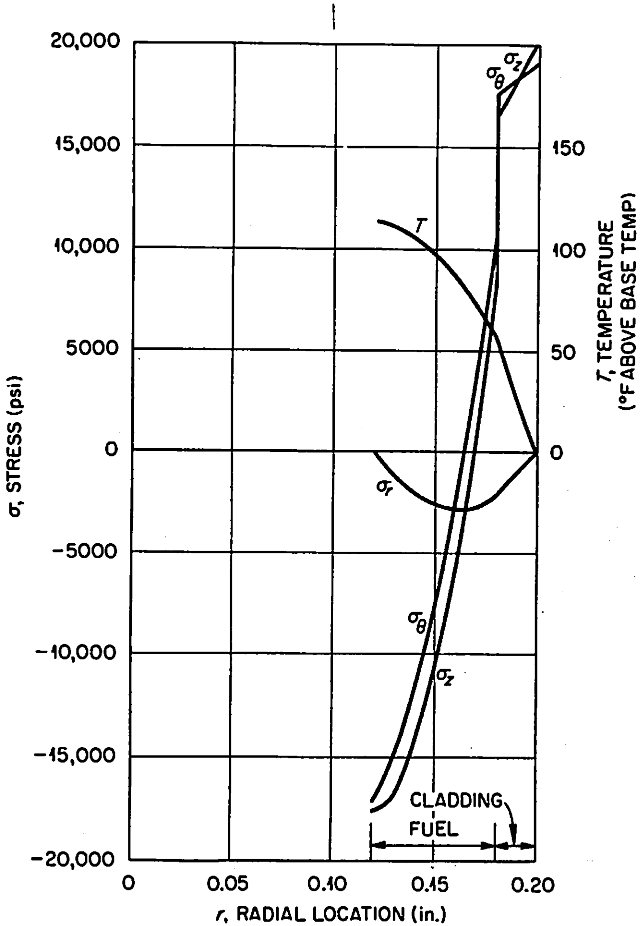

Fig. 4: Temperature and Thermal Stress Distribution in Externally Clad Annular Fuel Element

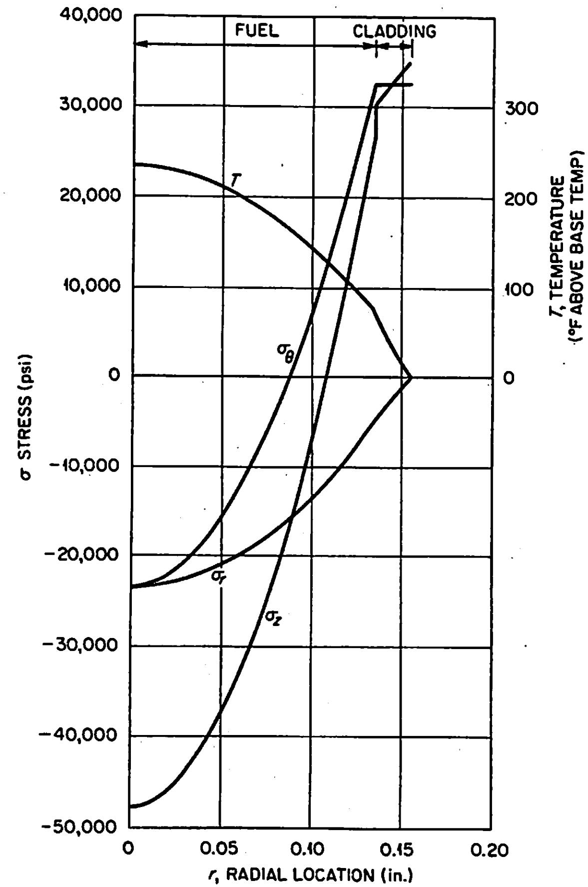

Fig. 3: Temperature and Thermal Stress Distribution in Clad Solid Cylindrical Fuel Element

