AN ELASTIC-PLASTIC THEORY FOR ARTIFICIAL GRAPHITE

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ABSTRACT

An elastic-plastic, time-independent phenomenological theory for the behavior of artificial graphite between 70°F and 750°F is presented. The theory is based on the observation that upon unloading plastic deformation appears practically immediately after the loading reversal occurs. Thus, there exists no yield surface which encloses a purely elastic region. The theory incorporates the feature that upon reloading the stress-strain curve approaches asymptotically the extension to the initial stress-strain curve which would have existed, provided unloading had not occurred. The theory is developed for radial paths only. The paper ends with an illustrative example.

1. INTRODUCTION

This paper develops an elastic-plastic, time-independent theory which presents a model for the behavior of artificial graphite in the temperature range from 70° to 750°F Although plastic deformation of graphite is not truly time-independent, the time dependence can be ignored in the same spirit as it is neglected for other engineering materials.

A characteristic feature of this theory is that upon reversal of the direction of loading (unloading or reloading) plastic deformation appears practically immediately after the loading reversal occurs. The consequence of such a behavior is that there exists no yield surface in the classical sense; that is, there exists no surface which encloses a purely elastic region. On the other hand the concept of a loading surface, distinct from a yield surface, can be introduced.

In the next section we introduce the basic concepts concerning the loading surface and we develop the constitutive relation. In the third section the constitutive relation is further developed. In particular we introduce the feature that upon a second reversal of the direction of loading (reloading) the stress-strain curve approaches asymptotically the extension to the initial stress-strain curve which would have existed provided the first loading reversal (unloading) had not occurred. In the fourth section we give an illustrative example. The theory is developed for radial paths only.

2. THE LOADING SURFACE AND THE CONSTITUTIVE RELATION

Figure 1 presents stress-strain curves for simple tension and compression showing the above described behavior. We observe that all segments, OA, AB, and BC, display entirely nonlinear behavior. During initial loading and at each reversal of loading, the elastic strains are overshadowed by the plastic strains which cause the nonlinear response. Plastic straining begins immediately upon initial loading from 0, and upon reversal of loading from A and from B. However, the initial slopes of the segments, OA, AB, and BC, are the same. Hence the rate of plastic straining is zero at the beginning of initial loading and of each loading reversal. The rate of plastic straining increases as loading increases, but upon reversal it starts again to increase from the zero value. We conclude that at the beginning of each loading reversal the behavior of the stress-strain curve is the same independently of the values of stress and of plastic strain at which the loading reversal occurs; in addition, we observe that this behavior is the same as at the beginning of the initial loading at 0.

The generalization of the behavior shown in Figure 1 to combined stresses necessitates the introduction of the six-dimensional stress space in which the stress point represents the actual stress. Since no experimental data for artificial graphite are available except for radial paths through the origin we shall restrict the theory to such paths only. Although a yield surface does not exist, we can define a loading surface. The loading surface is closed, passes through the stress point, and possesses the following property. A stress rate vector emanating from the stress point and directed radially toward the outside of the loading surface produces a plastic strain rate vector of non-zero finite magnitude which is also directed toward the outside of the surface; if the stress rate vector is directed toward the inside of the loading surface in the radial direction it will produce a zero plastic strain rate and represents loading reversal.

The loading surface passes always through the stress point. Consequently, as the stress point moves in stress space following the same loading path the loading surface will also move, changing position or form or both, as need be. Suppose that the stress point moves towards the outside of the loading surface. Then the plastic strain rate vector will generally increase in magnitude as the motion proceeds.

As shown in Greenstreet and Phillips [1] the loading surface for artificial graphite may be expressed by

$$f = \frac{1}{2} a_{KLMN} \sigma_{KL} \sigma_{MN} = \kappa \tag{1}$$

where the constants, a_{KLMN}, satisfy the relations, $a_{KLMN} = a_{MNKL}$. For a more general expression for f in this theory which allows for translation and deformation of the loading surface the reader is referred to Greenstreet and Phillips [1].

