STRESS ANALYSIS OF A BIMETALLIC CYLINDRICAL JOINT

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ABSTRACT

The paper describes the thermo-elasto-plastic stress problem of a bimetallic cylindrical joint, formed by an explosive welding technique of a Zr Nb pressure tube to a stainless steel primary circuit.

A finite element technique, using a triangular constant strain element, has been used. Local yielding is accounted for by assuming the von Mises'yielding hypothesis and an isotropic strainhardening rule.
I. INTRODUCTION

A pressure tube reactor consists of a bundle of pipes which are surrounded by the moderator and which contain fuel and coolant. As compared to the homogeneous solution with one big pressure vessel, the heterogeneous solution with pressure tubes possesses advantages concerning neutron economy due to the low temperature of the moderator and the free choice of coolant and moderator independent of each other. As concerns the characteristic of the in-core part of the pressure tube, for reasons of neutron economy a material with low neutron capture cross-section is preferred. Various pressure tube reactor projects throughout the world have selected zirconium alloys for the in-core part of the pressure tubes, while stainless steel is used for the remaining part of the primary circuit. A major problem that arises is how to realize the joint between the zirconium alloy tubes and the remaining part of the primary circuit, bearing in mind the relative small space available, due to the small pitch/diameter ratio of typical pressure tube reactor projects. Among other solutions which have been developed, the joint realized by an explosive welding technique \[1\] has shown advantages, especially with respect to the space needed (fig. 1a). This paper deals with the stress-analysis of an explosive joint between a Zircalloy 2.5 Nb pressure tube and a stainless steel tube AISI 304 L for the ESSOR reactor. Due to the difference in thermal expansion of the two materials, rather high thermal stresses are generated when the joint is heated up from fabrication temperature (20°C) to working temperature (300°C).

For an infinitely long joint conditions of plane strain exist: the stress analysis is straightforward (see appendix 1). For a joint of finite length a finite element technique has been employed to study the stress distribution in the joint. To this end an axisymmetric two dimensional code based on triangular ring elements has been developed.

Local yielding is accounted for by using the classical von Mises's criterion for yielding, together with the classical flow rule and an isotropic strainhardening rule.

II ELASTIC ANALYSIS

Finite element solutions for two dimensional elastic problems are now of common use. The code which has been developed (ROTEMF) enables to solve 2 dimensional problems including thermal stresses. The method for the solution which has been adopted is the partitioned stiffness matrix method, which inverts the stiffness matrix by blocks. By storing the inverted partitioned matrix in the auxiliary memory, little additional computer time is needed to solve various loading cases. \[2 \]. The mesh which has been used is depicted in fig. 2.
The mesh is generated automatically by the program MESHGEN. The local refinement is performed by the program MESHREF. This program can handle different blocks of meshes, and combine or overlap them. The program takes care of renumbering of nodes and elements in order to keep the band width small. The meshes are plotted automatically by the program MESHPLOT. The elastic calculation of the joint is performed for the conditions of 10 atm internal pressure and 300°C (that corresponds to 280°C temperature increase). The results of the calculation is plotted in fig. 5. These plots are obtained by the program STRESS PLOT which can handle the output tape of ROTTEMP, and makes a plot of the deformed contour and a plot of isovariables defined at the nodes or at the centroids of the elements.

III ELASTO-PLASTIC SOLUTION

Using the notation of Zienkiewicz and Cheung [2] the matrix equation governing the elasto-plastic solution can be written as:

\[
\begin{align*}
\sum_{k=1}^{\text{NELEM}} (B_k)^T [D_k] [B_k] \begin{bmatrix} u \end{bmatrix} &= [p] + [p_o] \\
[p_o] &= \sum_{k=1}^{\text{NELEM}} (B_k)^T [D_k] \begin{bmatrix} \varepsilon_{0,k} \end{bmatrix} , \begin{bmatrix} \sigma_k \end{bmatrix} = [D_k] \left( \begin{bmatrix} \varepsilon_k \end{bmatrix} - \begin{bmatrix} \varepsilon_{0,k} \end{bmatrix} \right) \\
[\varepsilon_k] &= [B_k] \begin{bmatrix} u \end{bmatrix} , \begin{bmatrix} \varepsilon_{0,k} \end{bmatrix} = [\varepsilon_{P,k}] + [\alpha T_k]
\end{align*}
\]  

The right hand side of eq. 1a consists of two parts: the loadvector \([p]\) due to external forces and the loadvector \([p_o]\) due to initial stresses. The initial strain vector \([\varepsilon_0]\) in a thermo-elastic plastic problem can be separated in plastic strains \([\varepsilon_P]\) and thermal dilatation strains \([\alpha T]\), as shown by eq. 1e. The latter is usually included in the linear elastic computer programs. The plastic strain depends on stress and stress history, which makes an incremental loading procedure necessary. The procedure followed in this paper to define the plastic strain increment due to a stress increment for elements in which the yielding criterion is violated is described in appendix 2. The program is called ROTPLAST.

