ON THE LIMIT PRESSURE OF BRANCH-PIPE TEE CONNECTION

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ABSTRACT

The lower bound to the limit pressure of a branch-pipe tee connection, is calculated on the basis of two widely used approximate yield surfaces. No exact bounds are available for one of the yield loci. The analysis performed here enables one to set a bound to this yield surface. The limit pressure calculations are based on a non-linear programming code (SUMT), and have been computed for a wide range of geometric parameters.

1. INTRODUCTION

The basic problem in the design of pressure vessels or pipes, may be stated as: (a) finding the stress distribution and (b) prescribing the allowable values of the stresses. The allowable stresses should be chosen so that they provide a satisfactory ratio between the failure pressure and the operating pressure. The thickness and type of material are then determined to provide the desired ratio.

It has been suggested that plastic collapse criterion (excessive plastic deformation) should form the main basis of design procedure for nozzles in pressure vessels, e.g., see Cloud and Rodabaugh [1]. Indeed, the principles of limit analysis were employed to set allowable stresses in section III of the ASME boiler and pressure vessel code for nuclear vessels [2].

In the accepted terminology of the theory of plastic-rigid solids,a "complete" solution consists of specification of the limit load, (collapse or yield-point load), associated stress, and velocity fields. The value of the critical load depends upon the choice of yield surface. A complete solution for one of them will be either a lower or upper bound for others, Ellyin [3].

The complexity of yield equations for general shells, has promoted several approximations, e.g. sandwich, limited interaction, mid-surface fulfillment, etc. It is the purpose of this paper to examine the relation between the limit pressures of a branch-pipe tee connection (or nozzle-to-cylindrical vessel attachment) obtained through employing two widely used yield surfaces.

2. STATEMENT OF THE PROBLEM

Consider two circular cylindrical shells of the mid-surface radii r and R, ($r \leq R$) normally intersecting, fig. 1. Hereinafter, the vertical cylinder will be termed "branch pipe" (or nozzle) and the horizontal cylinder, "run pipe" or (vessel), and the assembly as "tee connection" (or nozzle-vessel attachment). The shell material is assumed to be rigid-perfectly-plastic obeying a yield criterion to be specified later.

Let the tee connection be subjected to the internal pressure P, which is slowly increased from zero. For sufficiently small values of P, stresses everywhere will be below the yield, so that the strains are zero. In this stage, the stress distribution cannot be uniquely determined. When the pressure reaches a certain value, the tee connection will begin to deform near the junction (localized region of high stress concentration). As P is further increased, the plastic region near the junction will start to grow. However, the deformation will continue to be small until there is a sufficient amount of plasticity spread so that the surrounding rigid portions are unable to restrain plastic region from motion; the corresponding pressure is called limit pressure (collapse or yield-point pressure). The problem therefore, is to determine the limit pressure P_o, or a safe value of it.

Exact calculations of the limit pressure are only feasible for certain types of structures, however, the theorems of limit analysis provide a means of bounding P_o from above or below. The lower bound approach will be adopted in this paper. This theorem requires construction of an equilibrium state of stress, which satisfies the stress boundary conditions and does not violate the yield requirement. Such a stress distribution is termed "statically admissible stress field". This theorem, in essence, establishes the fact that material, if possible, will adjust itself to carry the applied load. It gives lower bound on, or safe value of, the limit pressure. Of all the pressure loads P^-, for which there exists a statically admissible stress field, the largest one is the limit pressure, i.e.

$$\max P^- = P_o$$

3. GENERAL RELATIONS

3.1. Differential equations of equilibrium.

The non-dimensional equilibrium conditions with the sign conventions of Timoshenko and Woinowsky - Krieger [4] have the form:
for the run pipe:

$$\frac{\partial n_y}{\partial y} + \frac{\partial n_{y\phi}}{\partial \phi} = 0$$

$$\frac{\partial n_\phi}{\partial \phi} + \frac{\partial n_{y\phi}}{\partial y} + h\,\frac{\partial m_{y\phi}}{\partial y} - h\,\frac{\partial m_\phi}{\partial \phi} = 0$$

$$n_\phi + h\,\frac{\partial^2 m_y}{\partial y^2} + h\,\frac{\partial^2 m_\phi}{\partial \phi^2} - p = 0 \tag{1}$$

$$q_y = h\left[\frac{\partial m_{y\phi}}{\partial \phi} + \frac{\partial m_y}{\partial y}\right]$$

$$q_\phi = \frac{\partial n_\phi}{\partial \phi} + \frac{\partial n_{y\phi}}{\partial y}$$

where $n_y = \frac{N_Y}{N_o}, \ldots, m_y = \frac{M_Y}{M_o}, \ldots, y = \frac{Y}{R}$, $h = \frac{M_o}{RN_o} = \frac{T}{4R}$, $p = \frac{PR}{N_o}$, $N_o = \sigma_{or} T$,