Suppose now that the stress point begins to move toward the inside of the loading surface so that loading reversal is initiated. The stress point will remain in contact with a loading surface during its motion inwards. This behavior is necessary since a reversal of loading is always permissible at any stage. The inward motion of the stress point may be considered at first as the sum of very small steps. On the basis of the concepts of classical plasticity theory, for each such step the plastic strain rate will be zero, and consequently there will be no plastic strain induced during the entire inward motion of the stress point. This behavior is, however, in contradiction to the behavior illustrated by Figure 1, and explained above, where it is seen that the plastic strain rate increases during the inward motion after an initial zero plastic strain rate.

To eliminate the contradiction, we shall postulate that upon reversal of loading a new loading surface is nucleated. The surface is initially of zero size and grows as the stress point moves; the new loading surface is so oriented that the path of the stress point is directed toward its exterior. In other words, that which was an inward direction with respect to the surface through the loading point at the initiation of loading reversal is now an outward direction for the new loading surface.

Consequently, we may say that the plastic strain rate towards the inside of the surface is initially zero because a motion towards the inside of the loading surface means instantaneous destruction of the existing loading surface and the nucleation of a new one which initially has zero size and is properly oriented towards the reversed path. Such behavior is compatible with the behavior at the beginning of loading at 0, Figure 1, where no loading surface other than zero size could have existed.

A reversal of loading at $\sigma_{KL} = \overset{*}{\sigma}_{KL}$ will start generating a loading surface given by

$$f_{(1)} = \frac{1}{2} a_{MLMN} \sigma_{KL_{(1)}} \sigma_{MN_{(1)}} = \kappa_{(1)} \tag{2}$$

where

$$\sigma_{KL_{(1)}} = \sigma_{KL} - \overset{*}{\sigma}_{KL} \tag{3}$$

Here $\overset{*}{\sigma}_{KL}$ represents a pseudovirgin state that exists at the instant of nucleation of the current loading surface; it plays the role of a new origin. The stress is measured from $\overset{*}{\sigma}_{KL}$ in the same way as it is measured from the zero state during the initial loading of the virgin body.

A new reversal of loading at $\overset{*}{\sigma}_{KL_{(1)}}$ will start generating again a new loading surface given now by

$$f_{(2)} = \frac{1}{2} a_{KLMN} \sigma_{KL_{(2)}} \sigma_{MN_{(2)}} = \kappa_{(2)} \tag{4}$$

where

$$\sigma_{KL_{(2)}} = \sigma_{KL} - \overset{*}{\sigma}_{KL_{(1)}} \tag{5}$$

The process is repeated for every loading reversal.

In Greenstreet and Phillips [1] it is shown that the relation between stress rates and plastic strain rates appropriate for graphite for first loading is

$$\dot{\varepsilon}^P_{KL} = \frac{1}{c[2f]^{\frac{1}{2}}} \frac{\partial f}{\partial \sigma_{KL}} \frac{\partial f}{\partial \sigma_{DE}} \dot{\sigma}_{DE} \tag{6}$$

For the first loading reversal we have

$$\dot{\varepsilon}_{KL_{(1)}} = \frac{1}{c_{(1)}[2f_{(1)}]^{\frac{1}{2}}} \frac{\partial f_{(1)}}{\partial \sigma_{KL_{(1)}}} \frac{\partial f_{(1)}}{\partial \sigma_{DE_{(1)}}} \dot{\sigma}_{DE_{(1)}} \tag{7}$$

and similarly for the successive loading reversals. In Eqs. (6) and (7) c and $c_{(1)}$ represent material constants and

$$\varepsilon^P_{KL_{(1)}} = \varepsilon^P_{KL} - \varepsilon^{P*}_{KL} \tag{8}$$

where ε^{P*}_{KL} is the plastic strain existing at the instant of reversal.

3. THE ASYMPTOTIC BEHAVIOR

An important feature of the curves illustrated in Figure 1 is that the curve obtained upon reversal of loading from B approaches asymptotically the extension to the initial loading curve which could have existed provided reversal of loading at A had not occurred.