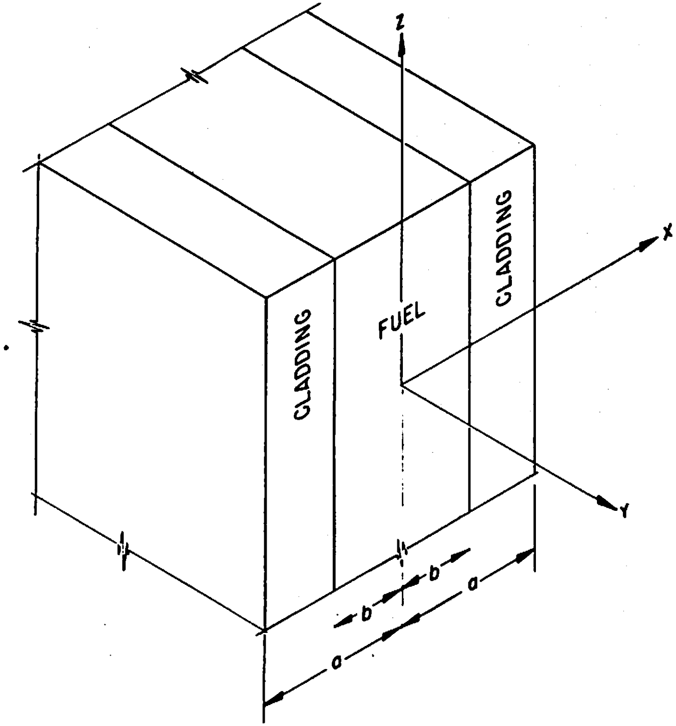

Fig. 6: Section of Clad Flat Fuel Plate

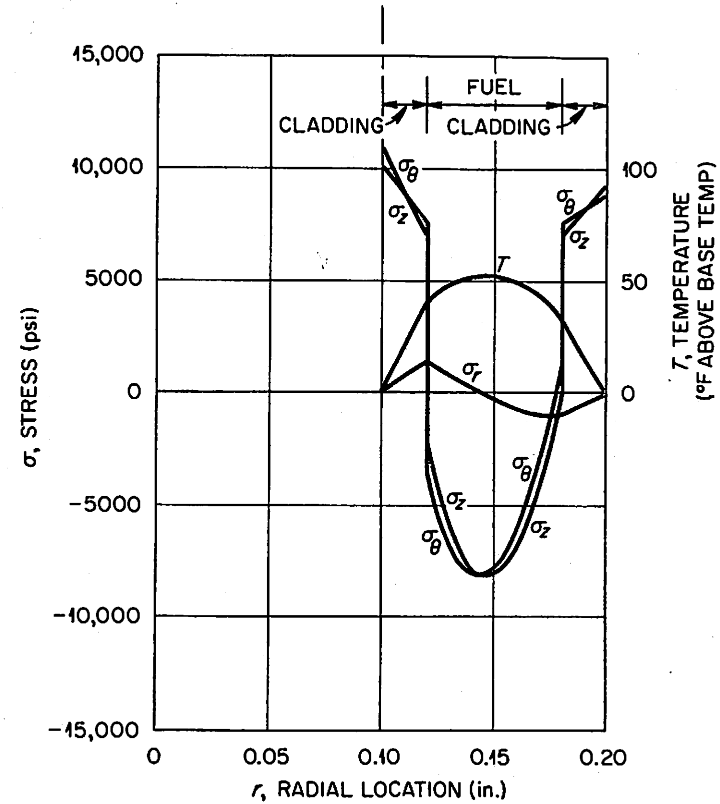

Fig. 5: Temperature and Thermal Stress Distribution in Internally and Externally Clad Annular Fuel Element

