HYDROELASTIC VIBRATIONS OF REACTOR FUEL RODS
SUBJECT TO PARALLEL TURBULENT FLOW

R.M. KANAZAWA*, A.P. BORESI**, B.G. JONES**,

*General Electric Company,
Nuclear Energy Division, San Jose, California, U.S.A.

**Nuclear Engineering Program,
University of Illinois, Urbana, Illinois, U.S.A.

ABSTRACT

The vibration of fuel rods excited by turbulent flow parallel to the rod axes is studied. A probabilistic mathematical model of a fuel rod is developed. The probable rod response, in the form of root-mean-square displacement and of normalized spectrum of rod response, is computed in order to demonstrate application of the mathematical model. Since rod surface pressures are as yet poorly determined, a boundary layer pressure distribution for flat plates due to Corcos is used as input to the model. The nature of the rod surface pressure in relation to the structure of the flow field is discussed. The interrelations of the fluid velocity and the fluid pressure fields with the wall pressure are examined. A possible technique for examining the surface pressure-flow field structure is presented.
1. INTRODUCTION

The interest in flow-induced motion of fuel elements in a nuclear reactor may be traced to early observations by Stromquist [1]. MTR-type fuel elements made in the form of long boxes with thin plates mounted inside with coolant flow through narrow channels between the fuel plates were observed to deflect under flow conditions. It was feared that for some critical velocity the plates would deflect sufficiently to close off the channel entirely. A theory was presented which showed that this occurrence was likely, but subsequent experiments did not reveal the flow channels to collapse. A recent review of plate-type fuel element instabilities is contained in a recent paper [2].

Vibrations of rod-type fuel elements were first shown to be significant in [3]. Not only was the amplitude of vibration shown to be significant, but also the "maximum" deflections appeared to increase in proportion with velocity raised to as high as the fifth power. Another deduction made from the experimental results was that the rod vibration was self-excited. This point of view was apparently held by succeeding investigators [4-7] until the publication of a landmark paper by Paidoussis in 1965 [8].

On the basis of theoretical and experimental results, Paidoussis asserted that self-excited vibrations, do not occur in stiff rods, except at very high fluid velocities. Paidoussis proposed that rod vibrations occurring at lower fluid velocities were caused by "cross-flow components of flow and other departures from steady, uniform and perfectly axial flow."

Clearly, the contention was made that rod vibrations are caused by turbulence in the fluid flow. Since it is well known that turbulence is a random process, capable of being described only in a probabilistic sense, and since it is assumed that the rods are acted upon by forces related to a random process, it follows that the rod response is a random process, also.

Paidoussis developed an empirical correlation of the "peak" displacements measured by different investigators. In a more recent article he proposed a revised correlation which takes into account his own "peak" displacement measurements [9]. However, since "peak" displacements of a random vibration are not a meaningful measure of the response,
measurements of peak amplitude show considerable scatter, and correlations based on the data are not particularly satisfactory.

Reavis has reported the results of a theoretical calculation for the root mean square rod displacement due to forced vibrations in parallel-flow [10]. The calculations incorporate the published results of pressure measurements made beneath a turbulent boundary layer for the random forcing function acting on the rod. No details of the theory or calculational procedure were presented. However, Reavis apparently used the calculated rod response to determine the relative turbulence level in the rigs of various experimenters.

Using the above results, he computed adjusted peak displacement data for the relative turbulence level in which they were measured and fitted an empirical correlation to the adjusted data. In spite of the above efforts, the peak displacement data still contain considerable scatter due to treatment as a deterministic variable.

A team of experimentalists recognized the random nature of the rod response in 1968 [11]. However, since proper electronic instruments to measure root mean square displacements directly were not available, an oscillograph of the displacement was made and "digitized" by hand. Only 50 points within a recording time of one to two seconds could be processed. A Gaussian probability density was assumed for the displacements and the 97.5\% (1.96 \sigma) confidence limits were used as the maximum values. An empirical correlation based on the data is compared to other existing correlations.

The results of another experimental investigation which included root mean square measurements of both rod displacement in parallel-flow and pressure fluctuations on a wall were presented in [12]. The mathematical analysis of the rod vibration incorporated probability theory and clearly ignored any interaction between rod and water. The significant contribution of the paper is that it represents the first attempt to explain the rod response on the basis of a measurement of the turbulent wall pressure in the vicinity of the rod. No attempt was made to predict probable rod displacement from knowledge of the pressure fluctuations.