This feature is introduced in the stress-strain relation by writing

$$\dot{\kappa} = h_{KL}\dot{\varepsilon}^P_{KL} + \xi \frac{\partial f}{\partial \sigma_{KL}} \dot{\sigma}_{KL} \tag{9}$$

where

$$h_{KL} = h_{KL}\left(\sigma_{MN}, \varepsilon^P_{MN}\right) \tag{10}$$

and

$$\xi = \xi\left(\sigma_{MN}, \varepsilon^P_{MN}\right) \tag{11}$$

It is assumed, see Greenstreet and Phillips [1], that $0 \leq \xi \leq 1$ and that $\xi = 0$ during first loading or whenever asymptotic behavior is not required.

Once expressions (9) and (11) are assumed we obtain for the second reversal

$$\dot{\varepsilon}^P_{KL_{(2)}} = \frac{1 - \xi}{c_{(2)}[2f_{(2)}]^{\frac{1}{2}}} \frac{\partial f_{(2)}}{\partial \sigma_{KL_{(2)}}} \frac{\partial f_{(2)}}{\partial \sigma_{DE_{(2)}}} \dot{\sigma}_{DE_{(2)}} \tag{12}$$

while Eq. (6) remains unchanged since during first loading, $\xi = 0$. For simplicity we shall assume here that the first loading reversal terminates at the origin so that also Eq. (7) remains unchanged.

To achieve the required asymptotic behavior we shall consider the uniaxial stress-plastic strain curve shown in Figure 2, which has been produced by initial loading (OK), first reversal of loading (KM), and second reversal of loading (MN). We shall write $\xi = \alpha^n$ and define a function of α for this case by

$$\alpha = \frac{\varepsilon^P - \varepsilon_o^P}{\varepsilon^{P*}} = \frac{(L)}{(L_o)} \tag{13}$$

where ε^P is the plastic strain for a given stress on second loading reversal, ε_o^P is the plastic strain for the same stress on initial loading, and ε^{P*} is the plastic strain at the first loading reversal point. It may be seen that α ranges from less than unity at the initiation of second reversal to zero as the second loading reversal curve approaches asymptotically the uninterrupted initial stress-strain curve. Also ε^{P*} is a known quantity for all $\sigma \gtreqless \sigma^*$, and ε_o^P is a given function of the prior deformation history. Thus, ξ is a known function of ε^P and σ at each stage of the process. In addition, by raising α to the power, n, where n is a positive real number, the rapidity with which the second loading reversal curve approaches the initial curve can be controlled. In Greenstreet and Phillips [1] a generalization of Eq. (13) is given which allows complete control of the asymptotic approach of the stress-strain curves for any radial loading. Expression (13) is similar to the one used by Eisenberg and Phillips [3].

4. ILLUSTRATIVE EXAMPLE

We consider a specimen loaded in tension to σ^* and subsequently the loading is reversed until the stress becomes zero at which stage the loading is again reversed and increases to reach values beyond σ^*.

During first loading we have

$$\sigma_{11} = \sigma, \ \varepsilon_{11}^P = \varepsilon, \ \varepsilon_{22}^P = -\mu_{12}^P\varepsilon, \ \varepsilon_{22}^P = -\mu_{13}^P\varepsilon \tag{14}$$

and the other σ_{KL} and ε_{KL}^P are zero. We obtain

$$\varepsilon_{11}^P = \frac{1}{2c} a_{1111}^{3/2}\sigma^2 = B_{11}\sigma^2 \tag{15}$$

$$\varepsilon_{22}^P = \frac{1}{2c} a_{1111}^{1/2}a_{1122}\sigma^2 = B_{22}\sigma^2 \tag{16}$$

$$\varepsilon_{13}^P = \frac{1}{2c} a_{1111}^{1/2}a_{1133}\sigma^2 = B_{33}\sigma^2 \tag{17}$$

where the meaning of B_{11}, B_{22}, B_{33} is obvious. Eq. (15) is identical to one proposed and verified experimentally by Jenkins [2].