In appendix 3 is described the method which has been adopted to devide the structure into substructures. Substructures which from the elastic solution are expected to remain elastic are considered as one macroelement, having nodes only at the intersection with another substructure. The elasto-plastic solution of the bimetallic joint is carried out by assuming
stress-strain relations for the two materials as given in fig. 3a. In fig. 6 and 7 are given the results as obtained by means of the program STRESS PLOT. Figure 6 shows the deformed contour and the equivalent stress levels for nominal conditions of 10 atm and 300°C. Figure 7 shows the deformed contour and the equivalent stress levels which remain in the joint after unloading due to plastic deformation.

Appendix 1

Stress distribution in an infinitely long cylindrical bimetallic joint

For the infinite bimetallic joint (fig. 1b) the stress distribution can be determined by elementary theory due to the conditions of plane strain.

We introduce strains (ε) and stresses (σ) with subscripts a and t for axial and tangential direction respectively and subscripts 1 and 2 for cylinders 1 and 2 respectively.

The stress and strain distribution for thin cylinders (δε/R ≪ 1) is defined by 8 unknowns:

\[ \varepsilon_{a,1}, \varepsilon_{a,2}, \varepsilon_{t,1}, \varepsilon_{t,2} \text{ and } \]
\[ \sigma_{a,1}, \sigma_{a,2}, \sigma_{t,1}, \sigma_{t,2} \]

To define the eight unknowns we have 8 equations resulting from compatibility, equilibrium and stress-strain relations.

1. Compatibility

From the condition of plane strain and radial contact

\[ \varepsilon_{a,1} = \varepsilon_{a,2} \]
\[ \varepsilon_{t,1} = \varepsilon_{t,2} \] (1)

2. Equilibrium

The classical equations for thin cylinders can be written as:

\[ \sigma_{t,1} \delta_1 + \sigma_{t,2} \delta_2 = pR \] (2)
\[ \sigma_{a,1} \delta_1 + \sigma_{a,2} \delta_2 = \frac{pR}{2} \]

3. Stress-strain relations

The generalized Hooke's law including thermal strains can be written as:

\[ \varepsilon_{t,1} = \frac{1}{E_1} \left( \sigma_{t,1} - \nu \sigma_{a,1} \right) + \alpha_1 \frac{T_1}{1} \quad i = 1, 2 \]
\[ \varepsilon_{a,1} = \frac{1}{E_1} \left( \sigma_{a,1} - \nu \sigma_{t,1} \right) + \alpha_1 \frac{T_1}{1} \quad i = 1, 2 \] (3)
4. Solution

The solution of the system can be written as:

\[ \alpha_{a1} = D_{12} \frac{dR}{d\alpha_1} - D_{21} \frac{E_1}{1-v} (\alpha_1 - \alpha_2) \]
\[ \alpha_{t1} = D_{12} \frac{dR}{d\delta_1} - D_{21} \frac{E_1}{1-v} (\alpha_1 - \alpha_2) \]
\[ \alpha_{a2} = D_{21} \frac{dR}{d\delta_2} + D_{12} \frac{E_2}{1-v} (\alpha_1 - \alpha_2) \]
\[ \alpha_{t2} = D_{21} \frac{dR}{d\delta_2} + D_{12} \frac{E_2}{1-v} (\alpha_1 - \alpha_2) \]

\[ D_{12} = \frac{E_1 \delta_1}{E_1 \delta_1 + E_2 \delta_2} \quad , \quad D_{21} = \frac{E_2 \delta_2}{E_1 \delta_1 + E_2 \delta_2} = 1 - D_{12} \]

The elastic part of the strains becomes:

\[ \varepsilon_{a1} = \frac{1-2v}{E_1} D_{12} \frac{dR}{d\delta_1} - D_{21} (\alpha_1 - \alpha_2) \]
\[ \varepsilon_{t1} = \frac{1-2v}{E_1} D_{12} \frac{dR}{d\delta_1} - D_{21} (\alpha_1 - \alpha_2) \]
\[ \varepsilon_{a2} = \frac{1-2v}{E_2} D_{21} \frac{dR}{d\delta_2} + D_{12} (\alpha_1 - \alpha_2) \]
\[ \varepsilon_{t2} = \frac{1-2v}{E_2} D_{21} \frac{dR}{d\delta_2} + D_{12} (\alpha_1 - \alpha_2) \]