In reviewing the past studies of hydroelastic rod vibration in parallel-flow, it is
apparent that the incorrect assertion that rods in parallel-flow executed self-excited motion retarded the study of flow-induced vibrations of fuel rods. However, with the results reported by Paidoussis and subsequent investigators, a set of logical steps to be taken appears to be the following:

1. A mathematical model for the hydroelastic rod must be derived. 2. The probable displacement of rods due to parallel-flow must be measured in a systematic experimental program. 3. The pressure fluctuations which caused the observed rod responses must be measured. 4. The probably response, based on the mathematical model, must be calculated from the pressure measurements. 5. The calculated rod response must be compared with the measured displacements to check the validity of the mathematical model.

In this paper we develop a probabilistic mathematical model for a hydroelastic rod which relies heavily on Paidoussis' theory for rod motion in water. The proposed model requires a method for evaluating an equivalent viscous damping coefficient. In an associated research program [13], a detonized water loop was designed and built to provide high-velocity, fully-developed turbulence in a concentric annular test section. Measurements were made to determine rod displacements due solely to fluid flow and an attempt was made to measure the random pressure field acting on the rod surface. The data was digitized and processed on a digital computer by using random data processing techniques. The results of statistical analyses enabled quantitative prediction of hydroelastic rod displacement in fully-developed turbulent flow and in flows downstream from obstructions. However, measurements of the pressure field on the rod surface were not totally successful. Consequently, in order to demonstrate the application of the mathematical model, a boundary layer pressure distribution for flat plates is used here as input to the model [14]. Finally, we discuss the nature of the rod surface pressure in relation to the structure of the flow field. The inter-relations of the fluid velocity and fluid pressure fields with the wall pressure are examined. A possible technique for examining the surface pressure-flow field structure is presented.
2. MATHEMATICAL MODEL OF FUEL ROD

Consider an elastic rod of length \( L \) simply-supported within a pipe so that the axes of the rod and the pipe coincide. An incompressible fluid flowing through the pipe at a constant velocity \( U \), provides energy which excites the rod and causes it to vibrate. The vibrational energy of the rod is dissipated through viscous forces which act on the rod while it is vibrating. The vibrations are assumed to be constrained to a horizontal plane. Lengths of continuously supported rod extend in directions both up- and down-stream creating a uniform, annular flow channel (Fig. 1). Fully-developed turbulent flow is assumed to exist. The mathematical modeling of the rod parallels the work of Paidoussis \([15]\). Elementary beam theory is employed.

The forces acting on a differential length of the rod are shown in Fig. 2.

For small deflections, the equations of motion are

\[
\frac{\partial v}{\partial x} + F_L = 0 \tag{1}
\]

\[
\frac{\partial v}{\partial x} + F_N + M \left( \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} \right) y + m \frac{\partial^2 y}{\partial t^2} - \frac{\partial}{\partial x} \left( T \frac{\partial y}{\partial x} \right) + F_D + F_T = 0 \tag{2}
\]

The viscous forces in the normal and longitudinal directions, are \( F_N \) and \( F_L \). They are assumed to be expressed by equations proposed by Taylor \([16]\).

\[
F_N = \frac{1}{2} \rho D U^2 \left( C_D \sin^2 \theta + C_f \sin \theta \right) \tag{3}
\]

\[
F_L = \frac{1}{2} \rho D U^2 C_f \cos \theta \tag{4}
\]

where \( \rho \) is the fluid density, \( D \) is the rod diameter and \( C_D, C_f \) are the profile and skin drag for a cylinder in cross-flow.

The above equations may be simplified by using Lighthill's expression for the relative velocity \( v \), between a cylinder and fluid flowing past it \([17]\).

\[
v = \frac{\partial y}{\partial t} + U \frac{\partial y}{\partial x} \tag{5}
\]

Then the angle \( \theta \) can be expressed as \( \theta = \sin^{-1} (v/U) \). With this fact and with
\[ C_D \sin^2 \theta \ll C_I \sin \theta, \text{ Eqs. (3) and (4) can be written} \]
\[ F_N = \frac{1}{2} \rho D U C_I \left( \frac{\partial y}{\partial t} + U \frac{\partial y}{\partial x} \right) \]  
\[ F_L = \frac{1}{2} \rho D U^2 C_I \]  

The expression for the net tension \( T \) can be obtained by superposing an arbitrary uniform tension \( T_0 \) and the solution to Eq. (1), using Eq. (7).
\[ T = T_0 + \frac{1}{2} \rho D U^2 C_I \left( \frac{L}{2} - x \right) \]  

The dissipation force \( F_D \) is the total drag force due to cross-flow; namely,
\[ F_D = \frac{1}{2} \rho D C_D \left| \frac{\partial y}{\partial t} \right| \frac{\partial y}{\partial t} \]  