At the instant of first loading reversal we have $\sigma = \sigma^*$ and $\varepsilon = \varepsilon^*$ from which we obtain

$$\varepsilon^{*} = B_{11}\sigma^{*2} \tag{18}$$

During first loading reversal we have

$$\varepsilon^{P}_{11_{(1)}} = \varepsilon^{P}_{11} - \varepsilon^{*} = - \frac{a^{3/2}_{1111}}{2c_{(1)}} \sigma^{2}_{(1)} = - B^{*}_{11}(\sigma - \sigma^{*})^{2} \tag{19}$$

where

$$B^{*}_{11} = a^{3/2}_{1111}/2c_{(1)} \tag{20}$$

The value B^{*}_{11} is normally different from B_{11} in order to allow for a permanent set $\varepsilon^{P}_{11} = \varepsilon^{**}$ at $\sigma = 0$.

We shall assume that $B^{*}_{11} = \frac{1}{2}B_{11}$ so that

$$\varepsilon^{**} = \varepsilon^{*} - B^{*}_{11}\sigma^{*2} = \frac{1}{2}B_{11}\sigma^{*2} \tag{21}$$

At $\sigma = 0$ and $\varepsilon = \varepsilon^{**}$ we have the second reversal of loading for which

$$\dot{\varepsilon}^{P}_{11_{(2)}} = \frac{1 - \xi}{c_{(2)}} a^{3/2}_{1111}\sigma_{(2)}\dot{\sigma}_{(2)} \tag{22}$$

which becomes

$$\dot{\varepsilon} = (1 - \xi)B^{**}_{11}\sigma\dot{\sigma} \tag{23}$$

We shall assume that $B^{**}_{11} = 2B_{11}$, and we recall that ξ is a function of σ and ε. Using the expression for ξ from the previous section and $n = 1$ we obtain

$$\xi = \frac{\varepsilon^{P} - \varepsilon^{P}_{o}}{\varepsilon^{P*}} \tag{24}$$

so that

$$1 - \xi = \frac{B_{11}\sigma^{*2} - \varepsilon + B_{11}\sigma^{2}}{B_{11}\sigma^{*2}} \tag{25}$$

from which with the assumed value of B^{*}_{11} we obtain

$$\dot{\varepsilon} = \frac{2(B_{11}\sigma^{*2} - \varepsilon + B_{11}\sigma^{2})}{\sigma^{*2}} \sigma\dot{\sigma} \tag{26}$$

This equation becomes

$$\frac{d\varepsilon}{d\sigma} + \frac{2\sigma\varepsilon}{\sigma^{*2}} = 2B_{11}\sigma + \frac{2B_{11}\sigma^{3}}{\sigma^{*2}} \tag{27}$$

The solution of (27) is

$$\epsilon = B_{11}\sigma^2 + v e^{-\left(\frac{\sigma}{\sigma^*}\right)^2} \tag{28}$$

where v is an integration constant.

When $\sigma = 0$ the strain is

$$\epsilon = \frac{1}{2} B_{11}\sigma^{*2} = \frac{1}{2}\epsilon^* \tag{29}$$

Thus, the stress-strain relation for this case is

$$\epsilon = B_{11}\sigma^2 + \frac{1}{2}\epsilon^* e^{-\left(\frac{\sigma}{\sigma^*}\right)^2} \tag{30}$$

This expression approaches

$$\epsilon = B_{11}\sigma^2$$

asymptotically for $\sigma \to \infty$.