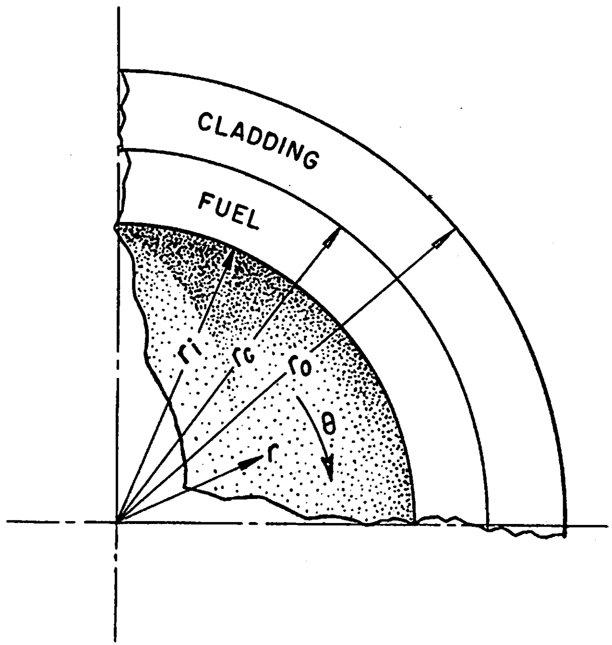

Fig. 8: Section of Externally Clad Spherical Element

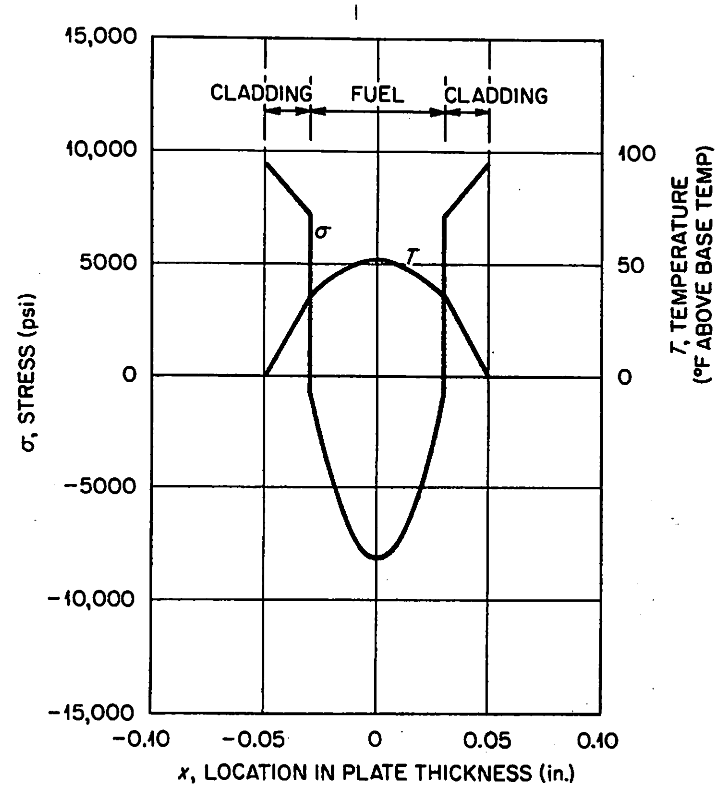

Fig. 7: Temperature and Thermal Stress Distribution in Clad Flat Plate Fuel Element