5. Numerical Application to the ESSOR reactor pressure tube

Tube 1 - Material AISI 304 L

\[ E_1 = 17 \times 10^3 \text{ kg/mm}^2 \] (300°C)
\[ v = 0.3 \]
\[ \alpha_1 = 18 \times 10^{-6} \text{ °C}^{-1} \]
\[ T = 300°C \]
\[ \delta_1 = 3 \text{ mm} \]
\[ R = 49 \text{ mm} \]

Tube 2 - Material Zr 2.5 Nb

\[ E_2 = 9 \times 10^3 \text{ (300°C)} \]
\[ v = 0.3 \]
\[ \alpha_2 = 6.5 \times 10^{-6} \text{ °C}^{-1} \]
\[ T = 300°C \]
\[ \delta_2 = 2.5 \text{ mm} \]
5.1 Thermal stresses

\[ D_{1,2} = 0.694 \quad D_{2,1} = 0.300 \]

\[ \sigma_{a,1} = \sigma_{b,1} = -24 \text{ kg/mm}^2 \]

\[ \sigma_{a,2} = \sigma_{b,2} = 28.8 \text{ kg/mm}^2 = -\sigma_{a1} \frac{\Delta t}{\Delta t} \]

5.2 Mechanical stresses

The pressure in the ESSOR reactor is rather low, due to the fact that an organic liquid is used as coolant agent.

The mechanical stresses for a pressure of 10 atm in the pressure tube of Zr 2.5 Nb are:

\[ \sigma_t = \frac{1}{2} \sigma_a = 2 \text{ kg/mm}^2 \]

For an infinite joint this stress level has to be multiplied by \( D_{1,2} \) and \( D_{2,1} \) for the stainless steel respectively Zr 2.5 Nb part of the joint.

5.3 Elasto-plastic solution

For the case of the ESSOR reactor the mechanical stresses are low, and by neglecting them a first guess of the plastic strains induced in an infinite joint due to thermal stresses can be obtained immediately. We consider the stress strain relations of the two materials as depicted in fig. 3a. We replott them in fig. 3b with different scales i.e. the stainless steel one unchanged in the first quadrant, the Zr 2.5 Nb one on an ordinate scale of \( \frac{2.2}{0.1} \) in the second quadrant. By drawing a horizontal line of length \( (\alpha_t - \alpha_a) \frac{T}{(1 - \nu)} \) between the two curves the stress level and the elastic and plastic strains in the tube are found. It is seen that the stress level in the infinite joint is reduced from (-24) resp 28.8 to-19.6 resp. 23.5. The plastic strain is equal to \( \frac{7.5}{10^3} \). The residual stress at \( T=0 \) is negligible (point 3)

Appendix 2

Finite Element formulation for elasto-plastic problems.

In chapter III the formulation of the thermo-elasto-plastic problem is given. It can be written as

\[ [K][u] = [F] + [p_o] \quad (1) \]

The vector \([p_o]\) represents the load vector due to initial strains

\[ [p_o] = [K_1][\varepsilon_o] = [K_1]\left([\varepsilon_p] + [\alpha T]\right) \quad (2) \]

The plastic strain vector \([\varepsilon_p]\) depends on stress history: an incremental loading procedure is necessary to follow up the evolution of the plastic strain \([3], [4], [5]\). The
procedure can be summarized as follows: A fraction of the load vector \([\mathbf{p}]\) is applied, giving rise to stresses beyond the yielding point in a limited number of elements. These elements are allowed to relax, by introducing appropriate plastic strains. The plastic strains are inserted in the right hand side member of eq. 1 according to eq. 2. A new stress distribution is found to be confronted with a new yielding point due to strain hardening. The iteration can be continued until the plastic strain increments become small, and a next load step can then be applied. This technique has been used in several papers\([3]\), \([4]\), \([5]\), the difference lying in the strategy to determine the plastic strain increments and the use of different hardening rules.

In this paper an isotropic hardening rule is used. The yield stress in a three dimensional state of stress depends then on one single parameter: the equivalent plastic strain. The relation ship can be determined from a stress-strain curve obtained by a monoaxial tensile experiment.