$M_o = \frac{1}{4}\,\sigma_{or} T^2$ and σ_{or} being the yield stress of the run pipe material.

For the branch pipe, the equilibrium equations will be found by replacing y, ϕ, R, T, h, p, and σ_{or} by z, θ, r, t, h_1, p_1, and σ_{ob} respectively. The relation between the two non-dimensional pressure quantities is:

$$p = \frac{t}{r} \cdot \frac{R}{T}\,\eta\, p_1 = n\eta p_1$$

where η is the ratio of the branch to run pipe yield stress, and n is the ratio of their membrane strengths when $\eta = 1$. The inverse of n is the hoop stress ratio $\frac{s}{S}$.

3.2 Boundary Conditions

If the pipes are considered to be "long", then for all practical purposes, the boundary conditions at the ends will approach the membrane state, i.e.,
at the ends of the run pipe:

$$y \to \pm\infty \quad n_y = \frac{p}{2}\ , \quad n_\phi = p$$
$$n_{y\phi} = m_y = m_\phi = m_{y\phi} = 0 \tag{2}$$

at the end of the branch pipe:

$$z \to \infty \quad n_z = \frac{p_1}{2}\ , \quad n_\theta = p_1 \tag{3}$$

$$n_{z\theta} = m_z = m_\theta = m_{z\theta} = 0 \tag{3}$$

The equilibrium of a small element along the intersection curve C, fig. 2, yields the following identities

$$s_t^b + s_t^r = 0 \qquad \text{(a)}$$

$$m_t^b - m_t^r = 0 \qquad \text{(b)}$$

$$n_t^b \cos\chi + q_t^b \sin\chi - n_t^r = 0 \qquad \text{(c)} \tag{4}$$

$$n_t^b \sin\chi - q_t^b \cos\chi - q_t^r = 0 \qquad \text{(d)}$$

where the appropriate values of s_t^b, $s_t^r, \ldots,$ q_t^r may be calculated from the construction of a Mohr's Circle, or simply by considering the equilibrium of each element in fig. 2. They may also be found in Eringen and Suhubi [5].

3.3 Conditions of Symmetry

The longitudinal (X = 0), and the transversal (Y = 0) planes, are planes of symmetry, fig. 1. The following conditions of symmetry therefore, must be imposed on the stress resultants:

$$\text{for } \phi = 0, \pi$$

$$n_{y\phi} = m_{y\phi} = q_\phi = 0 \tag{5}$$

$$\text{for } \theta = 0 \text{ (and } \pi) \text{ and } \theta = \frac{\pi}{2} \text{ (and } \frac{3\pi}{2})$$

$$n_{z\theta} = m_{z\theta} = q_\theta = 0 \tag{6}$$

3.4 Yield Conditions

The limit analysis of the title problem requires, in addition to the above relations, the specification of a yield criterion. Since the main object of the paper is the comparison of limit pressures based on various yield loci, for the sake of clarification, a more detailed account of yield surfaces will be given herein.

The state of stress at a generic point of the shell is assumed to be two dimensional. Tresca's yield condition may then be stated as:

$$\sigma_o - \max [| \sigma_1 | , | \sigma_2 | , |\sigma_1 - \sigma_2|] \geq 0 \quad (7)$$

whereas von Mises condition takes the form:

$$\sigma_o^2 - [\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2] \geq 0 \quad (8)$$

where σ_1, σ_2 are the principal stresses and σ_o is the tensile yield strength of the material. Figure 3, shows the yield conditions (7) and (8).