In Eq. (2), the force \( F_T \), exerted on the rod, is due to turbulent cross-velocities \( v' \) of the fluid. It is analogous to the viscous force \( F_N \) in Eq. (3), and is given by
\[ F_T = \frac{1}{2} \rho D U^2 \left[ C_D \left( \frac{v'}{U} \right)^2 + C_I \left( \frac{v'}{U} \right) \right] \]  

With the above results and the relationship between shear, bending moment and deflection from simple beam theory, \( V = \frac{\partial m}{\partial x} = EI \frac{\partial^2 y}{\partial x^2} \), the general equation of motion for a hydroelastic rod may be written as follows:
\[ EI \frac{\partial^4 y}{\partial x^4} + \left[ (MU - T_0) - \frac{1}{2} \rho D U^2 C_I \left( \frac{L}{2} - x \right) \right] \frac{\partial^2 y}{\partial x^2} + 2 M U \frac{\partial^2 y}{\partial x \partial t} + \rho D U^2 C_I \frac{\partial y}{\partial t} + \frac{1}{2} \rho D U C_I \frac{\partial y}{\partial t} + \frac{1}{2} \rho D C_D \left| \frac{\partial y}{\partial t} \right| \frac{\partial y}{\partial t} + \mu \frac{\partial^2 y}{\partial t^2} + \frac{1}{2} \rho D U^2 \left[ C_D \left( \frac{v'}{U} \right)^2 + C_I \left( \frac{v'}{U} \right) \right] = 0 \]  

where \( EI \) is the bending modulus for the rod, \( M \) is the virtual mass for the rod and is equal to the mass of the fluid displaced by the rod, and \( \mu \) is the sum of \( M \) and the rod mass \( m \).

To put Eq. (11) into a more tractable form without much loss of generality, we
1. Let the tension \( T_0 = 0 \).
2. Assume that an equivalent viscous damping force, \( c \frac{\partial y}{\partial t} \),
can be equated, by a method involving space and time averages, to the sum of forces

\[ F_S = -\frac{1}{2} \rho D U^2 C_f \left( \frac{1}{2} - x \right) \frac{\partial^2 y}{\partial x^2} + 2 M U \frac{\partial^2 y}{\partial x \partial t} + \rho D U^2 C_f \frac{\partial y}{\partial x} + \]

\[ + \frac{1}{2} \rho D U C_f \left( \frac{\partial y}{\partial t} \right)^2 + \frac{1}{2} \rho D C_D \left| \frac{\partial y}{\partial t} \right| \frac{\partial y}{\partial t} \]  \hspace{1cm} (12)

3. Represent the random forces arising from turbulent cross-flow by \( F(x, t) \).

\[ F(x, t) = -\frac{1}{2} \rho D U^2 \left[ C_D \left( \frac{\partial'^2}{\partial U}\right)^2 + C_f \left( \frac{\partial'}{\partial U}\right) \right] \]  \hspace{1cm} (13)

where \( F \) and \( \nu' \) are random variables.

Then, Eq. (11) may be written in the form

\[ EI \frac{\partial^4 y}{\partial x^4} + M U^2 \frac{\partial^2 y}{\partial x^2} + c \frac{\partial y}{\partial t} + \mu \frac{\partial^2 y}{\partial t^2} = F(x, t) \]  \hspace{1cm} (14)

Thus, the hydroelastic rod problem takes the form of a random forced vibration problem of a continuous, elastic beam-type structure, in which the interaction of the elastic rod and flowing fluid is incorporated by the equivalent viscous damping coefficient \( c \). A procedure for evaluating the damping coefficient \( c \) is presented in [13, 14].

The method of normal modes [18] is used to solve the random vibration problem represented by Eq. (14).

The procedure consists of representing a deterministic forcing function in terms of the normal modes of the system and determining the corresponding frequency response function of the system. The response of the structure to a random forcing function is obtained by decomposing the spatial distribution of the random forcing function into the normal modes of the structure and then computing a statistical average over time.

The frequency response function \( H_n(\omega) \) for the beam can be found by considering a forcing function of the form

\[ f(x, t) = e^{i\omega t} \sin \frac{nx}{L} \]  \hspace{1cm} (15)

where \( \omega \) is circular frequency, \( t \) is time, and \( \sin \frac{nx}{L}, n = 1, 2, 3, \ldots \) are the normal modes of the rod and by representing the rod deflection response \( y(x, t) \) as
\[ y(x, t) = H_n(\omega) e^{\text{rot}} \sin \frac{ntx}{L} \]  

(16)

Substitution of Eqs. (15) and (16) into (14) and solution for \( H_n(\omega) \) yields

\[ H_n(\omega) = (\lambda_n \frac{4}{E I})^{-1} \left[ 1 - \frac{\pi}{4} \frac{pD^2U^2}{E I \lambda_n^2} - \frac{\omega_n^2}{(\omega_n^2)^2} + 12 \frac{t}{(\omega_n^2)} \right]^{-1} \]  

