G 6/1*

MATRIX DISPLACEMENT METHODS IN FRACTURE MECHANICS ANALYSIS OF REACTOR VESSELS

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The technology of fracture mechanics is developing rapidly in response to increased requirements for integrity of engineering structures. It enables structural engineers to evaluate brittle failure resistance of structures within appropriate regimes of temperature, materials, and geometry. The evaluation includes the combined effects of material toughness, flaw characteristics, environment, and service loadings. Calculation of stress intensity factors associated with the flaws, geometry, and applied loading from the basis of fracture analysis and control procedures for reactor vessels.

With the corresponding rapid development and acceptance of the matrix displacement method (finite element method) for structural analysis, attention is being given to the use of this method in fracture analysis. Modern computers have made the matrix displacement method of structural analysis a convenient method for determination of stresses and displacements in complex structures. It appears logical to augment the method and provide capability for direct calculation of stress intensity factor for structures containing cracks. While much information has been published on stress intensity factors for simplified crack configurations under various loadings, this additional capability would be most helpful in fracture analysis of complex vessel structures. After the usual stress and displacement analysis of the vessel structure is performed, arbitrary cracks would be introduced in suspect areas. The augmented program would then permit calculation of stress intensity factors for each crack location. Further calculations would account for the case of fatigue enhanced crack growth during the lifetime of the vessel. Isolating crack locations in substructures which do not significantly influence the boundary stiffness at the joining points to the complete structure, would obviate the need to recalculate the complete structure.

Various investigators have shown reasonably good agreement of stress intensity factors calculated by the finite element method for standard laboratory test specimen configurations (mode I) where exact solutions are known. The work reported to date in the literature has been based upon rather simple, linear elastic programs having plane triangular elements with constant strain. The specimen configurations were analyzed in the two dimensional plane of symmetry, with appropriate corrections for plane stress or plane strain behavior. Direct calculation of the stress field in the immediate vicinity of a crack tip was limited in these programs by the crack tip stress singularity. Several schemes were devised to overcome this limitation. Finite element nodal point displacements and stresses were correlated with crack tip displacement and stress equations to give apparent values of the stress intensity factor. The approximate value of the factor was obtained by a tangent extrapolation of these apparent values to the crack tip (r = o). A rather fine mesh was required at the crack tip in order to obtain reasonable accuracy. It was also possible to estimate the stress intensity factor by a line integral evaluation over a path surrounding the crack tip.

The compliance method (strain energy release rate) has been shown by several investigators to obviate the need for the fine mesh at the crack tip. The method is straightforward and eliminates the need for elaborate treatment of the crack tip situation.

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DISCUSSION

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In order to avoid having to carry out a finite element LEFM analysis every time the fracture instability of a vessel nozzle with a defect is assessed, I would like to propose a simplified approach, and would be grateful for any comment you might have.

The approach is based on the fact that a stress gradient such as existing from say the crotch corner, to the weld toe, can be approximately represented by a moment plus a uniform stress. A second approximation used is the pessimistic application of the ASTM (STP410) notch stress intensity factors, (Figs. 6 and 7) to a strip along that stress gradient, using as "width of specimen, w" the distance between the inner and outer surfaces.

That this method gives pessimistic, i. e., too large, K_I values could be seen by comparing results obtained with those of the paper G 2/6 by Y. R. Rashid and J. D. Gilman, "Three-dimensional analysis of reactor pressure vessel nozzles". Using a 1 in. thick strip of material along the gradient, and integrating the stress distribution in Fig. 11 of that paper, a K_I value of 57 kpsi $\sqrt{\text{in.}}$ is obtained for a 0.65 in. crack. The value given in the paper is 50.6 kpsi $\sqrt{\text{in.}}$ For deeper cracks a varying thickness strip is used. With a 12.9 x 12.9 x 18.2 in. triangular "beam" cross-section, for cracks of 1.95, 3.25 and 4.45 in. deep the corresponding values are 75(66), 93(71) and 129(74) kpsi $\sqrt{\text{in.}}$, respectively.

The main attraction of this method is that only one computer analysis is required, or in cases where the stress distribution can be estimated by other means, no computer is required.

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Possibly at the present time, one might say that procedures for calculating stress intensity factors are a matter of personal preference. One might also say that this applies to the degree of conservation desired in the values obtained. Paris and Sih (6) have discussed estimation of stress intensity factors and believe that accuracies ± 10% or better can be reasonably achieved. They note, however, that this requires a certain amount of experience and intuition. I believe that with suitable experience one should be able to achieve ± 5% accuracy with finite element methods. In all calculation by any method, one should be aware of the relative magnitudes of the stress intensity values, the yield strength and the amount of plastic deformation at the crack tip.

In many design room situations, a series of preliminary calculations are performed in the process of narrowing-in to the final optimized answer. Then a detailed analysis is performed for confirmation of predictions. Approximate methods can be very useful in preliminary stages. I think, however, that in the final analysis one would want to be rather accurate.

Other than these comments, I do not really have much else to say about the superposition procedure discussed by Mr. Y. C. Wong. A more detailed description of his calculations would be necessary in order to make comparison of his results with those cited in paper G 2/6.