The relation between Tresca f_T, and von Mises f_M, conditions are:

$$0.866 f_M \leq f_T \leq f_M$$

or (9)

$$f_T \leq f_M \leq 1.155 \, f_T$$

However, since the equations of equilibrium and the boundary conditions are written in terms of stress resultants, the yield condition must also be formulated in these qualities. When such a formulation is carried out, the equations of yield surfaces become non-linear, even though the Tresca yield condition (7) in the principal stress space is piecewise linear. Sawczuk and Rychlewsk [6] have summarized the yield surfaces for plastic shells. The complexities of the exact yield loci make the solution of the practical problems very difficult, indeed, if at all possible. For this reason, attention has been focused on the approximation of the yield surfaces.

An approximation to the Tresca yield locus is proposed by Hodge [7], and is generally termed as "two-moment limited-interaction" yield surface, f_L. This approximation is based on the premise that in most shell problems, the moment and direct forces are not of simultaneous importance, so that the yield relation between moment and force are of limited interest. Although this surface was proposed for rotationally symmetric shells under axisymmetric loading, the general form will be valid for non-symmetric shells in the space of principal stress resultants. It has the Tresca hexagonal form for both moments and forces, and may be defined by twelve planes as:

$$1 - \max [|n_1|, |n_2|, |n_1 - n_2|] \geq 0$$

$$1 - \max [|m_1|, |m_2|, |m_1 - m_2|] \geq 0 \quad (10)$$

However, if $m_{y(\text{or } z)}$, $m_{\phi(\text{or } \theta)}$, $m_{y\phi(\text{or } z\theta)}$, $n_{y(\text{or } z)}$, $n_{\phi(\text{or } \theta)}$ and $n_{y\phi(\text{or } z\theta)}$ represent the state of generalized stresses at a generic point of shell, the yield condition (10) can be written as:

$$1 \pm \left(\tfrac{1}{2}\left[n_{y\,(\text{or } z)} + n_{\phi\,(\text{or } \theta)} \right] \pm \left\{ \tfrac{1}{4}\left[n_{y\,(\text{or } z)} - n_{\phi\,(\text{or } \theta)} \right]^2 + n_{y\phi\,(\text{or } z\theta)} \right\}^{\frac{1}{2}} \right) \geq 0$$

$$1 \pm \left\{ \left[n_{y\,(\text{or } z)} - n_{\phi\,(\text{or } \theta)} \right]^2 + 4\, n^2_{y\phi\,(\text{or } z\theta)} \right\}^{\frac{1}{2}} \geq 0$$

$$1 \pm \left(\tfrac{1}{2}\left[m_{y\,(\text{or } z)} + m_{\phi\,(\text{or } \theta)} \right] \pm \left\{ \tfrac{1}{4}\left[m_{y\,(\text{or } z)} - m_{\phi\,(\text{or } \theta)} \right]^2 + m^2_{y\phi\,(\text{or } z\theta)} \right\}^{\frac{1}{2}} \right) \geq 0 \qquad (11)$$

$$1 \pm \left\{ \left[m_{y\,(\text{or } z)} - m_{\phi\,(\text{or } \theta)} \right]^2 + 4\, m^2_{y\phi\,(\text{or } z\theta)} \right\}^{\frac{1}{2}} \geq 0$$

Part of the corresponding yield surface is shown in fig. 4. It consists of a cylindrical center part, and two nearly conical caps, for each set of resultant forces and moments. Note that there is no interaction between the forces and moments. The relation between this yield surface, and those of Tresca and von Mises are:

$$0.618\, f_L \leq f_T \leq f_L$$

$$0.618\, f_L \leq f_M \leq 1.155\, f_L \qquad (12)$$

An approximation to the von Mises yield surface is proposed by Rozenblium [8]. This approximation may be viewed from several directions:

a) a surface located between true upper and lower bounds to the von Mises yield loci,
b) a limited interaction approximation of the von Mises yield surface for a sandwich shell,
c) a close approximation of the von Mises yield condition, when satisfied only at the mid-surface of the shell, Ilyushin [9].

In the space of the generalized stresses, this yield condition may be written as

$$f_R = 1 - (Q_n^2 + Q_m^2) \geq 0 \qquad (13)$$

where

$$Q_n^2 = n^2_{y\,(\text{or } z)} - n_{y\,(\text{or } z)}\, n_{\phi\,(\text{or } \theta)} + n^2_{\phi\,(\text{or } \theta)} + 3n^2_{y\phi\,(\text{or } z\theta)}$$

$$Q_m^2 = m^2_{y\,(\text{or } z)} - m_{y\,(\text{or } z)}\, m_{\phi\,(\text{or } \theta)} + m^2_{\phi\,(\text{or } \theta)} + 3m^2_{y\phi\,(\text{or } z\theta)}$$

To the best of the writer's knowledge, no bounds are available for this yield criterion. Note however, that the Ilyushin yield surface is obtained for the "simplest" state of stress, and the yield locus (13) then becomes an inscribed one with the maximum discrepency of about 9%.