(17)

where

\[ 2t = \frac{c}{\mu \omega_n}, \quad \lambda_n = \frac{n \pi}{L}, \quad \omega_n^2 = \frac{4 E I}{\mu} \]

The forcing function \( F(x, t) \) is represented by an infinite sum of the normal modes in space as

\[ F(x, t) = \sum_{n=1}^{\infty} F_n(t) \sin \frac{ntx}{L} \]  

(18)

where

\[ F_n(t) = \frac{2}{L} \int_{0}^{L} F(x, t) \sin \frac{ntx}{L} \, dx \]  

(19)

The rod response to a deterministic loading can be expressed with the Duhamel integral [18]

\[ y(x, t) = \sum_{n=1}^{\infty} \sin \frac{ntx}{L} \int_{-\infty}^{\infty} F_n(\theta) h_n(t - \theta) \, d\theta \]  

(20)

\[ = \sum_{n=1}^{\infty} \sin \frac{ntx}{L} \int_{-\infty}^{\infty} h_n(t - \theta) \frac{2}{L} \int_{0}^{L} F(\xi, \theta) \sin \frac{ntx}{L} \, d\xi \, d\theta \]

where \( h_n(t) \) is the impulse response function related to \( H_n(\omega) \) by [19]

\[ H_n(\omega) = \int_{-\infty}^{\infty} h_n(t) e^{-\omega t} \, dt \]  

(21)

Coupling of modes due to damping is assumed to be negligible.

The response of the rod to a random forcing function \( F(x, t) \) is then determined
by forming the product \( y_1(x, t) \cdot y_2(x', t') \) of the displacement responses at two different locations \((x, x')\) and times \((t, t')\) and taking the statistical average

\[
E \left[ y_1(x, t) y_2(x', t') \right] = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \frac{m\pi x}{L} \sin \frac{n\pi x}{L} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_m(t - \theta) h_n(t' - \theta') e^{-i\omega(t - t') \phi} \, d\theta \, d\theta' 
\]

(22)

\[
\cdot \frac{4}{L^2} \int_{0}^{L} \int_{0}^{L} E \left[ f_1(\xi, \theta) f_2(\xi', \theta') \right] \sin \frac{m\pi \xi}{L} \sin \frac{m\pi \xi'}{L} \, d\xi \, d\xi' \, d\theta \, d\theta'
\]

With the assumption that the forces acting on the rod are due to fully-developed turbulence, the excitation process is taken as weakly stationary and weakly homogeneous. Thus,

\[
E \left[ f_1(\xi, \theta) f_2(\xi', \theta') \right] = R_{FF}(\xi - \xi', \theta - \theta')
\]

(23)

where \( R_{FF} \) is the cross-correlation function.

The cross-correlation function \( R_{FF} \) is related to the cross-spectral density function of the excitation, \( \Phi_{FF} \), by the Fourier transform pair

\[
\Phi_{FF}(\xi - \xi', \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{FF}(\xi - \xi', \theta - \theta') e^{-i\omega(\theta - \theta')} \, d\theta \, d\theta'
\]

(24)

\[
R_{FF}(\xi - \xi', \theta - \theta') = \int_{-\infty}^{\infty} \Phi_{FF}(\xi - \xi', \omega) e^{i\omega(\theta - \theta')} \, d\omega
\]

With Eqs. (23), (24), (22) and (21) the weakly stationary rod response, \( R_{YY}(x, x', \tau) \), may be written

\[
R_{YY}(x, x', \tau) = \frac{4}{L^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \frac{m\pi x}{L} \sin \frac{n\pi x'}{L} \int_{-\infty}^{\infty} H_m(\omega) H_n^{*}(\omega)
\]

\[
\int_{0}^{L} \int_{0}^{L} \Phi_{FF}(\xi - \xi', \omega) \sin \frac{m\pi \xi}{L} \sin \frac{m\pi \xi'}{L} \, d\xi \, d\xi' \, e^{i\omega\tau} \, d\omega
\]

(25)

where \( H_n^{*}(\omega) \) is the complex conjugate of \( H_n(\omega) \).