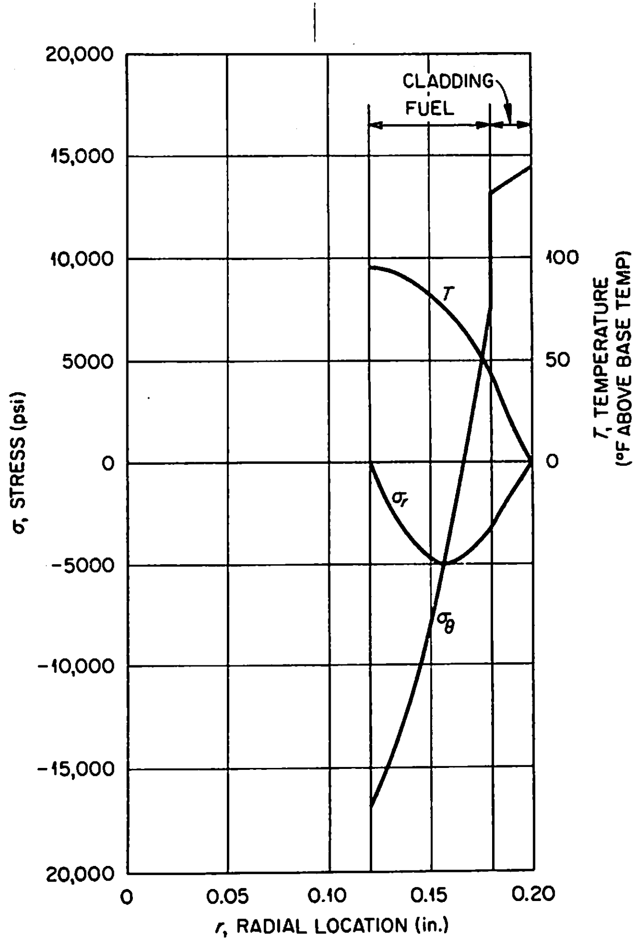

Fig. 10: Temperature and Thermal Stress Distribution in Externally Clad Hollow Spherical Fuel Element

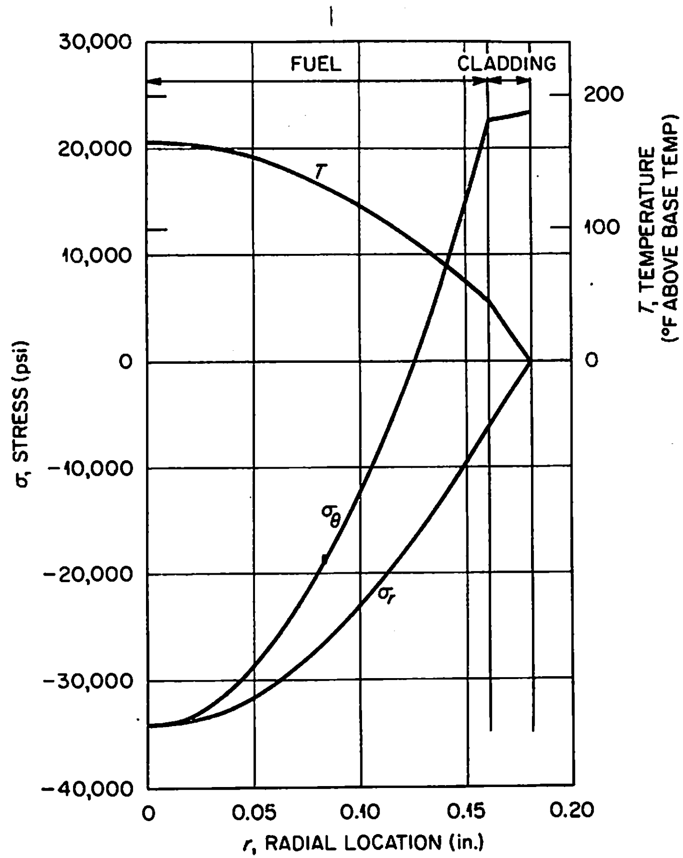

Fig. 9: Temperature and Thermal Stress Distribution in Clad Solid Spherical Fuel Element