4. OUTLINE OF LOWER BOUND COMPUTATION

The lower bound formulation for a nozzle-to-cylindrical shell attachment is given by Ellyin and Turkkan [10]. In order to make the paper self contained, a brief résumé of the method will be outlined herein.

For the run pipe, a set of stress resultants is chosen which satisfies equilibrium equations (1), boundary condition (2), and symmetry requirements (5). The stress resultants are functions of nine orbitrary parameters X_i, spatial coordinates y, θ, ϕ, and geometric variables, ρ (diameter ratio), and h. A similar set is determined for the branch pipe which fulfills the corresponding equations. The satisfaction of the stress continuity requirements at the junction, eqs (4), is achieved in the following manner. The tee connection of fig. 1, is subdivided into small segments by planes radiating through the Z-axis. For each segment, with a fixed θ, an expression for the pressure p is obtained by substituting the appropriate values of stresses at the junction, in eq. (4-a). Then, for each division, an admissible stress field is obtained through the solution of the following non-linear mathematical programming problem:

$$\text{maximize } p = K \sum_{i=1}^{18} C_i X_i \qquad \text{(a)}$$

subject to

$$R_j(X_i) \geq 0 \qquad j = i, \ldots, n, \qquad \text{(b)} \qquad (14)$$

$$R_j(X_i) = 0 \qquad j = n+1, \ldots, n+3, \qquad \text{(c)}$$

where K, and C_i's, are constant for a given θ, and geometric and material parameters of the tee connection. The inequality constraints are due to the yield requirement, and in addition, there is a physical restriction that the limit pressure of the tee connection should not exceed that of an unpierced run pipe, i.e.

$$1 - p \geq 0 \qquad (15)$$

For the Tresca limited interaction surface eqs (11), the number of inequality constraints are 24, which makes n = 25, whereas for the Ilyushin yield surface there are two inequality constraints, and thus n = 3.

The equality constraints are imposed by the continuity requirement at the junction, eqs (4-b, c, d.). The stress resultants are chosen so that they all attain their maximum value at the junction, regardless of the values assigned to X_i's. Thus it follows, that satisfaction of yield conditions (11) or (13) at the junction, will ensure fulfillment at any other point of the tee connection.

A set of X_i is found for each value of θ, which yields a maximum value for p subject to the prescribed constraints. It is shown in ref. [10], that the minimum p among these maximum p 's, is the lower bound, p^-, to the limit pressure of the tee connection.

The method adopted for the solution of the problem defined by (14), is the third version of the "Sequential Unconstrained Minimization Technique" (SUMT) developed by Fiacco and McCormick [11].

5. NUMERICAL RESULTS AND DISCUSSION

The branch-pipe tee connection geometry may be expressed by three non-dimensional parameters, e.g., $\rho = \frac{d}{D}$, $\frac{D}{T}$ and $\frac{s}{S} = \frac{d}{t} \cdot \frac{T}{D}$. In addition, the material variable η, branch to run pipe yield ratio, enters into the computation. The lower bound to limit pressure varies with respect to all these parameters. For the graphical representation, two of the parameters are fixed, and variation of the pressure against the remaining two are plotted. Figure 5, shows the variation of lower bound to limit pressure against the diameter ratios for different diameter to thickness ratio of the run pipe. The dotted lines on the figure correspond to the predictions based on von Mises approximation, whereas the solid ones belong to the Tresca limited interaction. For a representative $\frac{D}{T}$ ratio, the variation of pressure versus $\frac{d}{D}$, for various branch to run pipe hoop stress ratios, is shown in fig. 6.

From the results presented, it is clear that the limit pressure predicted on the basis of the von Mises approximation, f_R, is very close to that based on the limited interaction surface. For the wide range of parametric study, the maximum deviation is about - 6 per cent with an average of - 3 per cent.