Finally, the forcing function on the surface of the rod is defined as a random pressure
field which has both longitudinal and lateral structure. It is described by its pressure cross-spectral density function \( \Phi_{ pp } \). Then the rod response to a random pressure field is, in terms of the pressure cross-spectral density

\[
\Phi_{YY}(x, x', \omega) = \frac{4}{L^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \frac{m\pi x}{L} \sin \frac{m\pi x'}{L} H_m(\omega) H_n^*(\omega)
\]

(26)

\[
\int_0^L \int_0^L \int_0^L \Phi_{pp}(\xi - \xi', \eta - \eta', \omega) \sin \frac{m\pi \xi}{L} \sin \frac{m\pi \xi'}{L} d\xi d\xi' d\eta d\eta'
\]

The mean-square value of the displacement amplitude \( \bar{y}^2 \) is evaluated by integrating \( \Phi_{YY} \) over all frequencies. Thus

\[
\bar{y}^2(x, x') = \int_{-\infty}^{\infty} \Phi_{YY}(x, x', \omega) d\omega
\]

(27)

The random pressure field is the net transverse pressure field which acts on the rod. Since the rod is constrained to vibrate in a plane, the net pressure at any point along the length of the rod is the difference between the instantaneous resultant pressure acting over one-half of the rod circumference and the resultant pressure acting over the opposite half-circumference. Physically, two pressure fields are assumed to act on opposite sides of the rod along the entire length of the rod.

Two sets of related measurements are required to check the validity of the mathematical model, namely, 1. Systematic measurements of the rod displacement in the form of mean square spectral densities. 2. Measurements of the pressure field acting on the surface of the rod to determine the functional form of the cross-spectral density of the pressure.

The above measurements may be made in any test system, but both sets of measurements must be made under identical hydraulic conditions in that system. However, measurements made under conditions of fully developed turbulence do not confine the two sets of measurements to a specific test facility. Unfortunately, as of date, there have been no
successful measurements of the pressure acting on a rod in a pipe. In absence of such pressure data, to demonstrate the utility of the mathematical model, one may explore the feasibility of using the pressure field beneath a turbulent boundary layer as the random forcing function. In the next section, we follow this approach. A considerable amount of excellent experimental work has been done in boundary layer flows, and the information which is lacking for the pressure field on the inner wall of an annulus is readily available for pressure fluctuations on the wall of a pipe.

The choice of the plane boundary layer over that of curved surfaces such as inner walls of pipes avoids the unknown, but substantial, influences [25] due to surface curvature which has opposite curvature to that on the surface of the rod in an annulus.

3. RESPONSE OF ROD TO BOUNDARY LAYER PRESSURE FLUCTUATIONS

The general problem of parallel flow induced vibrations of rods is extremely complicated. The successful analysis of the response rests upon an accurate description of the interaction between the motion of the rod and the fluid forces that act on the rod. For some problems, such as systems of rods in parallel flow, it appears reasonable to initially ignore such interactions. Then, if theoretically calculated response does not agree with experimentally determined response, effects of interaction should be included in the analysis. Even with the exclusion of the interaction between the motion of the structure and the forces produced on the surface of the structure, the calculation of structural response is difficult. Numerous papers have been published on the response of plates to turbulent fluid pressure fields. A few of these papers have been noted in [20]. Clinch [21] has calculated the response of the wall of a pipe to turbulent flow and obtains very good agreement with experimental data for the averaged higher modes of shell vibration. Reavis [22] has presented a method for calculating the response of a rod to turbulent boundary layer pressure fluctuations which eliminates the requirement of a large digital computer but which is restricted to the fundamental bending mode of response. In this section, the identical problem is solved except that no assumptions are made which restrict the results to the fundamental mode of vibration.

As noted in the previous section, the response of a simply-supported rod to boundary layer pressure fluctuations can be expressed in the form of the cross-spectral density
function, \( \Phi_{YY}(x, x') \) and its integral over all frequencies, the mean-square value, \( \overline{y^2}(x, x') \) (See Eqs. 26 and 27).

To illustrate the use of the mathematical model, we employ an analytical expression for \( \Phi_{pp}(\xi, \eta, \omega) \) given by Corcos [23], namely,

\[
\Phi_{pp}(\xi, \eta, \omega) = \phi(\omega) A(\gamma) B(\beta) e^{-\omega\xi/U_c}
\]

where \( \phi(\omega) \) is the pressure spectral density function, \( A(\gamma) \) is the longitudinal (downstream) amplitude function, \( B(\beta) \) is the lateral (cross-stream) amplitude function, \( \gamma = \omega|\xi|/U_c \) and \( \beta = \omega|\eta|/U_c \). With data from [23], \( \phi(\omega) \) is approximated by the formula

\[
\phi(\omega) = \frac{\tau_o}{U} \left( \frac{\omega_s}{2\pi U} \right)^{-5} \quad \frac{\omega_s}{2\pi U} \leq 10
\]

with \( \phi(\omega) = 0 \), for the non-dimensional frequency \( \omega_s/(2\pi U) > 10 \). The shear stress \( \tau_o \) at the wall calculated by turbulent boundary layer theory [24] is