Adopting the von Mises' yield hypothesis the equivalent stress and the yield criterion can be written as:

\[
b^2 = \frac{1}{2} \left[ \left( \sigma_1 - \sigma_2 \right)^2 + \left( \sigma_2 - \sigma_3 \right)^2 + \left( \sigma_3 - \sigma_1 \right)^2 \right] \tag{3}
\]

\[
\sigma_y - b^2 = 0 \tag{4}
\]

The yield stress \(\sigma_y\) is a function of the equivalent strain \(\varepsilon_p\):

\[
\sigma_y = f(\varepsilon_p) \quad \text{(strain hardening coefficient)} = \frac{d\sigma_y}{d\varepsilon_p} \tag{5}
\]

The equivalent plastic strain is related to the normal plastic strain vector by identifying the speed of energy dissipation

\[
\dot{\varepsilon}_p = \mathbf{c}^T \left[ \dot{\varepsilon}_p \right] \tag{6}
\]

Finally we introduce the flow rule according to Drucker:

\[
[\dot{\varepsilon}_p] = \lambda \frac{\partial b^2}{\partial [\sigma]} \tag{7}
\]

The relaxation process can now be described as follows. Suppose an element with equivalent plastic strain \(\varepsilon_p\) and corresponding yield stress and strain hardening coefficient \(\sigma_y\) and \(c\) respectively.

A load step is applied and assume that according to the elastic solution described by eq. 1 the equivalent stress \(s\) becomes such that the yield criterion is violated:

\[
s^2 - \sigma_y^2 > 0. \tag{8}
\]

We introduce plastic strain increments defined by the flow rule

\[
[\Delta \varepsilon_p] = [\dot{\varepsilon}_p] (\Delta t) = \lambda (\Delta t) \frac{\partial b^2}{\partial [\sigma]} \mu \frac{\partial b^2}{\partial [\sigma]} \tag{9}
\]

The positive quantity \(\mu\) is still unknown, but it can be determined from the condition that at the end of the relaxation process the yield criterion should be satisfied.
Due to the plastic strain increments the stresses will relax by an amount:

\[
[\Delta \sigma] = -[D] [\Delta \varepsilon_p] = -\mu [D] \frac{\partial \sigma^2}{\partial \sigma} \tag{9}
\]

([D] stiffness matrix according to Hooke's law)

The equivalent stress will change by an amount:

\[
\Delta \sigma = \frac{\partial \sigma}{\partial \sigma} [\Delta \sigma] = -\mu \frac{\partial \sigma^2}{\partial \sigma} T [D] \frac{\partial \sigma^2}{\partial \sigma} \tag{10}
\]

Due to strainhardening the yield stress will change by an amount:

\[
\Delta \sigma_y = c \Delta \varepsilon_p \tag{11}
\]

According to the definition of equivalent plastic strain (eq. 6) we can rewrite eq 11

\[
\Delta \sigma_y = \mu \frac{c}{2} \frac{\partial \sigma^2}{\partial \sigma} T [D] \frac{\partial \sigma^2}{\partial \sigma} = 2 \mu c s \tag{12}
\]

In order that the yield criterion is satisfied we obtain:

\[
s + \Delta \sigma = (\sigma_y + \Delta \sigma_y) = 0 \tag{13}
\]

\[
s - \mu \frac{\partial \sigma^2}{\partial \sigma} T [D] \frac{\partial \sigma^2}{\partial \sigma} - \sigma_y - 2 \mu c s = 0 \tag{14}
\]

\[
\mu = \frac{2s (s - \sigma_y)}{\frac{\partial \sigma^2}{\partial \sigma} T [D] \frac{\partial \sigma^2}{\partial \sigma} + 4c s^2} \tag{15}
\]

From eq. 8 the plastic strain increment vector \([\Delta \varepsilon_p]\) can then be determined, and this vector is introduced into eq. 1. Equation (15) is similar to the expression given in [3], [4] and [5], the difference lying in the fact that for isotropic strain hardening the relaxation depends only on the equivalent of plastic strain and corresponding yield stress: the individual stress components before applying the step are not needed which gives advantages for program organisation.