The bounding surface lemma stipulates, that the relation between limit pressures calculated on the basis of two different yield loci, is the one which exists between the corresponding yield surfaces themselves. Therefore, the results presented here seem to indicate that the bounds of the approximate yield surface, (13) and that of the "exact" Mises surface, must be of the order:

$$0.618\, f_R \le f_M \le 1.227\, f_R \qquad (16)$$

At the first sight, this relation may be thought of as being too extreme. The popular belief has been that f_R, is much closer to the f_M, e.g. see Schroder and Sherbourne [12]. However, a closer look at the second definition of the yield locus f_R, as given in the section (3-3), will confirm the relation (16).

It has been shown by Ellyin [3], that the limited interaction yield surface is not a good approximation for a nozzle in a spherical pressure vessel with small $\frac{D}{d}$ and/or $\frac{T}{D}$ ratios. On the other hand, if the lower bound is calculated on the basis of the inscribed surface, and the upper bound on a circumscribed surface, the difference in the results will be so great that they will not be of any practical use. However, inevitable variability of material properties throughout the shell, reinforcement caused by fillet weld at the junction, and other uncertainties, seem to make the agreement, between the experimental limit pressure and the theoretical predictions based on the limited interaction, satisfactory for most cases, see Ellyin [13]. Drawing parallel conclusions, for the branch-pipe tee connection, it may be said that the limit pressure calculated on the basis Rosenblium [8] or Ilyshin [9] yield surface, will be adequate for design purposes. Indeed in ref. [10],it is shown that the theoretical predictionsfairly agree with the experimental limit pressure of test models with small fillet weld at the junction.

6. CONCLUSIONS

Lower bounds to the limit pressure of branch-pipe tee connections are calculated on the basis of two yield surfaces. The computations have allowed to set bounds on the Ilyushin approximation of the von Mises yield surface. It is shown that the bounding coefficients are of the same order of magnitude as the Tresca limited interaction yield locus. These bounds seem to be much wider than previously believed. However, it is argued, on the basis of experimental results and similiar solutions, that the limit pressure, predicted by employing Ilyushin yield surface, might be adequate for the design purposes.

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NOTATION

D, d = mid-surface diameter of run and branch pipe respectively

$N_o = \sigma_{or} T$ = maximum membrane plastic strength of run pipe cross section

$N'_o = \sigma_{ob} t$ = maximum membrane plastic strength of branch pipe cross section

$n = \frac{t}{d} \times \frac{D}{T}$ = the ratio of the membrane strength of run to that of branch pipe when $\eta = 1$

n_y, n_ϕ, $n_{y\phi}$ = non-dimensional membrane forces per unit length in run pipe

n_z, n_θ, $n_{z\theta}$ = non-dimensional resultant forces per unit length in branch pipe

$M_o = \frac{1}{4} \sigma_{or} T^2$ = maximum plastic bending moment capacity of run pipe cross section

$M'_o = \frac{1}{4} \sigma_{ob} t^2$ = maximum plastic bending moment capacity of branch pipe cross section

m_y, m_ϕ, $m_{y\phi}$ = non-dimensional moments per unit length in run pipe

m_z, m_θ, $m_{z\theta}$ = non-dimensional moments per unit length in branch pipe

P = pressure per unit area

P_o = limit pressure

P^- = lower bound to limit pressure

$p = \frac{PR}{N_o}$ = non-dimensional pressure

$p_1 = \frac{Pr}{N'_o}$ = non-dimensional pressure in branch pipe

p^- = non-dimensional lower bound to the limit pressure

q_y, q_ϕ = non-dimensional transversal shear forces per unit length in run pipe

q_z, q_θ = non-dimensional transversal shear forces per unit length in branch pipe

R = mid-surface radius of run pipe

r = mid-surface radius of branch pipe

$\frac{s}{S} = \frac{d}{t} \times \frac{T}{D} = \frac{1}{n}$ = ratio of nominal hoop stress in branch to that of run pipe