\[
\tau_o = \frac{1}{8} \lambda \rho U^2
\]

with \( \frac{1}{\lambda} = 2.0 \log(R_e \sqrt{\lambda}) - 0.8 \)

where \( R_e \) denotes Reynolds number. Equation (31) is the von Karman-Prandtl law of friction. It has been verified for \( R_e \) to \( 3.4 \times 10^6 \). In the present investigation, \( R_e \leq 3.1 \times 10^5 \). To evaluate \( \lambda \) by Eq. (31), the Blasius formula \( \lambda = 0.3164 R_e^{-0.25} \) was used to begin the iteration process of solving Eq. (31). Since Blasius' formula is accurate up to \( R_e = 10^5 \), the iteration procedure converged rapidly for \( R_e \leq 10^5 \).

The amplitude functions \( A(\gamma) \), \( B(\beta) \) were obtained by fitting curves through Corcos' data [25, 26] and are given by

\[
A(\gamma) = \exp(-0.1145|\gamma|) + 0.1145|\gamma| \exp(-2.5|\gamma|)
\]

\[
B(\beta) = 0.155 \exp(-0.092|\beta|) + 0.700 \exp(-0.789|\beta|) + 0.145 \exp(-2.910|\beta|) + 0.990|\beta| \exp(-4.0|\beta|)
\]

Finally, the convection speed of the random pressure field is determined by fitting experimental data [Fig. 9, Ref. 23], to obtain
\[
\frac{U_c}{U} = \begin{cases} 
0.9 - 0.06 \frac{\omega_s}{2\pi U}, & \frac{\omega_s}{2\pi U} \leq 3 \\
0.7, & \frac{\omega_s}{2\pi U} > 3
\end{cases}
\]  \hspace{1cm} (33)

The largest value of \(\omega_s/2\pi U\) used in the present calculation was 2.81.

With the above approximations, the spectral density \(\Phi_{YY}\) and the mean-square value of rod displacement \(\overline{y^2}\) may be computed. The details of the calculation are presented in [13, 14]. Here we present a few results. The results of the digital computer calculation are shown in Figs. 3 and 4.

Figure 3 is a series of plots showing the variation of root-mean-square displacement with a) water velocity, b) bending modulus, c) critical damping ratio, and d) rod natural frequency. Table 1 shows the rod response at the mid-point and quarter-point of the rod for the first three modes of vibration. All of the rms values have been divided by 32.4 mils which is the calculated response for the following case: \(U = 30 \text{ ft/sec.}, \ \xi = 0.05, f_0 = 15 \text{ Hz.}, \ \text{EI} = 200 \text{ lb-ft}^2\).

Figure 4 is a series of plots which shows the variation of the normalized spectrum of rod response with a) water velocity, b) bending modulus, c) critical damping ratio, and d) rod natural frequency. Only the first mode response at \(x/L = 1/2\) was calculated for the preceding plots. The higher mode response as well as displacement at \(x/L = 1/4, 3/4\) is shown in a separate plot e). All of the spectra have been normalized to unit mean square value.

The following parameter values were assigned:

\[
\begin{aligned}
s & = 0.125 \text{ ft.}, \quad \rho = 1.936 \text{ slugs/ft}^3 \ (\text{H}_2\text{O at 70}^\circ \text{F.}), \quad v = 1.06 \times 10^{-5} \text{ ft}^2/\text{sec.} \\
\text{(H}_2\text{O at 70}^\circ \text{F.}) \quad L & = 4 \text{ ft.}, \quad D = 0.0416 \text{ ft.}, \quad \text{and } E = 4.32 \times 10^9 \text{ lb/ft}^2
\end{aligned}
\]

The results show a strong dependence of rod displacement on water velocity. The slope of the line in Fig. 3 (a) is approximately 2. Also, the calculated displacement is inversely proportional to the bending modulus. Damping and natural frequency appear to play secondary roles in determining the rod response, at least over the range of parameters
used. Higher mode response to turbulent boundary layer excitation is quite small. It is noteworthy that the variation in rod response with damping is in agreement with the results for a single degree of freedom system excited by white noise [19].

The spectra show the relative frequency distribution of the mean square rod displacement. The spectra show a relative decrease in the lowest frequencies with increasing fluid velocity, and the results for increased damping show a flattening and broadening of the peak at the rod natural frequency, as one would expect. Also, the spectra for increasing rod natural frequency show a relative increase in the portion of the spectra just below the resonance peak. This result indicates that the excitation spectrum begins to drop in power density within the frequency range through which the rod natural frequencies were varied.