The plastic strain rate vector \([\dot{\varepsilon}_p]\) defined by the flowrule (eq. 7) will rotate during the relaxation process. In this paper it has therefore been preferred to apply small load steps, rather than taking a few big ones and iterate to satisfy the yielding criterion. Since the numerical work for an iteration is nearly equal to the one for a loading step, the strategy of a lot of smaller load steps without iteration gives more precise results for the same number of elastic solutions.
Appendix 3

In chapter III and appendix 2 it has been argued that for elasto-plastic problems an incremental loading procedure has to be applied, in order to follow up the evolution of the plastic strain. Each loadstep, and each iteration within a load step, is performed by an elastic solution. For cases where the plastic deformation is local, considerable computer time can be saved by introducing the concept of substructures, consisting of parts of the structure which remain elastic. The substructures can be considered as macro finite elements, to be defined by nodes. For proportional loading, it is sufficient to define them by the nodes of the intersection. If the load is not proportional, nodes with loads have to be included in the definition. The determination of the stiffness matrix of the macroelement and the conversion of physical loads on the substructure into nodal forces can be carried out by a standard finite element program. A mesh is generated in the substructure and by means of the program MESHREF a renumbering is carried out, such that only the degrees of freedom corresponding to the nodes defining the macroelement (see above) belong to the last partitioning. By means of a block elimination (see Zienkiewicz [2]) we can write:

\[
[K_{i+1}] = [K_{i+1}] - [C_i]^T[K_i]^{-1}[C_i] \quad (1)
\]

\[
[P_{i+1}] = [P_{i+1}] - [C_i]^T[K_i]^{-1}[P_i] \quad (2)
\]

\[
[K_N][u_N] = [P_N] \quad (3)
\]

Equation (3) is completely compatible with the concept of finite elements.

In the example of this paper the concept of substructures is also used to represent the long cylinders attached to the weld, which might be subject to modifications. The stiffness matrix of a long cylinder can be found by iteration since \([K_1] = 2 \rightarrow N\) and \([C_1] = 1 \rightarrow N-1\) (fig. 2) remain unchanged: it is sufficient to generate \([K_1][K_2]\) and \([C_1]\) and repeat the procedure described by equations (1), (2) and (3).
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Elasto-plastic solutions of engineering problems
Initial Stress finite element approach

Discrete Element Methods for the Plastic Analysis of Structures subjected to Cyclic Roading (NASA CR 803, 1967.)
GEOMETRICAL LAYOUT OF EXPLOSIVE WELD

FIG. 1a

INFINITE CYLINDRICAL BIMETALLIC JOINT
(CONDITION OF PLANE STRAIN)

FIG. 1b

PARTITIONINGS

LONG CYLINDER CAN BE REPRESENTED AS SUBSTRUCTURE

FIG. 1c
STRESS-STRAIN CURVE AISI 304L AND Zr Nb at 300°C

FIG. 3a
FIG. 5 ELASTIC SOLUTION

scale of contour : 15/1
displacements scale : 100/1

levels : 1 - 0.0 Kg/cm²
2 - 1.03 "
3 - 2.06 "
4 - 3.09 "
5 - 4.12 "
6 - 5.15 "
7 - 6.18 "
8 - 7.21 "
9 - 8.24 "
10 - 9.27 "
FIG. 7 RESIDUAL STRESSES:

scale of contour : 15/1
displacements scale : 100/1

levels:
1 - 0.0 Kg/sm²
2 - 7.52 "
3 - 15.04 "
4 - 22.57 "
5 - 30.09 "
6 - 37.62 "
7 - 45.14 "
8 - 52.66 "
9 - 60.19 "
10 - 67.71 "
DISCUSSION

Q R.A. VALENTIN, U.S.A.

1. Would you comment on the relative effectiveness of your first, simplified, engineering approximation as compared to the "exact" finite element approach? In particular, was the time involved in development of the finite element solution time well spent - in an economic sense - or could you have designed the problem out of existence?

2. Did you experiment with alternate ways of iterating the time steps (load steps) in the elastic-plastic solution?

3. Could you extend your finite-element work to include a bimetal joint where one metal obeyed isotropic hardening and the other kinematic hardening?

A J. REYNEN, JRC Ispra, Italy

1. The finite-element code which has been developed can now be used for other 2-dimensional thermo-elastic-plastic problems. I agree with you that if I had to develop a code for one single problem, there are other means to solve the problem more economically. As a matter of fact, thermal cycling experiments showed us that there was no fatigue problem, even before we started the plastic code.

2. Not in a consequent way, yet. Since we have a proportional loading, larger load increments could be applied probably. In this particular example we took 10 increments and 3 iterations, except for the last one where we took 5 iterations.

3. We are working on a code to take into account kinematic hardening due to Prager-Ziegler. With such a code you can also deal with elements having isotropic hardening, I suppose, by making a proper choice of the parameters defining the translation of yield locus.