T = wall thickness of run pipe

t = wall thickness of branch pipe

X, Y, Z = Cartesian coordinate system

X_i = arbitrary parameters contained in the stress fields

ϕ = cylindrical coordinate in run pipe

θ = cylindrical coordinate in branch pipe

$\rho = \frac{r}{R} = \frac{d}{D}$ = branch to run pipe diameter ratio

$\eta = \sigma_{ob} / \sigma_{or}$ = branch to run pipe yield strength ratio

σ_{ob} = yield stress of branch pipe

σ_{or} = yield stress of run pipe material

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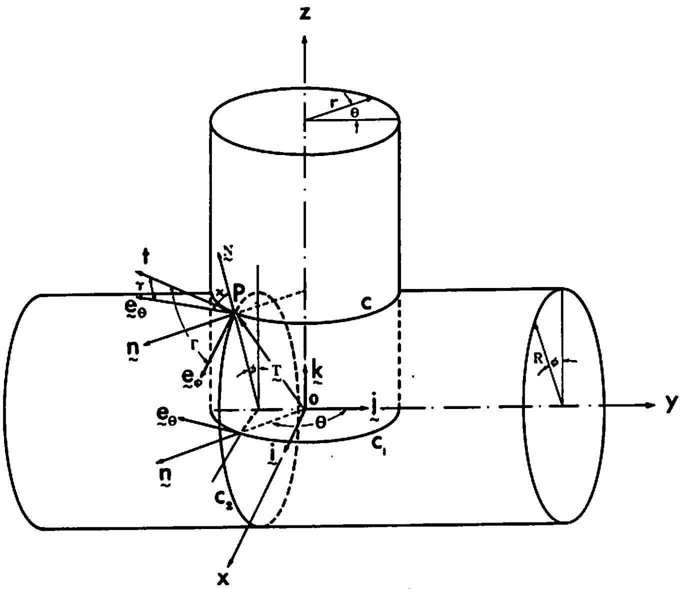

Fig. 1- Sketch of a typical mid-surface of branch-pipe tee connection. The unit vector $\underset{\sim}{t}$ is tangent to the intersection curve C. The set of unit vectors $\underset{\sim}{e}_\phi$, $\underset{\sim}{N}$, and $\underset{\sim}{e}_\theta$, $\underset{\sim}{n}$, are tangent and normal to circles, C_2 (run pipe) and C_1 (branch pipe) respectively.

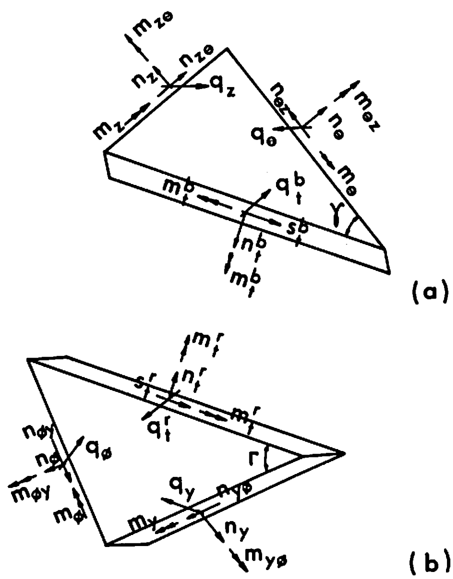

Fig. 2- Stress resultants acting on the elements taken along the intersection curve: a) branch pipe; b) run pipe.

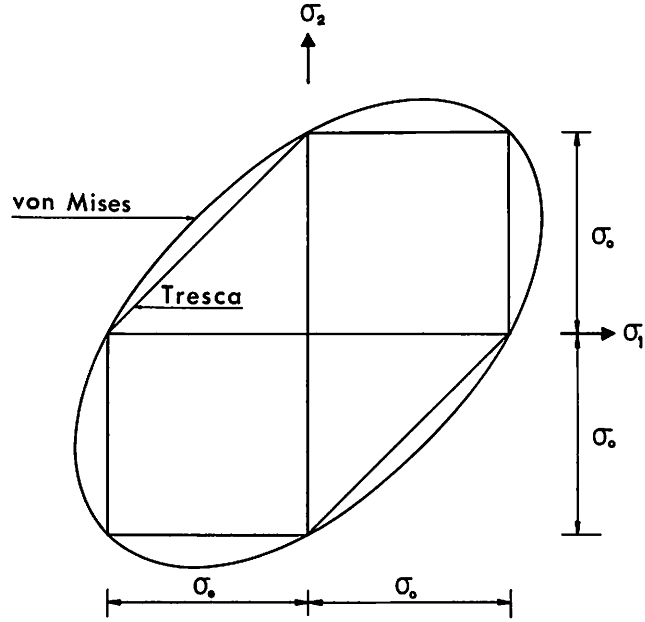

Fig. 3- Mises and Tresca yield condition for plane stress.