It should be noted that effectively, the above calculations are equivalent to the computation of the response of a flat strip loaded on one side. The data for the flat plate measurements of Corcos should probably be warped to conform to the boundary layer that exists on a rod. Unfortunately little reliable data exists for the rod. Furthermore, if the strip is loaded on both sides at the same time, the question of correlation of the pressure fluctuations on opposite sides of the strip becomes important.

Again, these types of measurements have not been successfully carried out.

4. CONSIDERATIONS OF ROD PRESSURE-FLUID VELOCITY RELATIONS

Since means have been available to examine the structure of turbulent fluid velocity fields but have not been available to measure the local static pressure field in these flows, there has been only limited work conducted to explain the interrelation of these two closely coupled fields. However, Corcos [23] has attempted to interrelate these two fields by comparing the predictions with experimentally observed wall pressure data beneath turbulent boundary layers on plane surfaces. Although some success was realized, there was serious
discrepancy in the low frequency, large scale portion. Unfortunately, this is the portion of particular interest in the exciting of fundamental modes in structures.

Although considerable experimental study has been performed on the structure of wall pressure \[25, 23, 28\], the majority of these have been conducted in low speed air flows over plane surfaces. However, a recent study by Willmarth and Yang \[29\] in the same media but on a long circular cylinder has demonstrated some sizeable differences attributed to the curvature of the surface. Such large differences suggest that in the determination of wall pressure within the relatively complicated structure of reactor fuel bundles there may be additional factors which would prevent the direct utilization of either flat plate or curved rod wall pressure data as the excitation for rod vibration evaluations. This would be particularly suspect since the measured data are obtained for boundary layers for which the free stream flow is unconfined and undisturbed whereas the reactor fuel assemblies have confined, fully developed internal flow fields. Not only is detailed wall pressure data relatively scarce in internal flows, but there is also little detailed velocity turbulence structure data available on annular flows, which closely approximate flows in rod bundles. In the latter case the majority of the structural measurements have been performed in circular tubes and in plane 2-dimensional channels, principally following the classical articles by Laufer\[30, 31\]. Although these geometries provide the extremes of the circular tube and thin annulus, the effects of both convex and concave curvature on the detailed velocity structure of turbulent annular flows is not documented. Similarly, the wall pressure structure for confined flows is limited to circular tubes\[32\] and outer walls in annuli\[33\], neither of which provide the correct excitation for rod vibrations, namely, the wall pressure on the inner wall of the annulus. This is an area of research which requires further experimental work, using established techniques to provide not only single point measurements, but also two-point space correlations which will properly identify the statistical nature of the rod excitation.

Ideally the designer would like to be able to predict the excitation on the rod bundle elements from the fluid flow field characteristics which can be readily measured, such as
the turbulent velocity field. Although the necessary interrelations are not yet known, the classic work of Corcos [23] and the recent work of Jones and Spencer [34, 35] provide some interesting concepts and observations on the interrelation of the pressure and velocity fields and their relation to the excitation on a bounding surface. In the classic model of turbulence generated pressure fields the total pressure field is a combination of the acoustic pressure field, which propagates as sound waves in the medium, and the pseudosound pressure field which is coupled to the fluid and propagates with fluid convection speeds. In highly turbulent, incompressible flow fields the pseudosound dominates. Its spatial correlation structure combines as a volumetric source to provide the local pressure field at a point in the flow and adjacent to the flow field. With this concept Corcos [23] has separated the wall pressure field on a flat plate below a turbulent boundary layer into two components. One component is due primarily to the very small structure in the near wall turbulence. It is dominated by high frequency content. The second component is due to the outer boundary layer turbulence, which is characterized by much lower frequency content. To complicate the picture further, observations of the convective speed show significant differences between the high and low frequency content of the pressure and velocity fields.

In a study of both the velocity and static pressure turbulence characteristics in a two-dimensional free shear layer, Spencer and Jones [36] observed that the turbulent structure of the local static pressure field was most closely related to that of the velocity component in the flow direction. Figure 5 shows the rms distributions of the fluctuating pressure and both the transverse and the axial velocities in the fully developed mixing region. Figure 6 compares the power density distributions for the same quantities. The strong coupling of both the transverse and axial velocity fluctuations with the static pressure fluctuations is displayed in Fig. 7. Comparisons of the Eulerian integral time scales for static pressure and axial velocity show these to be of the same magnitude. It is, therefore, concluded that the Eulerian structure of the local static pressure field may be estimated from that of the axial velocity component. However, in recently completed two-point velocity and pressure correlations
the Lagrangian structure revealed that the pressure field is convected appreciably faster than the velocity field in the two-stream mixing layer.