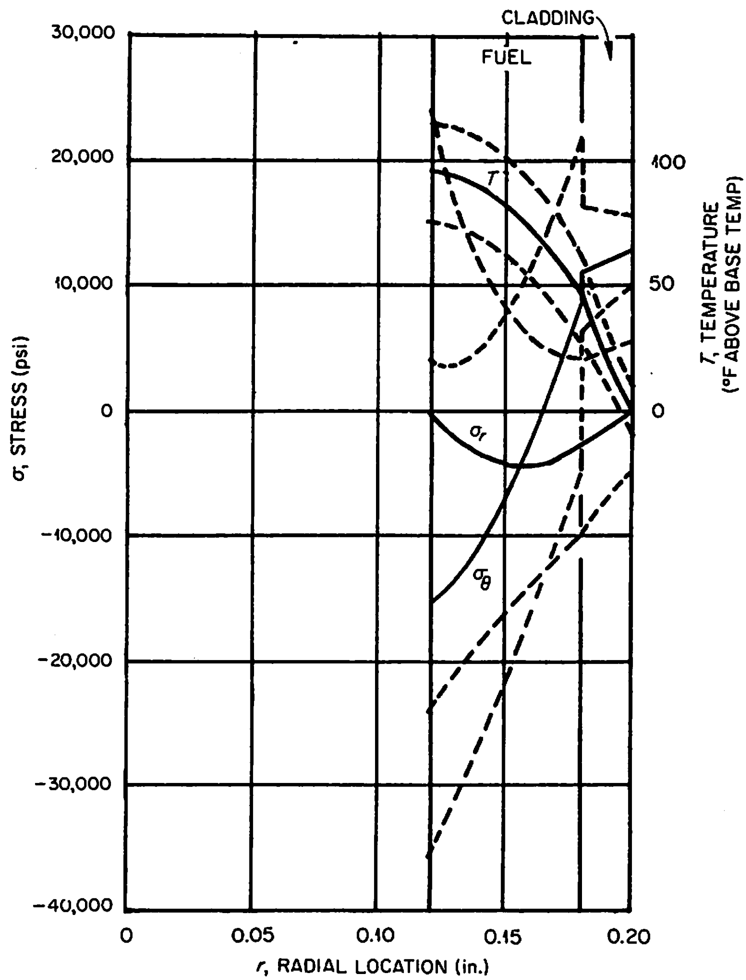

Fig. 11: Ninety Percent Range of Temperature and Thermal Stress Distribution in Externally Clad Hollow Spherical Fuel Element

DISCUSSION

Q R. A. VALENTIN, U. S. A.

In your examples you refer to uniform internal heat generation to develop the temperature fields for which your analysis was performed. Even for a perfectly homogeneous, isotropic material one should consider another geometric effect when comparing cylindrical, spherical, and annular elements. That is, in a thermal flux one always has some self-shielding and hence true uniform internal heat generation is not possible. While perhaps of minor importance, it is primarily a geometric effect and, as such, would make an interesting added parameter in thermal stress calculations - at least for simple geometries where the diffusion approximations are known in closed-form.

A C. O. SMITH, U. S. A.

The assumption of uniform heat generation was made only for the numerical examples. The only assumption on temperature which restricts the equations is one of radially symmetric temperature. The assumption of uniformity was made only to simplify the calculations.

Q R. HAUSERMANN, Switzerland

1. Is it right that your model does not include a gap ? If yes, this at the BOL is not realistic.
2. Particular interest in PWR fuel, where the cladding is not freestanding and we have or might have a line contact between fuel and cladding. For that we would have a temperature as follows: $T = f(r, \theta)$ and not, as you assume, $T = f(r)$.

A C. O. SMITH, U. S. A.

1. No gap is assumed. If there is a gap, then the two segments can be treated separately and the results combined as appropriate.
2. The equations as developed do not take line or point contact nor angular variation of temperature into account. I believe the angular (or axial) variation is generally smaller than the radial variation of temperature. In any event, the equations could be used as approximations.

C Z. J. HOLY, Australia

I would like to draw attention to a report by J. Whatham, which deals with a shelled type spherical fuel element with heat producing kernel. Here axisymmetric heat transfer variation in the gap is treated including the heat radiation effects. A paper by Whatham and Hawker "A study of the shell thermal stress distribution in a loose kernel type of spherical fuel element for a pebble bed reactor" describing this work is appearing in Nuclear Engineering and Design, 21 (1972) No. 1.

Q H. WALTHER, Italy

This is more a comment than a question. I was impressed by a concluding remark of Prof. Pister he made yesterday: We should always try to give to the computer the task of doing the routine work. I find the foregoing questions to be connected with this remark. It might be convenient not to try always to arrive to a final equation as result, as it may happen that you need to explore just a case that the equation cannot: e. g. , inhomogeneous or anisotropic conditions, or a gap. Introducing just the general solution for a spherical zone for instance, and the boundary conditions separately into the computer would be the method of discussing almost all cases: in spherical geometry for instance anisotropy of elasticity, plasticity, creep, thermal- and irradiation induced expansion.

A C. O. SMITH, U. S. A.

I concur that I like the approach of a rather general solution which can then either be specialized or can be studied for the effects of perturbations of various parameters.