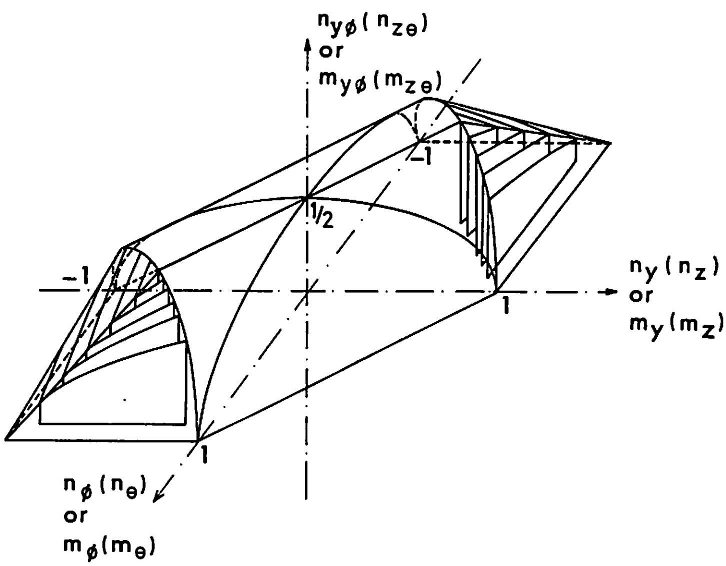

Fig. 4- Representation of part of limited interaction yield surface in the generalized membrane forces (or moments) space; symmetrical with respect to the horizontal plane.

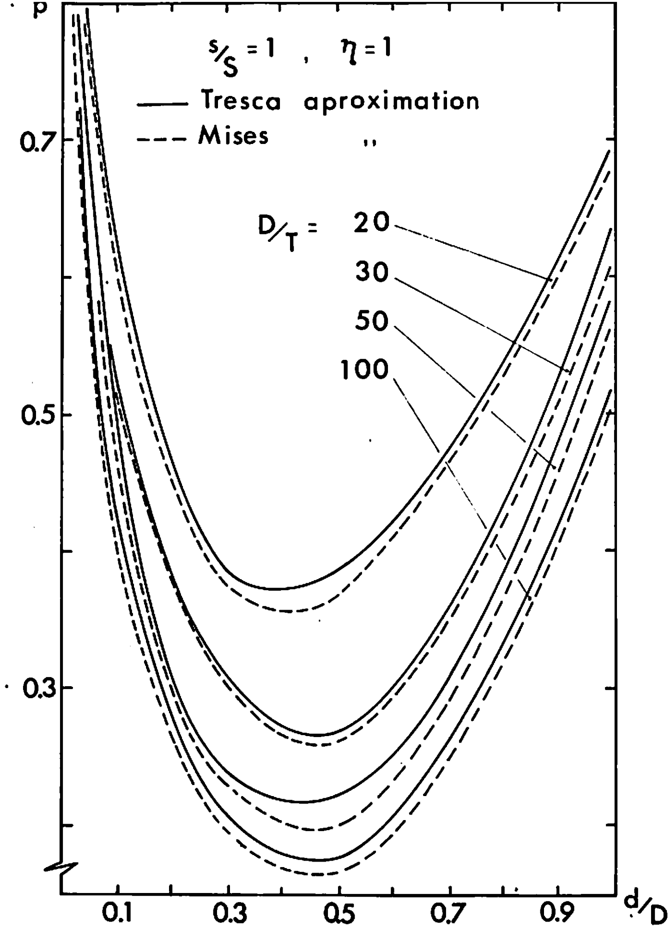

Fig. 5 - Variation of lower bound to the limit pressure of branch-pipe tee connection, against diameter ratios, for different thickness to diameter ratio of run pipe.