Spencer and Jones obtained the local instantaneous static pressure variations using a newly developed bleed-type transducer [37] which has been designed for use in air. Through the application of this instrument, it now is possible to study directly the interrelation of the flow field pressure with that of the adjacent wall pressure. Since the transducer is small (0.035 inches outside diameter), it can be readily adopted for internal flow probing. Interpreting the results for water should be straightforward since incompressible flows would be used and the dominance of the pseudosound would negate the effect of the difference in acoustic velocities in the two media. On the other hand, there is a good possibility that the pressure transducer can be modified for use in water, which would be more acceptable for studying directly applications of reactor fuel rod behavior.

5. CONCLUSIONS

The response of rods to a random pressure field can be computed provided certain statistical descriptions of the pressure field are known. No approximations are necessary in the computation other than standard methods of numerical analysis assuming that the analytical representation of the pressure field lends itself to closed formed integration over the space variables [13]. Although Bakewell [27] has indicated that pressure distribution over a body of revolution in water flow is similar to air flows over flat plates and in pipes, one must not assume that the pressure description employed here accurately describes the pressure field acting on a rod in parallel water flow. In particular, whether or not instantaneous diametrically opposite pressures on the rod are independent or uncorrelated is not known. In general, precise measurements required to assess the response of structures to the highly disturbed flows that exist in nuclear reactor are lacking. Consequently, considerable judgment is required in using available information to estimate fuel-rod response to fluid-flow.

Some hope of obtaining estimates of the pressure acting on a fuel rod rests upon development of a relationship between the pressure and velocity characteristics of the flow and the pressure on the rod.
REFERENCES


TABLE 1
NORMALIZED DISPLACEMENT AT MID-AND QUARTER-POINTS
FOR FIRST THREE MODES OF VIBRATION

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Normalized Displacement</th>
<th>Mid-Point</th>
<th>Quarter-Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1.000</td>
<td>0.707</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0</td>
<td>0.0475</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0.0078</td>
<td>0.0056</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1.0078</td>
<td>0.760</td>
</tr>
</tbody>
</table>
Fig. 1  Hydroelastic Rod Supported Inside Pipe

Fig. 2  Differential Element of Hydroelastic Rod
Fig. 3 Variation of rms Displacement with
(a) Water Velocity,  (b) Bending Modulus,
(c) Damping, and  (d) Rod Natural Frequency
Fig. 4  Normalized Spectra of Rod Response to Turbulent Boundary Layer Pressure Fluctuations. Effects of (a) Water Velocity, (b) Bending Modulus, (c) Damping, (d) Rod Natural Frequency, and (e) Position Along Rod Showing Higher Modes.
Fig. 5  Lateral Distributions of Turbulence Intensities in Fully Developed Free Plane Shear Layer with Velocity Ratio, r, of 0.3 (r = Secondary Mean Velocity \( U_0 = 30 \text{ ft/sec} \)/Primary Mean Velocity \( U_p = 100 \text{ ft/sec} \)); \( \Delta U = U_p - U_0 \); \( \Delta H = \text{Mean Dynamic Pressure Difference Between} \ U_a \text{ and} \ U_b \); \( u, v = \text{Axial and Transverse Velocity Fluctuations, Respectively}; \ p = \text{Static Pressure Fluctuations}; \ \eta = \text{Normalized Shear Layer Coordinates} \ ( = y/(x - x_0), \text{where} \ y = \text{lateral displacement,} \ x = 22 \text{ inches} \text{ is the axial location and} \ x_0 = \text{virtual origin of shear layer}); \text{ and} \ \eta_0 \text{ is the Lateral Coordinate of the Shear Layer Centerline Defined for} \ U = (U_a + U_b)/2. \ \text{See Ref. 36.}
Fig. 6  Power Spectral Densities for Static Pressure Fluctuations, $p$, and Axial and Transverse Velocity Fluctuations, $u$ and $v$, respectively, at $x = 22$ inches, $r = 0.3$, and the Shear Layer Centerline.
Fig. 7  Normalized Pressure-Velocity and Shear Stress Correlations Across the Shear Layer at $x = 22$ inches and $r = 0.3$. 
DISCUSSION

J. A. DEARIEN, U. S. A.

Was any attempt made to establish the effect of annulus dimension on the results obtained?

A. P. BORESI, U. S. A.

No. We used one rod size and one pipe size for the fluid flow. The effects of rod size were not studied, but as you know, this is indeed a parameter of importance. Our model, I believe, has been superseded by the Argonne model. However, the principle contribution of our paper, I believe, is the suggestion that there appears a hope of obtaining estimates of the pressure acting on a fuel rod by the development of a relationship between the pressure and velocity characteristics of the flow and the pressure on the rod.