5. ACKNOWLEDGMENT

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6. REFERENCES

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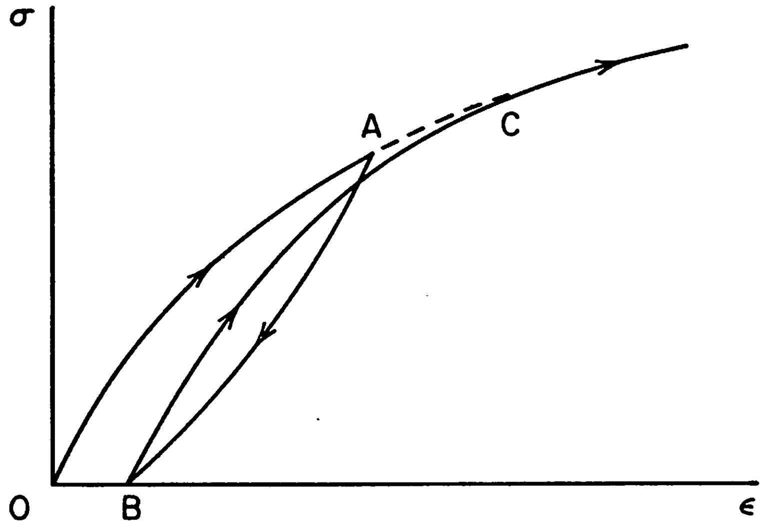

Figure 1

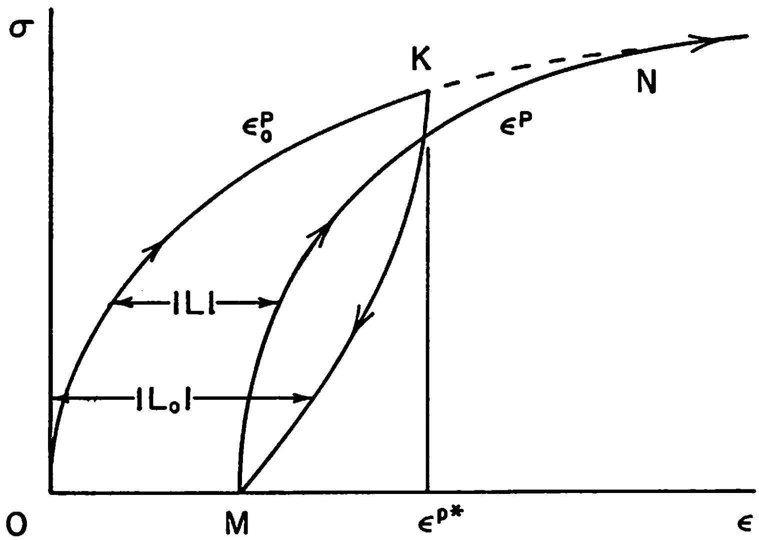

Figure 2

DISCUSSION

Q

F. C. WEILER, U. S. A.

I feel that the phenomenological model that you have presented is correct in so far as describing the behaviour of graphite. However, from practical and economical considerations, can one use your theory in practice? A tremendous number of experiments must be run to obtain all the constants needed.

A

W. L. GREENSTREET, U. S. A.

Practical and economic considerations do not preclude the use of the theory presented in practice. This is especially true when finite element analysis methods are used; the equations given are no more difficult to use than mathematical models for describing non-linear behaviour of metals that are in current use.
It is true that there are a number of constants in the theory. They are brought in partially through the anisotropic character of the material. For example, 5 "elastic" and 5 "plastic" material constants plus an additional constant to characterize unloading and reloading are needed for a transversely isotropic material. Except for the latter characterization constant, uniaxial specimens subjected to monotonic loading will yield both sets of constants, and the number is no greater than needed for anisotropic characterization in the elastic case.

Q

J. L. HEAD, U. K.

Reactor designers are concerned, not with virgin graphite, but with heavily irradiated material which has undergone very considerable damage to its microstructure. I would like to ask Dr. Greenstreet whether the theories describe the behaviour of the irradiated material.

A

W. L. GREENSTREET, U. S. A.

The consideration of irradiation induced effects can be a very important item for nuclear reactor applications, but we have not attempted to include these effects in the theory or to examine the model in comparison with irradiated graphite behaviours. There are many applications in which irradiation effects are unimportant or nonexistent, however, and the theory can be very useful in these cases.

C

A. PHILLIPS, U. S. A.

In reply to Mr. Weiler's comments I like to say that there is no shortcut to experimental research. It is necessary to get the facts even if it is expensive to do so. No theory is better than the facts we enter into it.