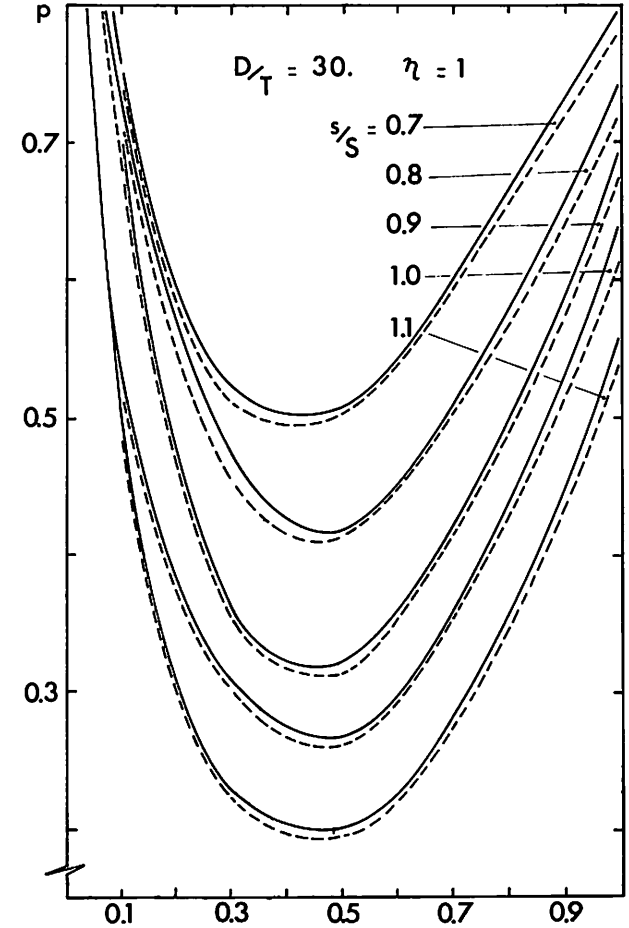

Fig. 6 - The lower bound to the limit pressure versus thickness to diameter ratio of run pipe, for various hoop stress ratios.

DISCUSSION

Q D. G. H. LATZKO, The Netherlands

No lower bound limit analysis is better than the elastic formulation from which it departs. I should therefore like to hear your opinion on the claim of general applicability of this basis made in your paper, i. e. the independence of the d/D ratio. In this connection I should like to refer to some of the papers and part of the discussion of Session G2 of this conference.

A F. ELLYIN, Canada

I am not quite sure what is meant by the "elastic formulation". There is no connection between the solution presented here and the elastic solution given by others, e. g. those of Session G2, except that both solutions have common requirements of satisfying equilibrium equations, stress boundary conditions, and stress continuity relations. However, an elastic solution is more restrictive in the sense that it must also satisfy compatibility equations and any prescribed geometric boundary (displacements and their derivatives). It is the latter requirements which generally impose restriction on an elastic solution e. g. a shallow shell approximation, etc. In contrast in a lower bound solution one needs only to insure that the assumed yield condition is not violated.
Any restriction on d/D ratio would therefore come from simplifying the continuity equations (4), i. e. replacing the space curve C of Fig. 1, by a plane curve. Since no such assumption was made, the solution is therefore applicable to all d/D ratios.

Q A. N. SHERBOURNE, Canada

Do you obtain any influence of geometry changes for this class of unsymmetric problems or conversely for what ranges of geometric shell size will geometry changes be unimportant.

A F. ELLYIN, Canada

A nozzle in a cylindrical shell exhibits some change in geometry. The experimental results, however, show that the influence of geometric change is of second order for cylindrical shells with diameter to thickness ratio $100 > D/T > 20$, which covers almost all the practical ranges. I should perhaps mention that for very thin shells the influence of change in geometry would be appreciable. However, for this range of parameters other failure modes e. g. buckling, may prevail, before occurence of limit pressure.

Q J. P. LAFAILLE, Belgium

Is the method that you described applicable to the computation of the nozzles of the pressure vessel ?

A F. ELLYIN, Canada

Yes, the method outlined in the paper is applicable to nozzles in cylindrical, spherical or any other configuration of vessel shells.

C T. ARIMAN, U. S. A.

I noticed that the author assumed that both lower and upper shells are infinitely long. This assumption may be acceptable for the lower shel. but I do not think it is realistic to make the same assumption for the much shorter upper shell. In that case the solution differs quite a bit from the infinite shell solution. Because of the finite length even for the internal uniform pressure loading the numerical results might be different than those of the author. The second point is related to the plastic analysis. I believe the efforts based on the work (or strain) hardening material case might produce much more realistic results for the problem under consideration. Incidentally in recent years (since 1965) there are a number of studies related to the cylindrical shells with cutouts and intersecting cylindrical shells. Some of them consider large holes or large diameter ratios. Even a most recent one develops a nonlinear elasto-plastic (strain hardening material) analysis of shells with cutouts.